Ain Temouchent University – Belhadj Bouchaib

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Educational Handout

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Title:
Plasticity and Damage

Course intended for students of:
Master's (specialty and level): Master 2, Structural Engineering option

Year: 2024-2025

Introduction

The primary goal of this program is to provide Master 2 students in Civil Engineering (Structure option) with a solid theoretical and practical understanding of **material and structural behavior beyond the elastic range**. The course aims to develop the ability to analyze, model, and design structural elements that exhibit **plastic deformation and damage** under various loading conditions, in accordance with modern design philosophies.

The present document has as objectives:

- Introduce students to the fundamental principles of **anelastic behavior**, distinguishing between elastic, plastic, and damage responses of materials.
- Enable students to **understand the physical meaning of yielding, hardening, and failure**, and how these phenomena affect the structural performance and safety margins.
- Clarify the differences between **traditional elastic design** and **plastic or damage-based design approaches**, and the reasons for adopting advanced methods in modern engineering practice.

Pedagogical Structure is as :

- **Continuous assessment (40%)** emphasizes active learning through assignments, problem-solving sessions, and discussions.
- **Final examination (60%)** evaluates comprehensive understanding, analytical ability, and the capacity to apply theoretical concepts to structural design problems.
- The handout serves as a **structured learning guide**, alternating between concise summaries for clarity and detailed explanations for in-depth comprehension, in full alignment with the official *cavevas* program.

The document contains four chapters:

Chapter 1: Introduction to anelastic structural design

Chapter 2: Plastic structural design

Chapter 3: Limit analysis applied to structural design

Chapter 4: Damage

Chapter 1: Introduction to anelastic structural design

1.1 Introduction

Deformations and distortions are calculated within the proportionality limit (Elasticity limit) of the structure's material. Elastic designs are based on the assumption that structural elements behave elastically.

Safety factors have been made available based on yield stress. But strictly providing a safety factor based on yield stress is not a consistent indication of the safety factor in relation to the element's maximum capacity.

In other words, if the elastic limit is reached at one point, this does not necessarily imply total failure of the element. Due to plastic deformation and strain-hardening of the material, less-stressed particles will be brought into action, so that the structure is in fact capable of withstanding greater loads. In modern design, the principle explained above is introduced and the calculation method is called plastic design.

An important property of steel is its ability to withstand large deformations without breaking. A large proportion of deformation occurs during the elastic phase (yieding). A relatively large proportion of deformation occurs during strain hardening.

Although plastic analysis and design normally apply to steel structures, the idea can also be applied to reinforced concrete structures which are designed to behave elastically in a ductile mode at ultimate loads close to reinforcement plastification.

1.2 Notion of behavior law (Stress-strain diagram for mild steel)

Figure 1.1 shows the stress-strain diagram for mild steel. The section from O to A is a straight line. The stress corresponding to point A is called the *proportional limit*.

If the specimen is stressed beyond the limit of proportionality to the state represented in B, the material still remains elastic. But in the range from A to B, the relationship between stress and strain is non-linear. The stress at B is called the *elastic limit*.

If the specimen is loaded beyond its elastic limit, plastic deformation will occur. At point C, considerable extension occurs, corresponding to the decrease in load. The stress at point « C » is called the *upper yield strength*.

As the material is subjected to further deformation, the corresponding stress decreases, and in the state represented at “D”, the material offers high resistance to further deformation. The stress at “D” is called the *lower yield strength*.

As strain is increased, stress increases and in the state indicated by “E”, a “neck” is developed. The stress corresponding to “E” is called the *ultimate tensile strength*.

As the strain is further increased, the stress decreases and the specimen breaks in the state represented by “F”. The stress at “F” is called the *stress at break*.

For the purposes of plastic analysis of structures, the stress-strain diagram is taken in a modified form. We assume that there is a defined elastic range, and beyond this range, the material is assumed to have evolved during the plastic state.

Figure 1.2 shows the modified ideal elasto-plastic stress-strain diagram. In this diagram, OC represents the elastic domain and CD represents the plastic domain.

For all practical purposes, for some steels, after reaching the lower yield point, the material deforms plastically up to a strain of 1% to 1.5%, without any increase in stress.

After the plastic range, strain-hardening begins, and once again an increase in stress is required to produce an increase in strain.

The lower yield strength is generally considered to be the yield strength used in structural design.

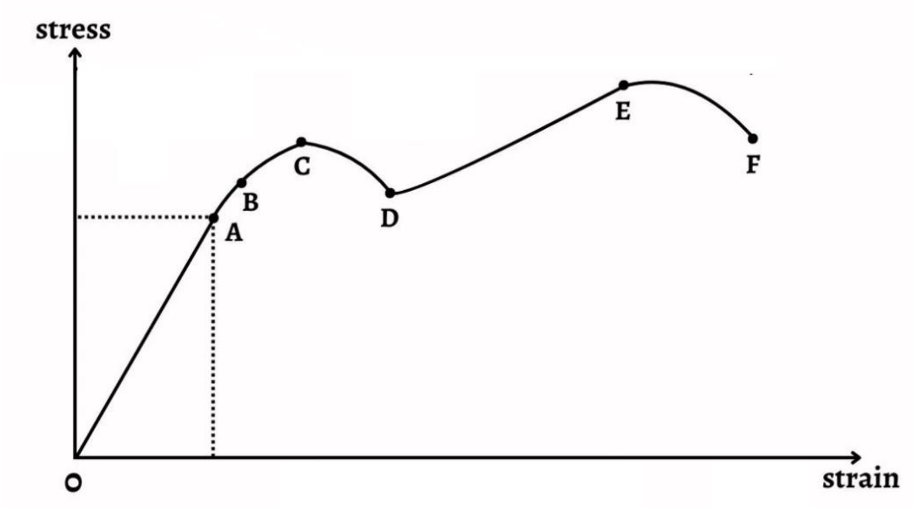

Figure 1.1: Stress-strain diagram for mild steel

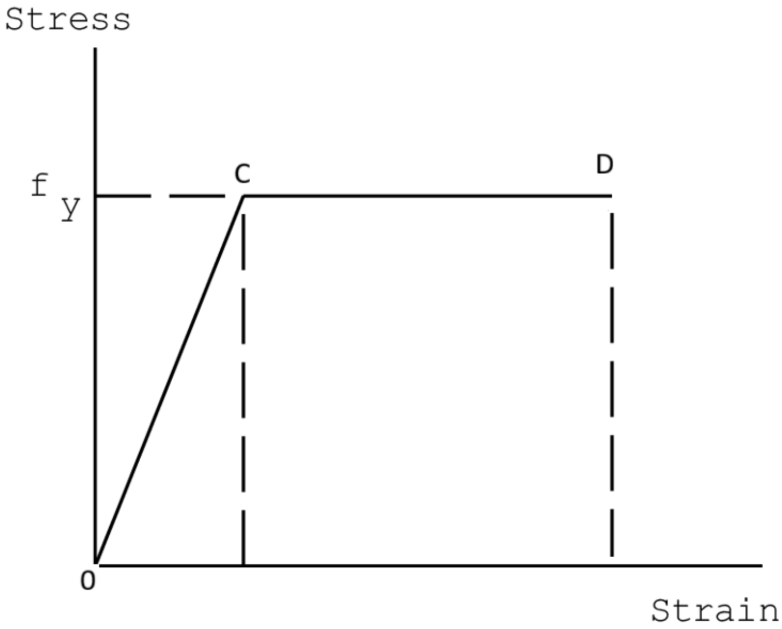

Figure 1.2: Ideal elasto-plastic stress-strain diagram

1.3 The need for plastic calculations

Compared to predictions based just on elastic theory, plastic analysis offers a clearer evaluation of the actual safety margin and a more accurate and realistic representation of structural behavior at the point of failure. Plastic analysis provides a more accurate estimate of a structure's ultimate capacity than elastic estimates because it accounts for the redistribution of stresses beyond the elastic limit.

Furthermore, by using the maximum strength potential of materials, plastic design results in more cost-effective and optimized structural dimensions. The limit loads at which a structure is most likely to fail due to excessive deformation can be predicted using plasticity theory.

It is crucial to remember that the ultimate load established by plastic design and the ultimate limit state (ULS) are two distinct ideas. Finding the actual failure loads and associated failure mechanisms is the main goal of plastic analysis. This information offers important insight into the true safety reserve of certain structural components or the structure overall.

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1.4 conclusion

The basic ideas of anelastic structural design were presented in this chapter, with a focus on the shift from conventional elastic analysis to plastic design methodologies. In order to demonstrate the difference between the elastic and plastic domains, as well as the part that yield strength, strain hardening, and ductility play in determining a material's ability for deformation, the stress-strain behavior of mild steel was thoroughly investigated.

The chapter made clear that depending only on elastic theory offers a constrained perspective on structural safety by going over behavior laws and the necessity of plastic computations. On the other hand, plastic analysis allows for a more accurate assessment of the ultimate load-bearing capacity by capturing the redistribution of stresses beyond the elastic limit.

By presenting the fundamental assumptions, limit states, and analytical techniques used to determine the ultimate load capacity and collapse mechanisms of steel and reinforced concrete structures, the following chapter, Plastic Design of Structures, will build upon these foundations and apply them to structural systems.

Chapter 2: Plastic design of structures

2.1. Introduction

Plastic design of structures provides a rational method for assessing their ultimate limit state. Unlike elastic design, it considers the ability of materials—particularly steel and reinforced concrete—to undergo plastic deformations without immediate failure. This approach enables engineers to predict the limit load beyond which the structure can no longer sustain additional loads due to the formation of plastic hinges and the development of a collapse mechanism. By allowing for redistribution of internal forces, plastic design leads to a more realistic evaluation of structural capacity and promotes efficient, safe, and economical designs.

2.2 . Plastic Traction

Generally speaking, tensile test specimens are standardized ($\phi \geq 4mm$ ou $e \geq 3mm \ldots$). All the parameters derived from the tensile test reflect the material properties in the direction of the test.

This involves working the specimen in tension until it breaks. The stress-strain diagram (behavior law) is used to locate the plasticity threshold (le seuil de plasticité).

2.3 Plastic flexion

When the section of a member has become fully plastic under the effect of a bending moment, any attempt to increase the moment causes the element to act like a hinge.

That is, at the section where the elastic limit is reached (figure 2.1) , large rotations are possible without significant changes in the resisting moment. This phenomenon is described as a “plastic hinge", but unlike a hinge, rotation is resisted by a constant moment .

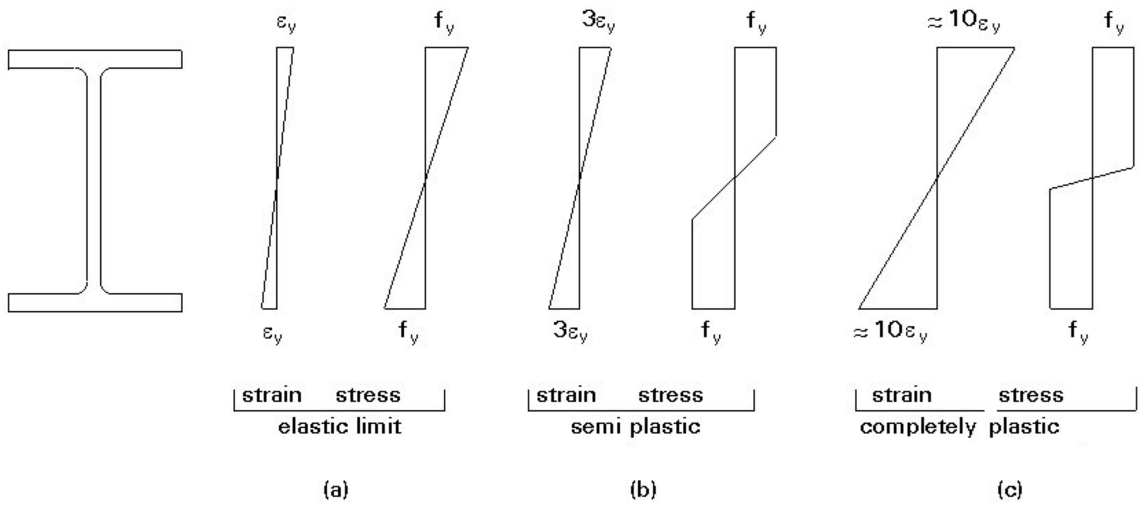


Figure 2.1: Development of plasticity in a section

For the purposes of simple plasticity theory, the diffusion of plasticity along the element is neglected. Plastic deformation is assumed to be limited to the cross-section of maximum moment M_{max}, giving the moment at the plastic hinge.

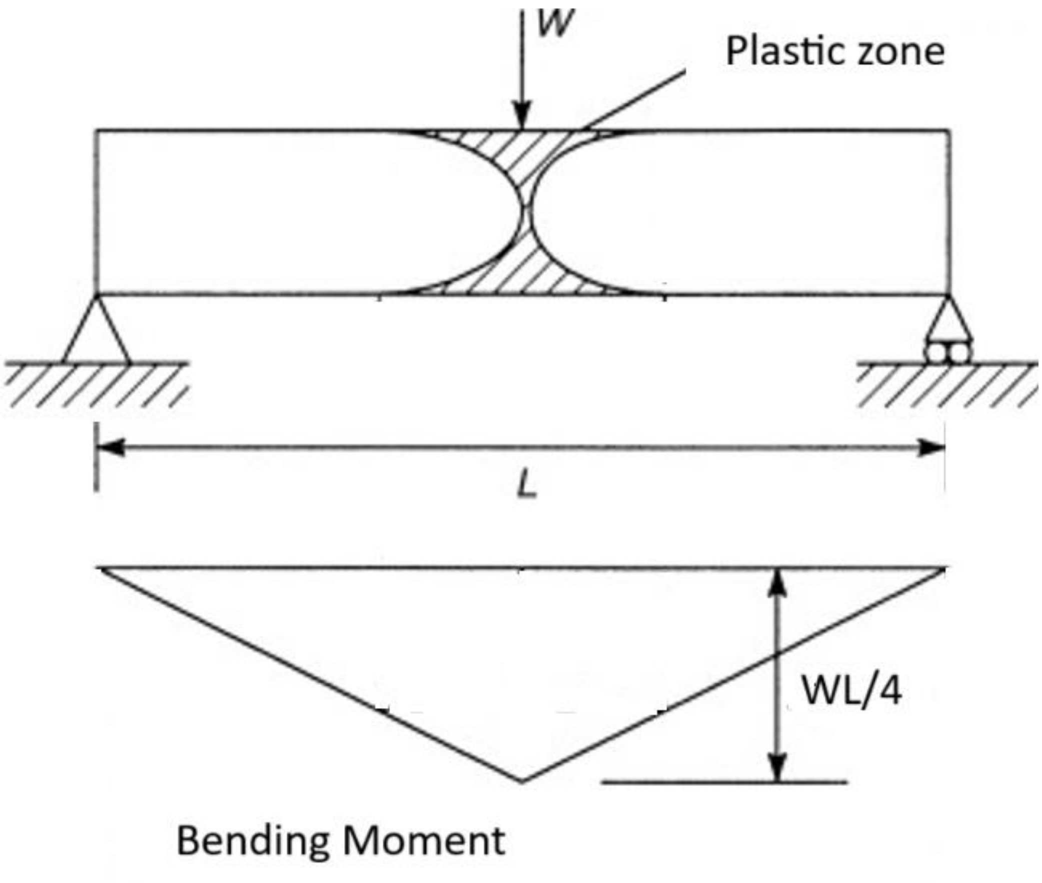


Figure 2.2: Plasticity diffusion at the plastic hinge

2.4 Notion of plastic hinge

when loads increase in a critical section of a structural element, plasticity starts with the outer fibers and progresses fairly rapidly until it reaches the central fibers (figure 2.1).

When all the fibers (tensioned and compressed) surrounding the neutral axis of the section are at the plasticity threshold, the element undergoes a rotation at this point, i.e. a plastic hinge (see Figure 2.3).

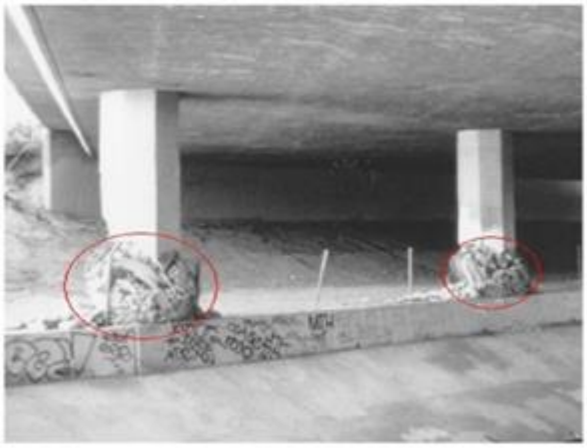


Figure 2.3: Plastic hinges in RC bridge columns (source: google image)

2.5 Notion of moment – curvature

Plasticity propagation through a section can be analyzed from the material's stress-strain curve. For bending members, this is done by finding the relationship between the moment and the curvature of the section.

Assuming an initially straight beam length δx, after deformation it takes on the shape of a circular arc.

ε_c and ε_t are the compressive and tensile strains respectively.

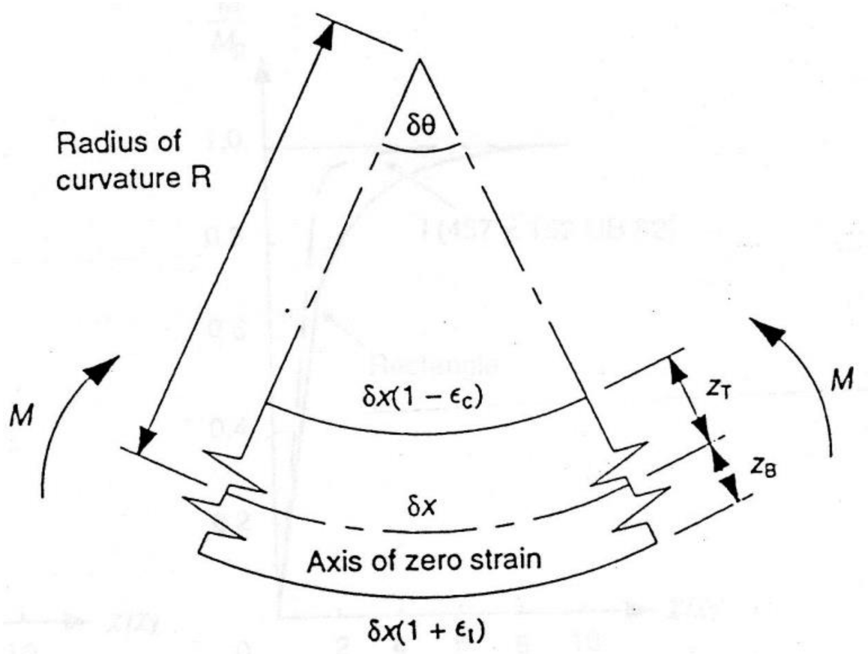


Figure 2.4: diagram illustrating the moment-curvature notion

From figure 2.4:

$$R.\delta\theta = \delta\, x$$
$$(R + Z_B).\delta\theta = \delta\, x.(1 + \varepsilon_t)$$
$$(R - Z_T).\delta\theta = \delta\, x.(1 - \varepsilon_c)$$

From which :

$$R + Z_B = R(1 + \varepsilon_t)$$
$$R - Z_T = R(1 - \varepsilon_c)$$

After subtraction :

$$Z_T + Z_B = R(\varepsilon_t + \varepsilon_c)$$

So the curvature is :

$$\chi = \frac{\varepsilon_t + \varepsilon_c}{Z_T + Z_B} = \frac{\textit{deformation rate}}{\text{section depth}}$$

An ideal diagram of the moment-curvature relationship is shown in figure 2.5.

As soon as the plastic moment is reached, the plastic hinge is formed and the curvature increases without any change in moment, this is known as "plastic rotation of the hinge".

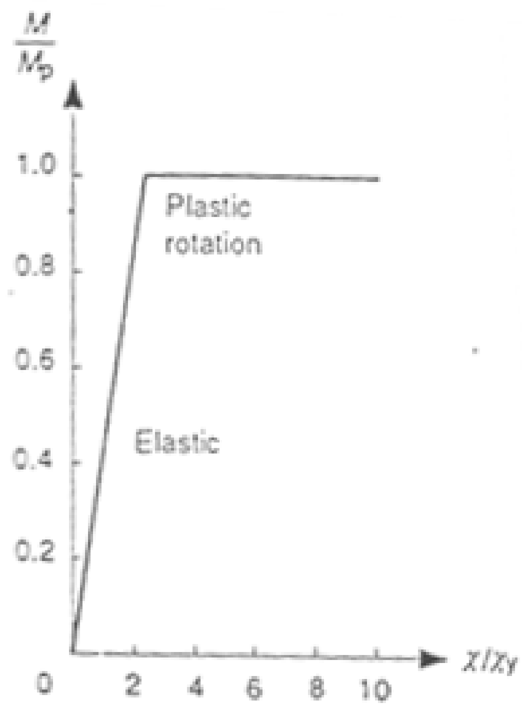


Figure 2.5: Perfect elasto-plastic diagram

In practice, there is always a small region of elasticity in the middle of the section. The assumption of ideal behavior introduces a very small error for sections with a form factor (the ratio of plastic modulus to elastic modulus) close to unity. Sections with large form factors are far from perfect (see figure 2.6).

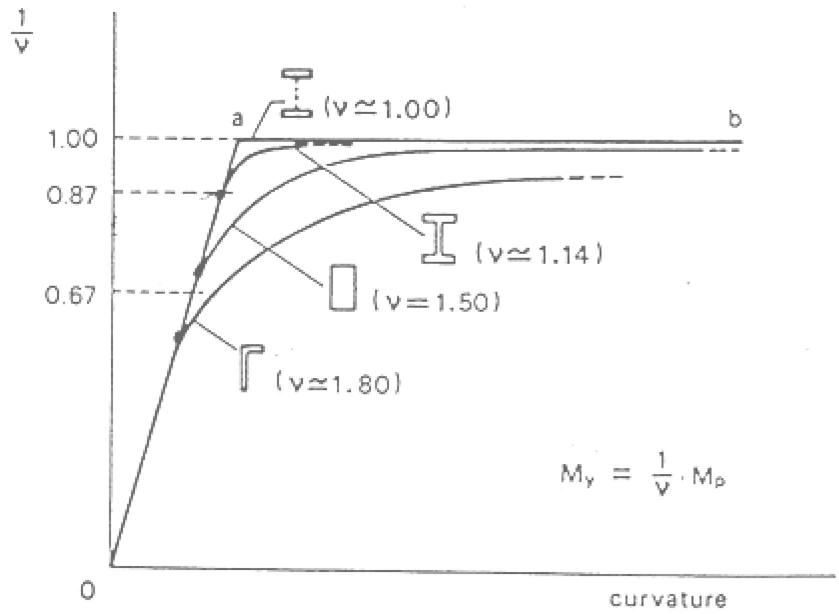


Figure 2.6: Characteristics of the moment-curvature diagram for different cross-section shapes

2.6 Notion of Plastic Moment :

The bending moment corresponding to the formation of a plastic hinge is called the plastic moment.

The bending moment (in the absence of any axial, shear or torsional force) that produces a plastic hinge in a member is the plastic moment, M_p :

$$M_p = \text{Plastic Modulus x elastic limit stress} = \text{Z}_\text{p} \text{ x } \sigma_\text{y}$$

For a rectangular section (b x h):

$$M_p = \frac{bh^2}{4}.\sigma_y \text{ with } Z_p = \frac{bh^2}{4}$$

The plastic modulus is the sum of the moments of all surfaces composing a section with respect to the axis of equal surfaces (axis of equal areas).

The ratio of plastic modulus to elastic modulus is known as the “form factor”.

$$\upsilon = Z_p / Z_e$$

In a simple bending test on any cross-sectional shape, plasticity does not start until the bending moment reaches a value of M_p / υ . For a section of I, it is about $0{,}87M_p$.

2.7 Reinforced concrete section design

The moment-curvature law of a section depends on its geometrical characteristics, the mechanical characteristics of the materials of which it is made (concrete, steel....), as well as longitudinal and transverse reinforcement (the reinforcement ratio...) and the normal force of the section.

We consider the Euler-Bernoulli hypothesis, which assumes that flat sections remain flat after deformation.

An analysis of a reinforced concrete section shows 3 main states (figure 2.7):

- cracking of concrete after a brief elastic phase (uncracked),

- plastification of steels, after crack opening,

- In the ultimate state and after section failure, which results in concrete crushing in compression and tensile steel failure.

It is important to highlight that other classifications may also exist.

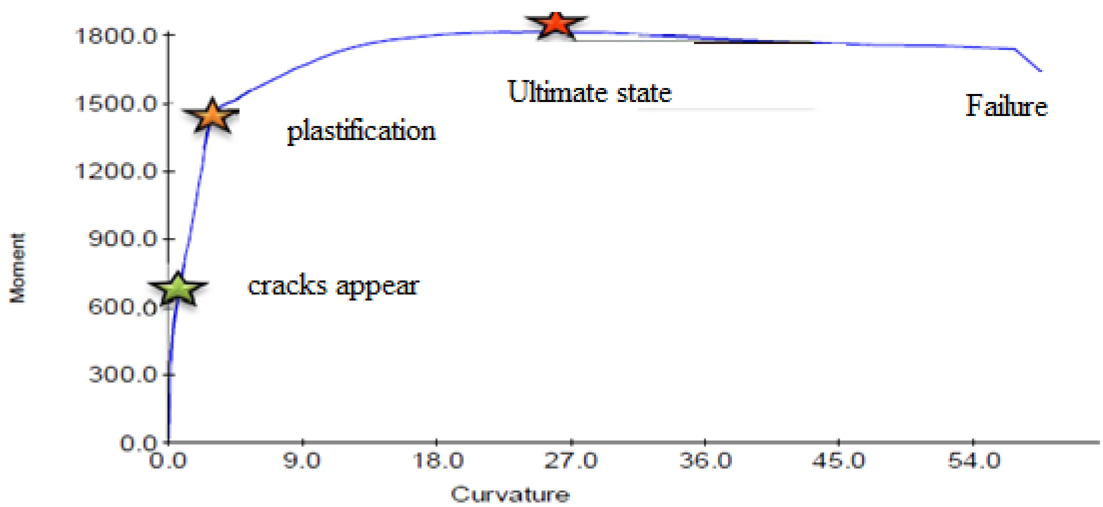


Figure 2.7: Moment-curvature diagram for concrete

In the elastic phase, stresses are small and the behavior of steel and concrete can still be assumed to be linear-elastic.

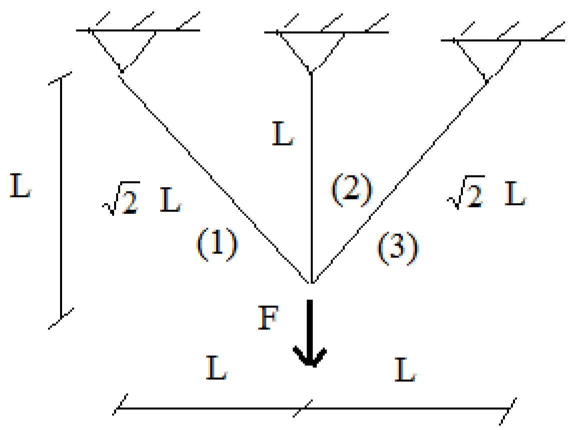
2.8 Determination of capacity curves (force-displacement) of structures (trusses, beams, portals) by incremental analysis

2.8.1 Capacity curve

This curve is an intrinsic characteristic of the system from the point of view of the effect of external actions. It gives an estimate of the expected plastification mechanisms and the distribution of progressive damage, as a function of the intensity of forces and displacements.

2.8.2 Truss system

The following example explains how to determine the capacity curve (force-displacement) of a truss structure.



The degree of hyperstaticity of the truss system is calculated as follows:

$R = m - 2.j + r$

with:

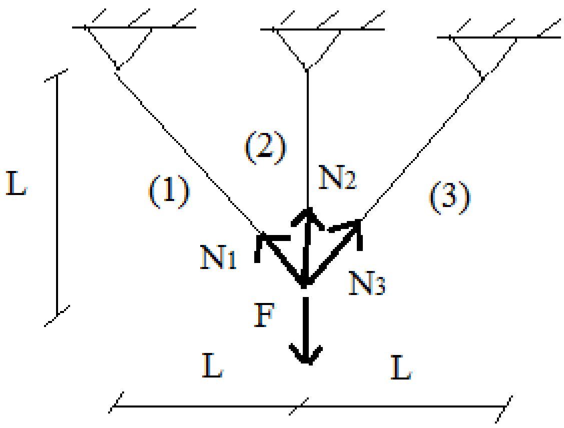
R : degree of hyperstaticity

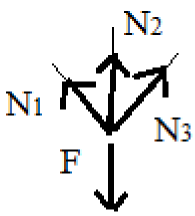
m : number of bars

j : number of nodes

r : number of support reactions

for this example, the degree of hyperstaticity is 1.





applying the balance of forces along the two axes x and y, we get:

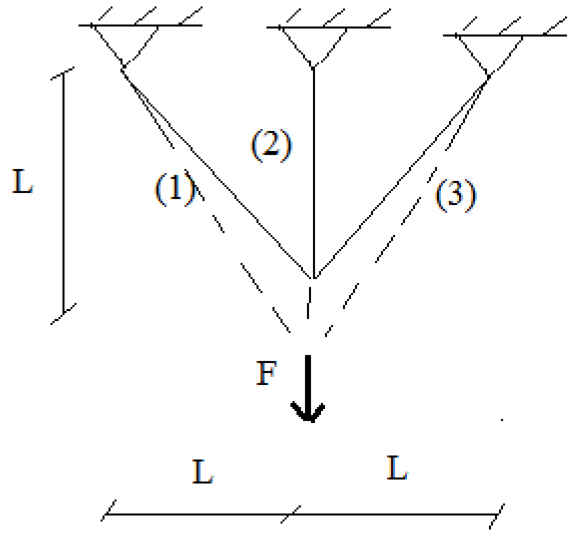
$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases}$$

$$\begin{cases} \sum F_x = 0 \Rightarrow N_2 + N_1.\dfrac{\sqrt{2}}{2} + N_3\dfrac{\sqrt{2}}{2} - F = 0 \\ \sum F_y = 0 \Rightarrow -N_1\dfrac{\sqrt{2}}{2} + N_3.\dfrac{\sqrt{2}}{2} = 0 \Rightarrow N_1 = N_3 \end{cases}$$

$$\Rightarrow \begin{cases} N_1 = N_3 \\ N_2 + \sqrt{2}.N_1 = F \end{cases}$$

So all normal internal forces are a function of the external force F .

$$N_1 = f(F)$$
$$N_2 = f(F)$$
$$N_3 = f(F)$$



knowing that:

the elongations of bars 1, 2 and 3 are u_1, u_2, u_3 respectively, so:

for bar 1

$$\left(l\sqrt{2}+u_1\right)^2 = l^2 + \left(l+u_2\right)^2$$
$$2.l^2 + 2.\sqrt{2}.l.u_1 + u_1^{\,2} = l^2 + l^2 + 2.l.u_2 + u_2^{\,2}$$

using the small deformation hypothesis:

$$u_1^{\,2} = 0$$

we get:

$$\sqrt{2}.u_1 = u_2$$

The compatibility equations are expressed as:

$$\begin{cases} \varepsilon = \dfrac{\Delta l}{l} = \dfrac{u}{l} \\ \sigma = E.\varepsilon \\ N = \sigma.S \end{cases}$$

we have:

$$\varepsilon_2 = \frac{u_2}{l}$$

$$\varepsilon_1 = \frac{u_1}{l.\sqrt{2}}$$

So :

$\varepsilon_2.l = \sqrt{2}.\varepsilon_1.l.\sqrt{2} = 2.\varepsilon_1.l$ knowing that:

$$\sigma_2 = E.\varepsilon_2$$
$$\sigma_1 = E.\varepsilon_1$$

$$\frac{\sigma_2}{E}.l = 2.\frac{\sigma_1}{E}.l$$
$$N_2 = \sigma_2.S$$
$$N_1 = \sigma_1.S$$

$$\frac{N_2}{E.S}.l = 2.\frac{N_1}{E.S}.l$$
$$N_2 = 2N_1$$

the conditions of compatibility can be summarized as follows:

$$\begin{cases} N_1 = N_3 \\ N_2 + \sqrt{2}.N_1 = F \\ N_2 = 2.N_1 \end{cases}$$

$$N_1 = \frac{F}{2+\sqrt{2}} = N_3$$
$$N_2 = \frac{2.F}{2+\sqrt{2}}$$

The truss capacity curve is:

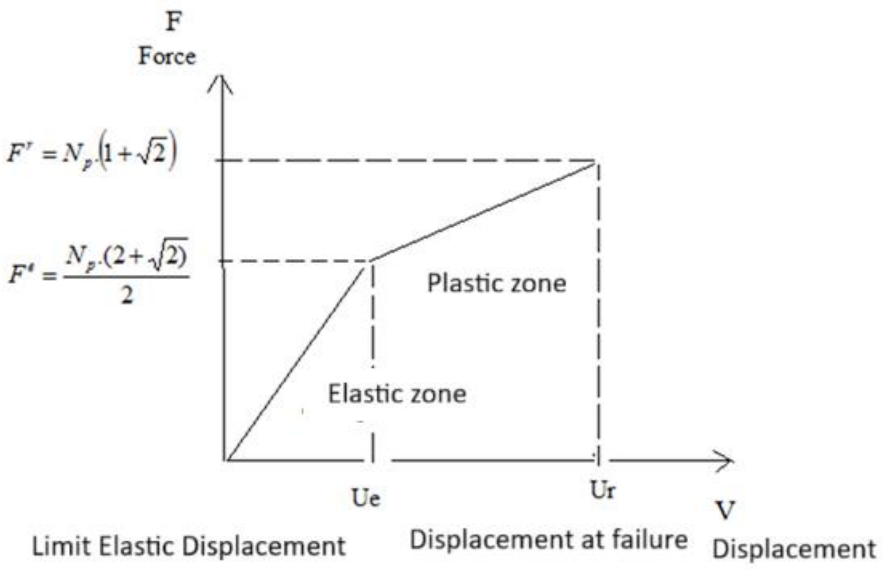


Figure 2.8: Capacity curve

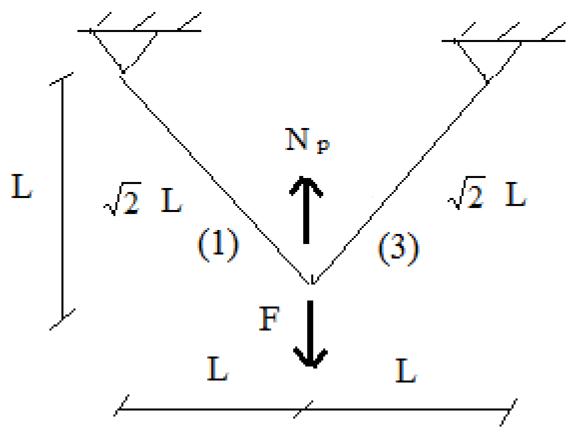
The normal plastic stress is:

$$N_2 = N_p \Rightarrow N_p = \frac{2.F}{2+\sqrt{2}}$$

$$F = \frac{N_p.(2+\sqrt{2})}{2}$$

So the second bar is plastified the first $(N_2 = N_p)$ and then the system becomes isostatic.

Overcoming $F^e = \frac{N_p.(2+\sqrt{2})}{2}$; we get the following diagram:



Applying equilibrium equations:

$$\begin{cases} \sum F_x = 0 \Rightarrow N_1 = N_3 \\ \sum F_y = 0 \Rightarrow \left(N_1 \cdot \frac{\sqrt{2}}{2} \right) .2 + N_p = F \Rightarrow N_1 . \sqrt{2} + N_p = F \end{cases}$$

As a result, the two bars will plastify and the system will become unstable, leading to total failure.

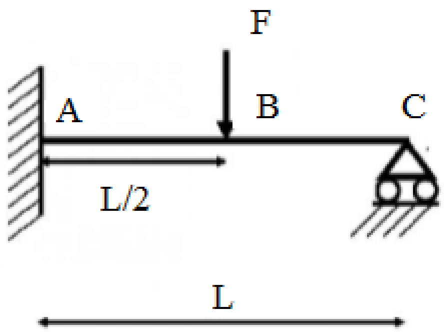
$$N_1 = N_p \Rightarrow N_p . \sqrt{2} + N_p = F$$
$$F^r = N_p . \left(1 + \sqrt{2}\right)$$

With:

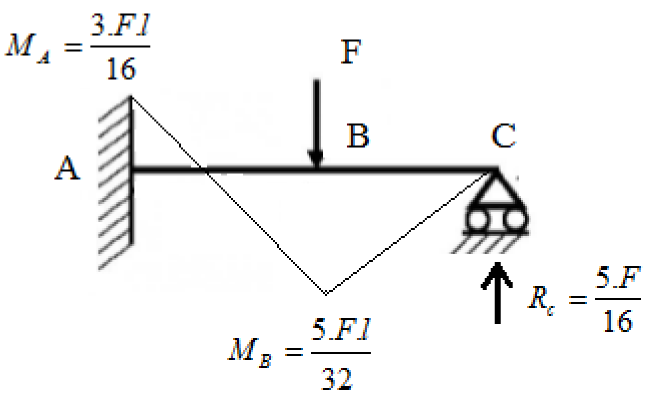
F^r : Failure force.

2.8.4 Beam and portal system

The same principle as for the truss system applies to beams and portals taking the following beam as an example:



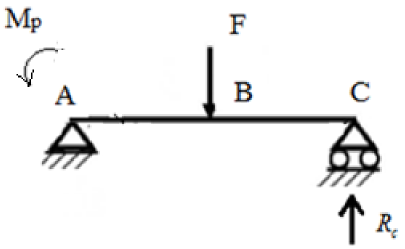
The elastic bending moment diagram is:



In this case, the plastification is at support B:

$$M_A = M_p \Rightarrow \frac{3.F.l}{16} = M_p$$

Once the plastic hinge joint has formed at the A support, the beam becomes isostatic, doubly supported at A.



The elastic force is then: $F_e = \frac{16.M_p}{3.l}$

Capacity curve:

Calculation of M_B

If $M_B = M_P$, beam failure occurs:

$$\sum M_{F/A} = 0 \Rightarrow R_c.l - \frac{F.l}{2} + M_p = 0$$

$$R_c = -\frac{M_p}{l} + \frac{F}{2}$$

So :

$$M_B = R_c.\frac{l}{2} = -\frac{M_p}{2} + \frac{F.l}{4}$$

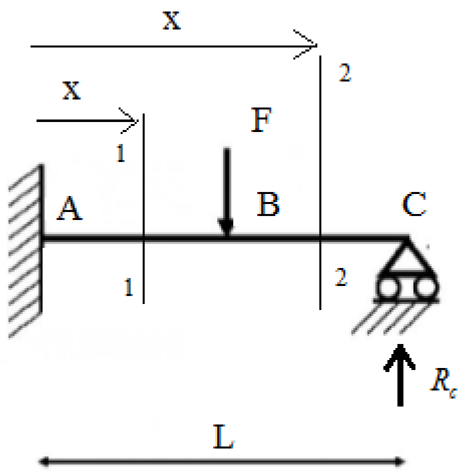
After plastification in B:

$$M_p = -\frac{M_p}{2} + \frac{F.l}{4}$$

$$\frac{F.l}{4} = M_p + \frac{M_p}{2} = \frac{3M_p}{2}$$

$$F_r = \frac{12M_p}{2.l} = \frac{6.M_p}{l}$$

Deflection calculation



Knowing that:

$u'' = \frac{M(x)}{E.I}$ is the curvature

$u' = \theta(x)$ is the rotation

$u(x)$ is the deflection

Deflection in section 1-1 $0 \leq x \leq \frac{l}{2}$

$$M_1(x) = R_c(l - x) - F\left(\frac{l}{2} - x\right)$$

Deflection in section 2-2 $\frac{l}{2} \leq x \leq l$

$$M_2(x) = R_c(l - x)$$

After integration, we get :

$$u_{max} = \frac{7.F.l^3}{768.E.I}$$

Knowing that :

$$F_e = \frac{16.M_p}{3.l}$$

the maximum displacement can be expressed as:

$$u_{\max} = \frac{7.M_p.l^2}{144.E.I} = V_e$$

Capacity curve is:

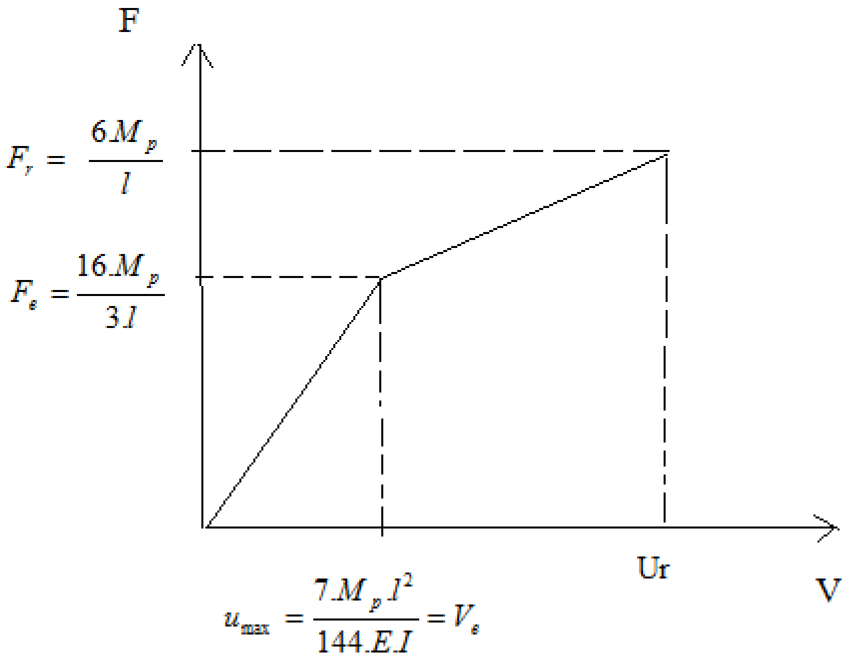


Figure 2.9: capacity curve for the beam under study

2.9 Limit Analysis and Regulations (Ultimate Limit State; Seismic Design)

Limit analysis represents an advanced stage of structural assessment that focuses on determining the ultimate load-carrying capacity of a structure before collapse. It is based on the principles of plasticity and is closely related to the Ultimate Limit State (ULS) design philosophy adopted in modern structural codes such as the Eurocodes and international seismic regulations. The main objective of limit analysis is to identify the collapse mechanisms and corresponding limit loads that define the structural safety margin beyond the elastic range.

In seismic design, this approach becomes particularly relevant, as structures are expected to undergo significant inelastic deformations during strong ground motions without suffering total failure. The capacity curve, established through incremental plastic analysis, characterizes the global behavior of the structure under increasing lateral forces. To evaluate the seismic performance, this curve must be superimposed on the seismic response spectrum, which represents the earthquake demand in terms of acceleration, velocity, or displacement. The intersection of these two curves defines the performance point, corresponding to the equilibrium between structural capacity and seismic demand.

Before this comparison, it is necessary to transform the axes of the capacity curve into compatible units with those of the response spectrum—typically by converting base shear into spectral acceleration and roof displacement into spectral displacement. This transformation allows for a consistent evaluation of the structure's performance level (operational, life-safety, or collapse prevention) according to design codes.

Limit analysis therefore provides a rational framework for assessing structural adequacy under extreme loading, ensuring compliance with regulatory safety margins and promoting performance-based seismic design. These aspects will be explored in greater depth in Chapter 3, which focuses on the application of limit analysis to structural design and its integration into modern seismic and regulatory frameworks.

Conclusion

This chapter introduced the fundamental principles of plastic design and demonstrated how structures behave beyond the elastic limit through mechanisms such as plastic traction, flexion, and the formation of plastic hinges. It established the relationship between moment and curvature, explained the notion of plastic moment, and extended these concepts to reinforced concrete sections. Furthermore, it presented methods for determining capacity curves in truss, beam, and portal systems, highlighting how these curves describe the progressive plastification and ultimate strength of structures.
The objectives of this chapter were achieved by providing a clear understanding of how plastic design enables engineers to exploit material ductility for safer and more economical structures while accurately assessing their load-bearing capacity at failure.
Building on these foundations, the next chapter—"Limit Analysis Applied for Structural Design"—will explore how the principles of plasticity are used to establish limit theorems and design criteria that define the ultimate limit states of real structures under various loading conditions.

Chapter 3: Limit analysis applied to structural design

3.1 Introduction

Limit analysis is a fundamental approach in structural design that aims to evaluate the ultimate load-carrying capacity of a structure beyond its elastic range. It is primarily based on the theory of plasticity, which enables the prediction of structural behavior up to collapse. This approach provides a rational and efficient means of designing structures capable of undergoing significant inelastic deformations without sudden failure.

In this chapter, the principles of limit analysis are first developed with reference to steel structures, which exhibit a distinctly ductile behavior allowing plastic redistribution of internal forces. Subsequently, the applicability and limitations of the same approach in reinforced concrete (RC) structures are discussed, considering their partially ductile nature and composite behavior.

3.2 Fundamental Assumptions of Plasticity Theory

The theory of plasticity, as applied to structural steel design, is founded on the following key assumptions:

1. **Ductile Behavior of Steel**

   Steel is considered a ductile material, capable of sustaining large plastic deformations without fracture. This ductility allows the formation of plastic hinges and redistribution of internal moments, enabling the structure to reach an ultimate load before collapse.

2. **Plane Sections Remain Plane**

   The distribution of strains across a section is assumed to remain linear. For beams, this means that sections initially plane before bending remain plane after bending — an assumption consistent with classical beam theory.

3. **Idealized Stress–Strain Relationship**

   The steel stress–strain curve is simplified as **ideal elasto-plastic**: the material behaves elastically up to the yield stress, and thereafter, it yields at a constant stress irrespective of further strain increase.

4. **Symmetry in Tension and Compression**

   The relationship between stress and strain is assumed to be identical in tension and compression.

5. **Incremental Loading**

   External loads are assumed to be applied gradually in constant steps, allowing the structure to redistribute stresses as plastic regions develop.

6. **Continuity at Connections**

   Structural connections are assumed to provide full continuity and sufficient strength to transmit the **plastic moment** between members.

7. **Neglect of Elastic Deformation**

   Elastic deformations are neglected in limit analysis since the primary objective is to determine the ultimate load corresponding to plastic collapse.

3.2.1 Application to Steel Structures

For steel structures, the above assumptions are well justified due to the high ductility of steel and its predictable post-yield behavior. Plastic design enables engineers to exploit the material's full strength potential, leading to efficient and economical structures. Through the formation of plastic hinges at critical sections, the structure develops a mechanism at collapse, which defines its ultimate load capacity. Such an approach ensures safety while allowing for optimal use of material, particularly in beams and frames where moment redistribution plays a key role.

3.2.2 Extension to Reinforced Concrete (RC) Structures

While plastic analysis principles were originally developed for steel structures, they can be extended to reinforced concrete (RC) with certain limitations. Reinforced concrete behaves as a composite material, where concrete is strong in compression but weak in tension, and steel reinforcement provides tensile resistance.

The plastic design of RC members assumes that:

- Concrete in compression and steel in tension can reach their respective ultimate strains simultaneously.

- The section behaves as an idealized rectangular stress block in compression.
- Redistribution of moments can occur within the ductility limits provided by the reinforcement ratio and confinement.

However, due to the limited ductility of concrete and possible brittle failure modes, the plastic analysis of RC structures requires stricter control of detailing, confinement, and strain compatibility.

Therefore, the application of limit analysis to RC structures is mainly restricted to statically indeterminate beams and frames with sufficient rotational capacity at plastic hinges.

3.3 Limit analysis theorems

Limit analysis is built upon two fundamental theorems — the Static (Lower-Bound) Theorem and the Kinematic (Upper-Bound) Theorem.
Together, these theorems provide a powerful framework for evaluating the ultimate load capacity of structures, ensuring both safety and efficiency in design.

3.3.1 Static (Lower-Bound) Theorem

The **static theorem**, also known as the **lower-bound theorem**, states that:

If a distribution of internal stresses can be found that is in equilibrium with the applied external loads, does not exceed the yield criterion at any point, and satisfies all boundary conditions, then the structure is safe against collapse under those loads.

In other words, the structure will not fail as long as a feasible stress field exists within the yield limits of the material.

This theorem is essential from a **safety perspective**, since it guarantees that the actual collapse load will be at least as high as the calculated value.
Designs based on the static theorem are therefore **conservative**, ensuring that the structure can safely carry the applied loads without reaching the collapse state.

In the case of a **steel frame**, the static theorem can be applied to:

- Verify that internal bending moments and axial forces remain within the yield limit.
- Design members such that the overall system remains in equilibrium under **service or design loads** (e.g., gravity and wind loads).

3.3.2 Kinematic (Upper-Bound) Theorem

The **kinematic theorem**, or **upper-bound theorem**, states that:

If a kinematically admissible collapse mechanism can be assumed, and the external work done by loads equals the internal plastic work at yield, then the corresponding load represents an upper bound to the true collapse load.

This theorem focuses on **mechanisms** — possible ways in which a structure can deform plastically to the point of collapse.
It is particularly valuable from an **efficiency point of view**, as it helps identify how the structure might fail and allows the designer to use material economically while still achieving sufficient ductility.

For instance, in a **steel frame**, the kinematic theorem can be used to:

- Identify potential **collapse mechanisms**, such as plastic hinges forming in beams and columns.
- Evaluate the ultimate load capacity under **extreme conditions** (e.g., seismic or accidental loads).
- Optimize the distribution of material to ensure plastic hinges form in desired locations, providing ductile and controlled failure modes.

3.6.3 Complementarity and Practical Use

In practical structural design, both theorems are used in a complementary way:

- The **static theorem** ensures that the design remains on the **safe side** by not underestimating strength.
- The **kinematic theorem** ensures **efficiency**, avoiding unnecessary overdesign by providing an upper estimate of the collapse load.

When both conditions are satisfied — that is, when the lower-bound load (from the static theorem) and the upper-bound load (from the kinematic theorem) coincide — the **exact collapse load** of the structure is obtained.

In summary:

- The **static approach** ensures safety and conservatism.

- The **kinematic approach** promotes efficiency and optimal material use.

Together, they form the foundation of **plastic design methods** used for steel and reinforced concrete structures, particularly in the context of **ultimate limit state design**.

3.2.1 Notion of limit load :

Limit load is the external load that is sufficient for the structure to behave according to a mechanism is called the limit load.

3.2.2 Kinematic theorem

For a given structure and load P. Structural failure occurs under loads $\lambda_l P$:

The ultimate failure multiplier (λ_l) is equal to the maximum value of the factor λ such that under loads λP a distribution of internal forces can be found :

1) In equilibrium with loads λP ;
2) Respecting the plasticity criterion of the structure's elements.

To calculate the ultimate (limit) load of a structure, we need to start from two of the above conditions. The aim is to find a statically admissible moment diagram that transforms the structure into a mechanism.

Here are the steps to follow:

1. make the structure isostatic;
2. draw the bending moment diagram of the isostatic fundamental structure loaded by the effective loads;
3. draw the moment diagram for each hyperstatic quantity;
4. combine these moment diagrams in such a way as to reach, but not exceed, the value Mp.
5. deduce the value of the ultimate failure load as a function of Mp.

3.2.3Kinematic method

Determining the limit load using the kinematic method involves successively considering all possible failure mechanisms. According to the kinematic theorem, the true limit load is the smallest of the loads found. The principle of virtual work is to be applied to a mechanism that deforms under constant load.

3.3 Application to structural failure load calculations

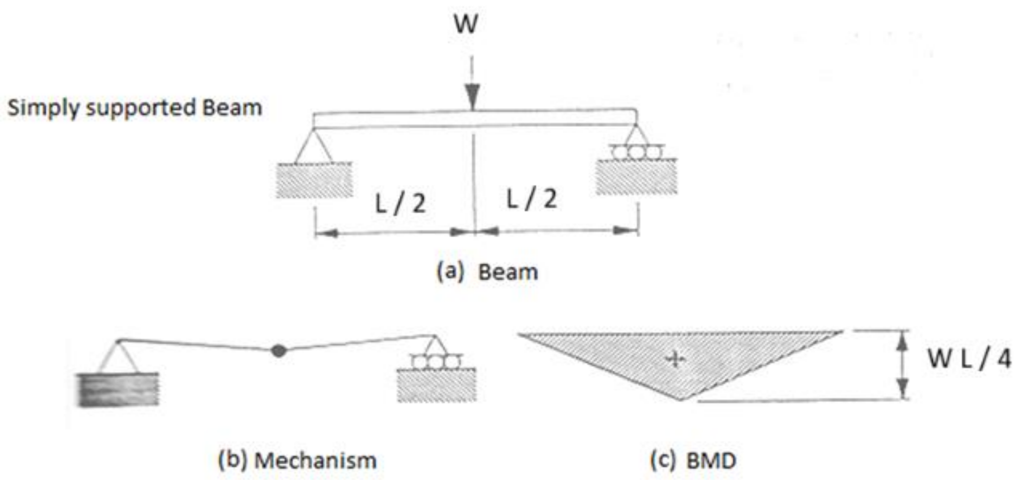
3.3.1 using the kinematic theorem

a) Simply supported beam

Rule 1:

The plastic hinge is formed under the point of application of the load.

$$\frac{W_c L}{4} = M_P \Rightarrow W_c = \frac{4M_P}{L}$$



Note: BMD is Boundary Moment Diagram

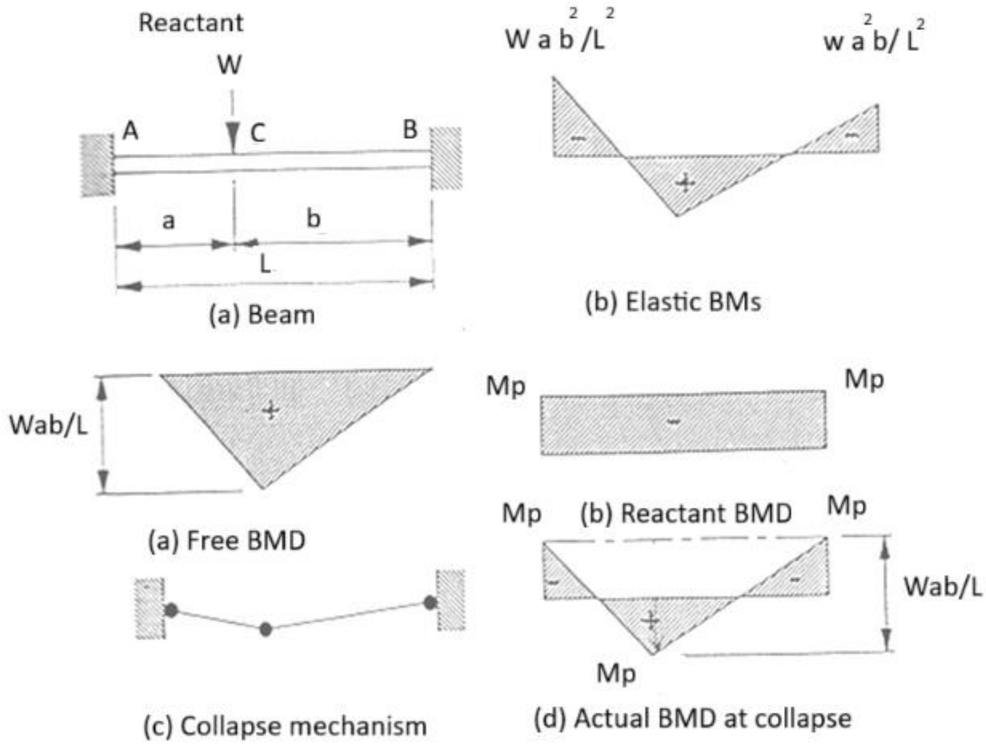
b) Beam embedded at both ends

Rule 2:

The plastic hinge is formed at the clamped ends of the beam.

$$\underbrace{M_P}_{Actuel} + \underbrace{M_P}_{Reactif} = \frac{W_c ab}{L}$$

$$W_c = \frac{2M_P L}{ab}$$

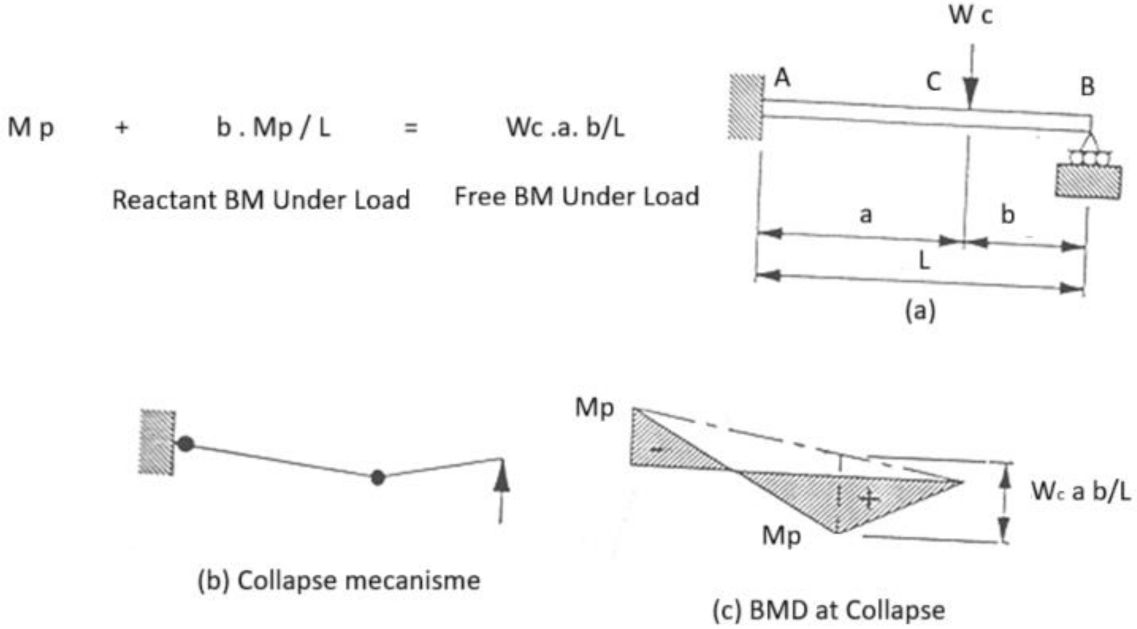


Note: BMD is Boundary Moment Diagram

c) Fixed-supported beam

$$\underbrace{M_P}_{Actuel} + \underbrace{\frac{b\,M_P}{L}}_{réactif} = \underbrace{\frac{W_c ab}{L}}_{Libre(sous\,la\,charge)}$$

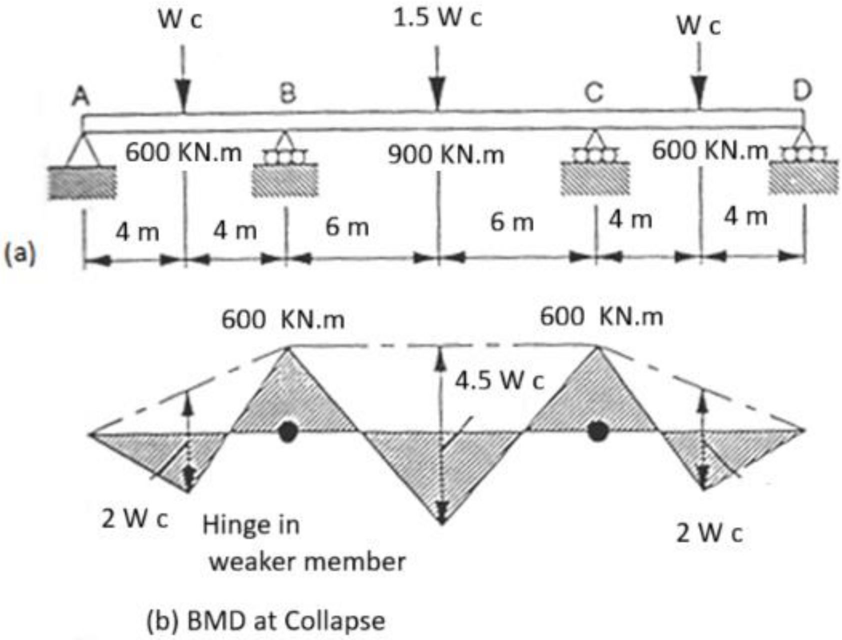
$$W_c = \frac{M_P(L+b)}{ab}$$



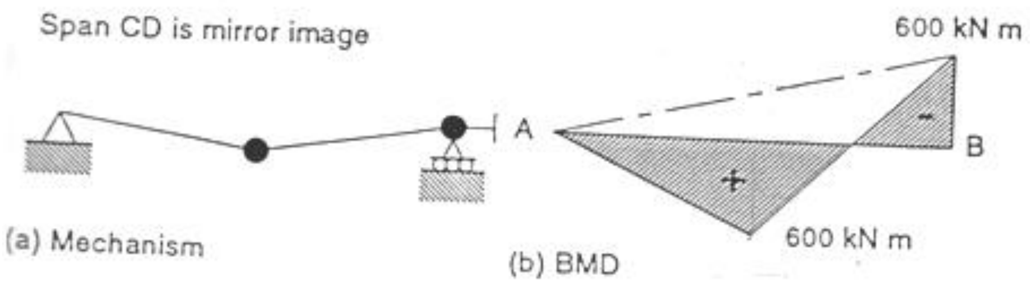
d) Continuous beam

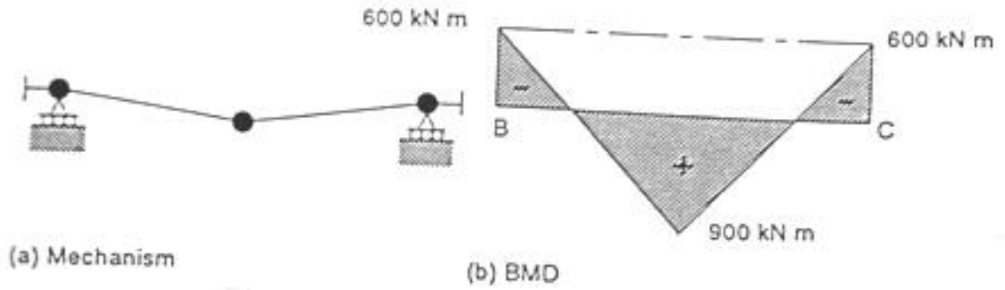
Rule 3:

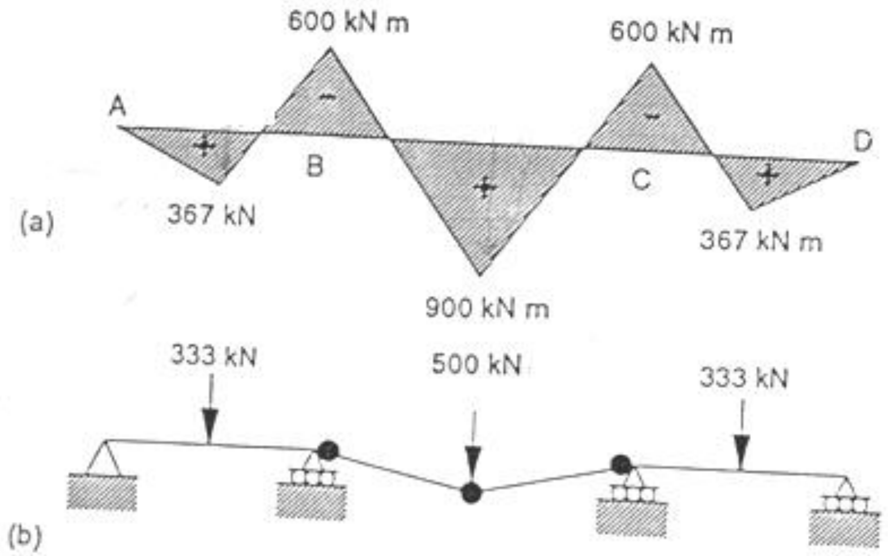
At support level, the plastic hinge is formed at the plastic moment of the weakest element in terms of bending stiffness (EI).



Note: BMD is Boundary Moment Diagram

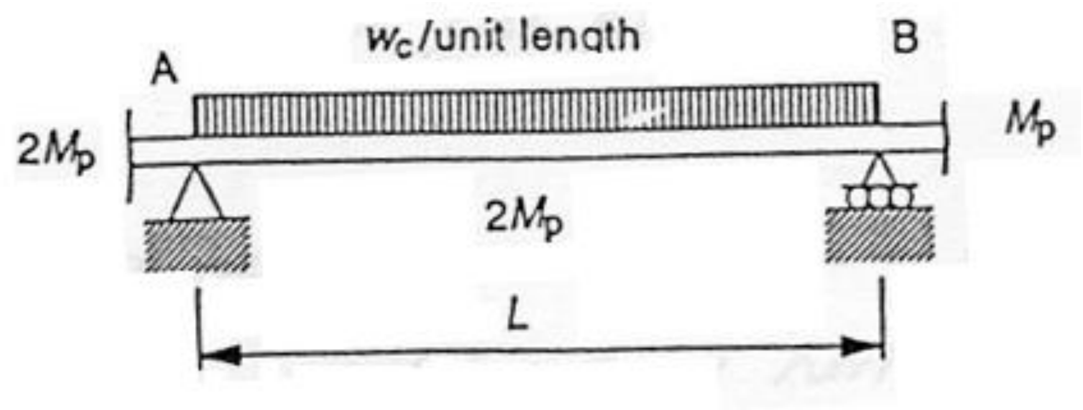






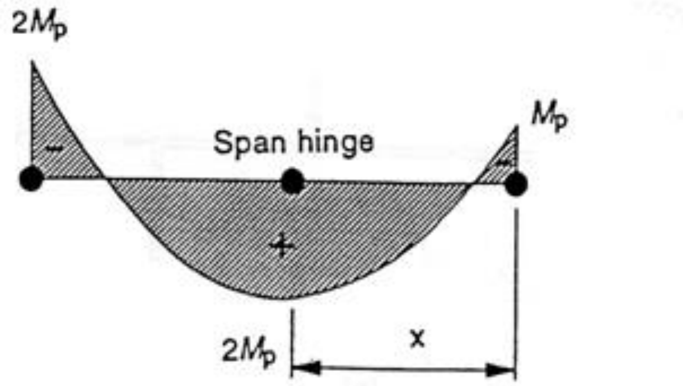
a) Beam with distributed loading

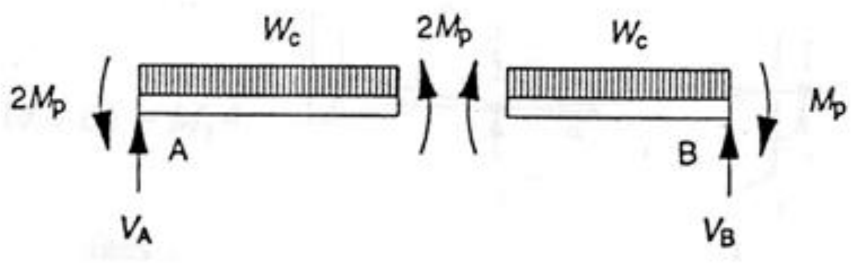
The position of the plastic hinge in the case of distributed loading cannot be deduced by a simple rule.



The plastic hinge is formed where the moment is maximum, ie : $\frac{dM}{dx} = 0$.

Applying the equilibrium equations after cutting the system at the hinge joint, we get :





$$\sum M_B = 0, \qquad \frac{w_c x^2}{2} = 3M_p$$

$$\sum M_A = 0, \qquad \frac{w_c (L-x)^2}{2} = 4M_p$$

Two equations with two unknowns, giving the following results:

$x = 0.464\,L$

and : $w_c = \dfrac{27.86}{L^2} M_p$

f. Simple frame

In the elastic analysis of a hyperstatic beam under static loading, three conditions must be considered:

- Continuity condition: the deformation must be continuous.

- Equilibrium condition: sum of moments and sum of forces are zero.

- Moment limit: the elastic moment is the limit moment (not to be exceeded).

In the plastic analysis, three similar conditions must be considered:

- Mechanism condition: plastic hinges interrupt the continuity of slope deformation, requiring the formation of plastic hinges to allow the structure (or part of the structure) to deform as a mechanism compatible with the remaining boundary conditions (mechanism: ability of the system to deform without increasing loading).
- Equilibrium condition: similar to that in the elastic case
- -Plastic moment condition: moments beyond the plastic moment cannot be resisted.

The behavior of the frame shown in figure 2.3 as it increases is given in table 2.1.

Assuming initially that: V=H=1

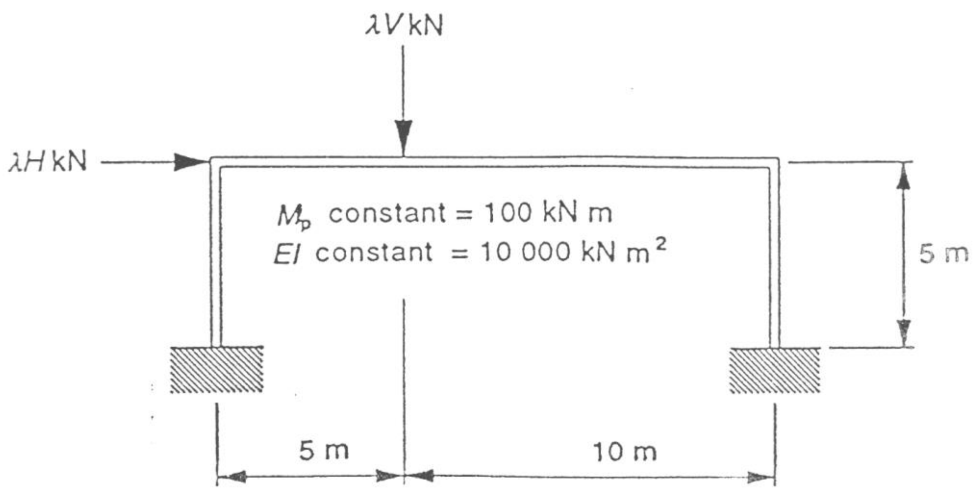


Figure 2.3

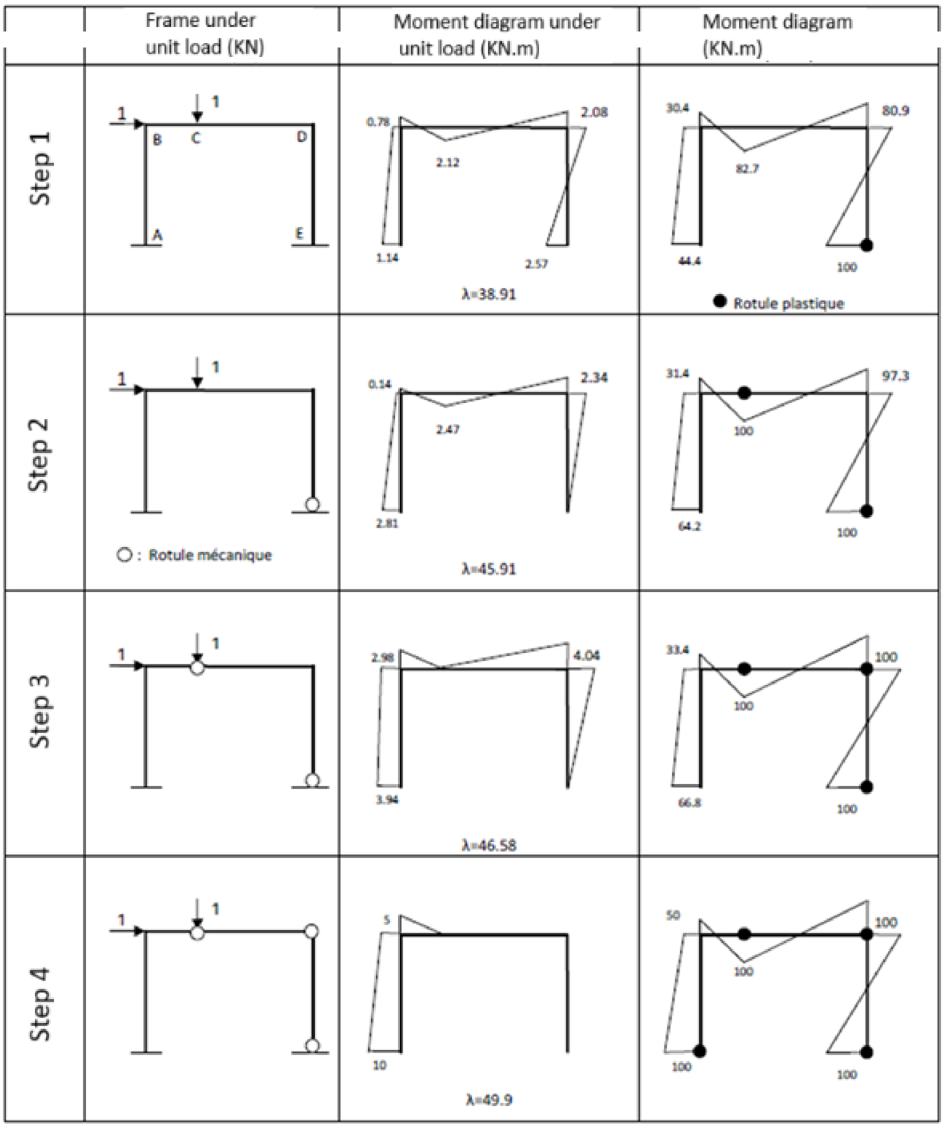


Table 2.1

- ❖ When λ =39, the E moment reaches the value of the plastic moment and the plastic hinge is formed.
- ❖ For step 2, the structure can be analyzed as in step 1 using the same elastic method, but taking into account the plastic hinge formed at E.
- ❖ To have a moment of :
- ❖ $M_c = 82.7 + 2.47\,\lambda'$ with λ' is the change in $\lambda = 46 - 39 = 7$
- ❖ As soon as λ reaches the value of 50 is reached, the fourth plastic hinge forms and the structure becomes a mechanism. The structure is about to break.

- In this case, there are enough plastic hinges to have a mechanism, a condition known as the "Mechanism Condition".
- The coefficient of change at which the structure changes into a mechanism is called the "Loading factor at failure λ_c".
- The mechanism in step 4 is the current mechanism for this frame. It is possible to try other mechanisms and repeat the above calculations to find the value of λ_c (the coefficient of change for which the structure changes into a mechanism).

Let's consider the same frame mechanism discussed earlier (see figure 2.4), with the same number of plastic hinges, but in different positions. The bending moment diagram is given in figure 2.4 b.

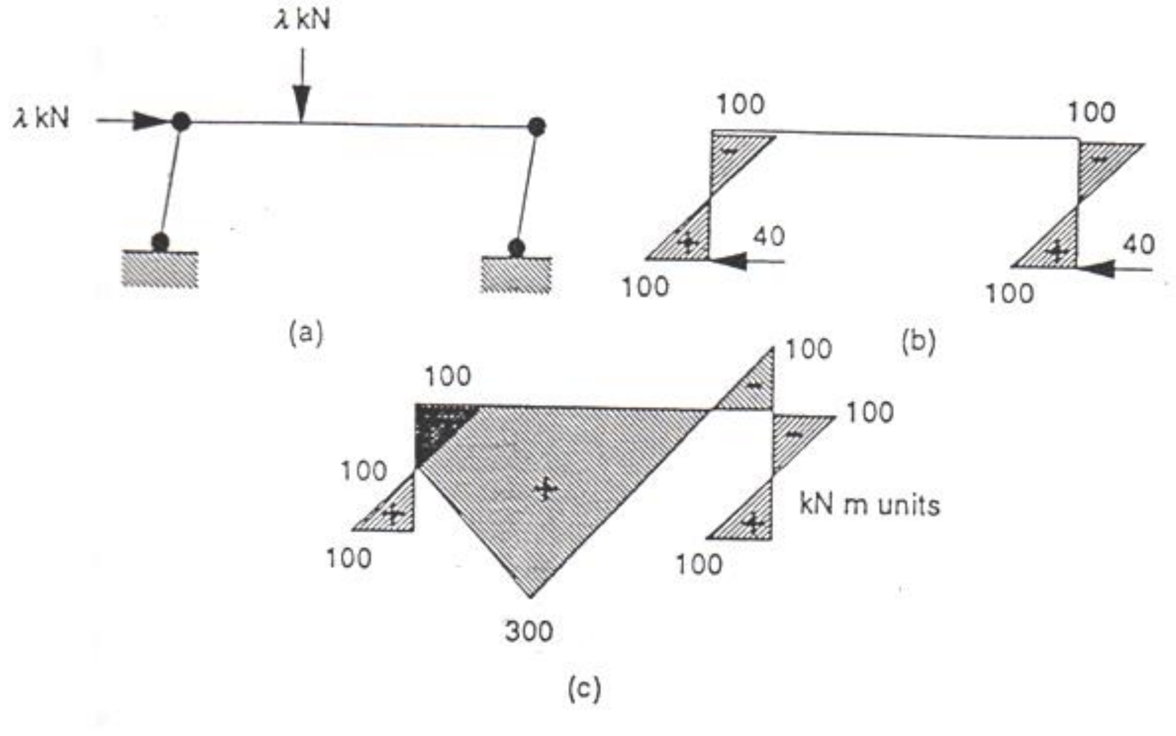


Figure 2.4

The horizontal reaction is H=40, so we get $\lambda = 80$, which means that the mechanism takes effect as soon as λ reaches the value of 80, which is higher than the true value of the loading factor at failure. Completing the bending moment diagram (see figure 2.4 c), we find that the moment at "C" is greater than the value of M P =100.The temptation of the mechanism is false, but its moment diagram satisfies the equilibrium equations and the mechanism conditions. In this case, λ is said a higher band, with respect to λ_c .

i) Partial collapse

3.7.1 Partial Collapse

Partial collapse occurs when only a portion of the structure reaches the collapse condition while the remainder remains stable and unyielded.
In such a case, insufficient plastic hinges are formed to produce a full mechanism throughout the entire structure.

This situation typically arises when:

- Only one span or bay of a multi-bay frame forms the necessary number of plastic hinges.
- Some members reach their plastic moment capacity earlier than others, causing localized collapse while the rest of the frame still resists loads.
- Uneven load distribution or non-uniform stiffness causes certain regions to yield prematurely, concentrating deformations in a limited area.

From a design standpoint, partial collapse indicates that energy dissipation is not evenly distributed across the structure.
This may lead to localized damage rather than global ductile behavior — an undesirable situation in seismic or ultimate limit design, where uniform ductility is preferred.

3.7.3 Premature Beam Failure

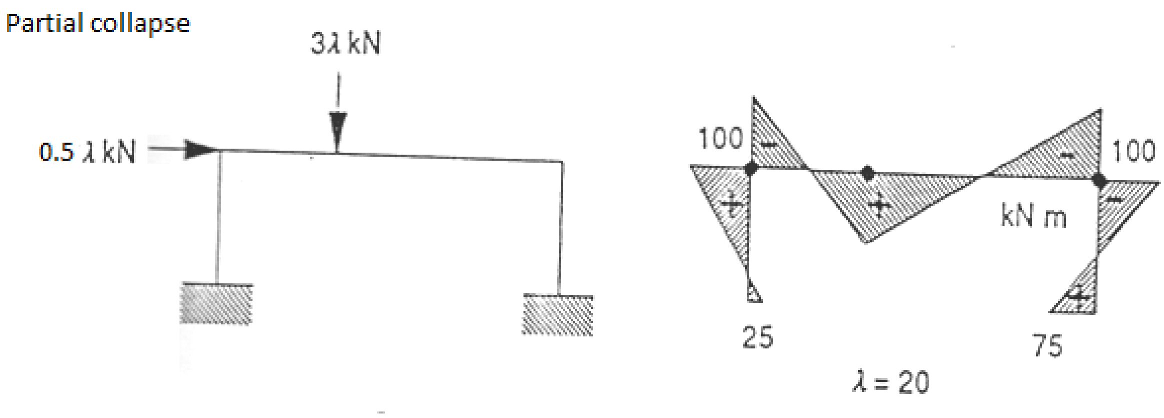
Premature beam failure is often the root cause of both partial and over-collapse.
In a well-designed frame, plastic hinges should ideally form first in the **beams**, allowing for a **ductile beam-sway mechanism**, while the **columns** remain largely elastic.
This ensures that energy is dissipated in controlled regions, preventing sudden or brittle failure.

However, premature beam failure occurs when:

- The **beam sections** are **under-designed** relative to the columns (low moment capacity).
- There is **insufficient rotation capacity** due to local buckling or inadequate detailing.
- **Connections** between beams and columns are not strong enough to transmit the full plastic moment.
- **Unbalanced loading** or **stiffness irregularities** cause one beam to yield before others.

When beams fail prematurely:

- Plastic hinges may form too early in specific beams, leading to **localized collapse** rather than a global mechanism (partial collapse).
- If these hinges propagate excessively before other parts reach yield, the frame can lose stiffness abruptly, resulting in **over-collapse** or **progressive failure**.



Here we have premature beam failure

ii) Over-collapse

3.7.2 Over-Collapse

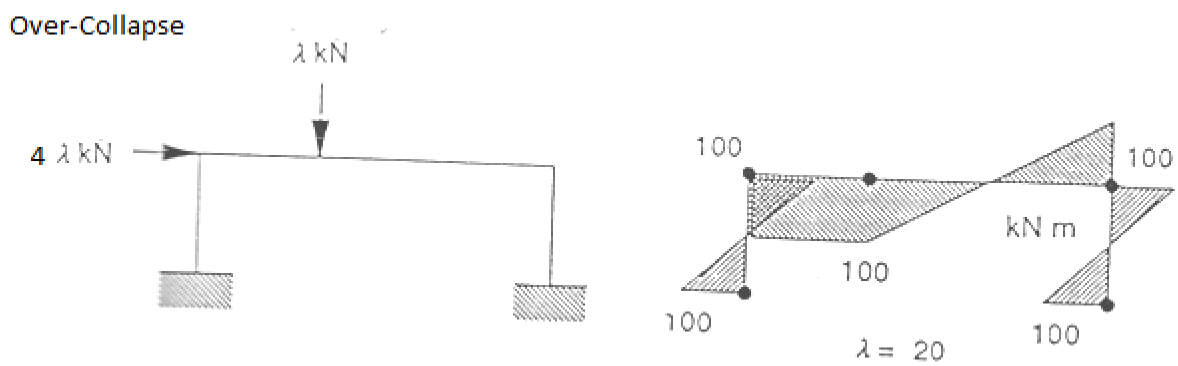
Over-collapse occurs when more plastic hinges form than are required to create the minimum collapse mechanism.

In this situation, the structure not only forms the intended mechanism but continues to develop additional plastic hinges in secondary members or regions not critical to the collapse mode.

While the structure technically collapses once the necessary mechanism is formed, additional hinge formation beyond that point can lead to:

- Excessive deformations or instability.
- Redistribution of moments that exceed member capacities.
- In some cases, a loss of global stability (especially in multi-storey frames).

Over-collapse typically suggests that the structure possesses more ductility than needed, but if not properly controlled, it can cause unintended failure sequences or excessive rotations incompatible with serviceability requirements.



Here, the last two last hinges form instantly

3.3.2 Using the static theorem

a. Ruin of a simple frame

In frame analysis, other possibilities besides beam failure are to be considered, such as the « sway ».

The virtual work method is powerful for beams and portals. The method is based on :

1) When frame failure occurs, all deformations are due to rotation at the joints (plastic hinges);

2) The principle of virtual work can be applied to these deformations.

Consider the frame shown in figure 3.1 (a). The deformation of the frame shown in Fig. 3.1 (b) is based on the application of a small horizontal deformation to the left corner, while neglecting axial deformations (buckling).

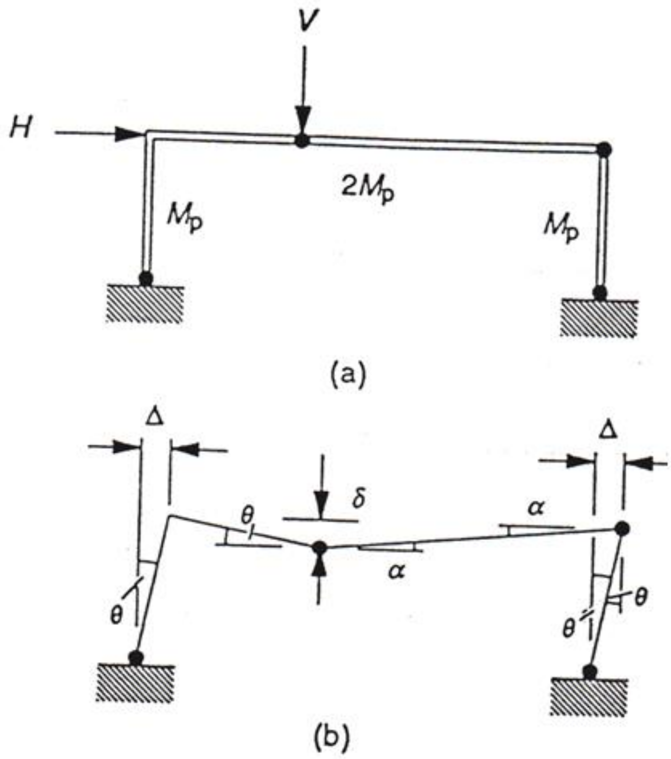


Figure 3.1

Deformations and rotations can be considered as virtual displacements, such as :

External (virtual) work done by applied forces:

$$H\Delta + V\delta = \sum W\delta$$

Internal (virtual) work, absorbed by rotation of the plastic hinge:

$$M_p\theta + 2M_p(\theta+\alpha) + M_p\theta = \sum M_p\theta$$

At the point of failure, the structure is in equilibrium and the following work equation can be applied assuming that:

1) At the moment of failure, the moment diagram remains constant as the structure deforms;
2) All axial loads are ignored

Outside (virtual) work = inside (virtual) work

$$\underbrace{\sum W\delta}_{sur\ toutes\ les\ charges} = \underbrace{\sum M_p\theta}_{sur\ toutes\ les\ articulations}$$

$$\sum force \text{ x } \begin{matrix} deplacement \\ virtuel\ dans\ la\ direction\ de \\ la\ force \end{matrix} = \sum \begin{matrix} moment\ plastique\ à \\ chaque\ rotule\ plastique \end{matrix} \text{ x } \begin{matrix} \text{rotation virtuelle} \\ \text{correspondante} \end{matrix}$$

Since axial deformation is ignored, premature failure due to buckling is also ignored.

δ and θ can be linked geometrically, and W can be calculated in terms of M_p

b. Beam fixed at both ends

The procedure is as follows (see the system in figure 3.2):

1) Choose the failure mechanism (given previously);

2) Impose virtual plastic rotations on the hinges (note signs);

3) External work :

$$W_c \text{ x } \delta = \text{force x virtual displacement distancet}$$

1) Internal work: Absorbed by the hinge joints

$$(\text{-M}_p).(-\theta) + M_p.(\theta+\phi) + (-M_p).(-\phi) = 2M_p.(\theta+\phi)$$

1) Mechanism geometry; $\phi = \frac{a}{b}\theta$

2) Apply balance and use mechanism geometry:

External Work=Absorbed Work

$$W_c.\delta = 2M_p(\theta+\phi)$$

$$W_c.a.\theta = 2M_p.(1+\frac{a}{b}).\theta$$

$$W_c.a.\theta = 2M_p\frac{(a+b)}{b}.\theta$$

$$W_c = \frac{2M_pL}{a.b}$$

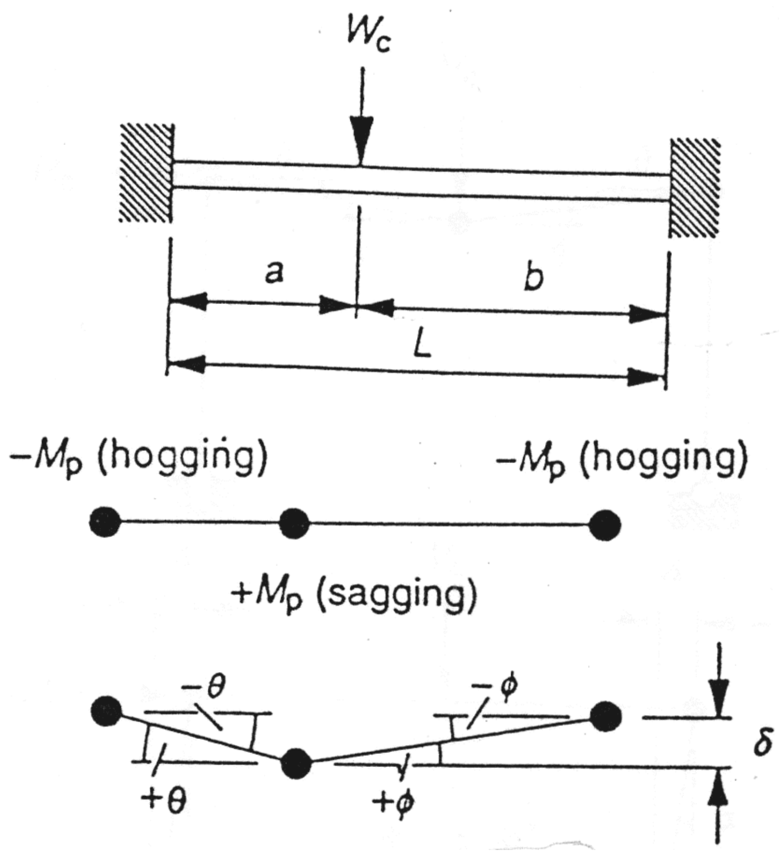


Figure 3.2

c. Double-supported frame (two double supports)

The frame shown in figure 3.3 is subjected to both horizontal and vertical loads, which means it will have several possible mechanisms (each requiring a different calculation).

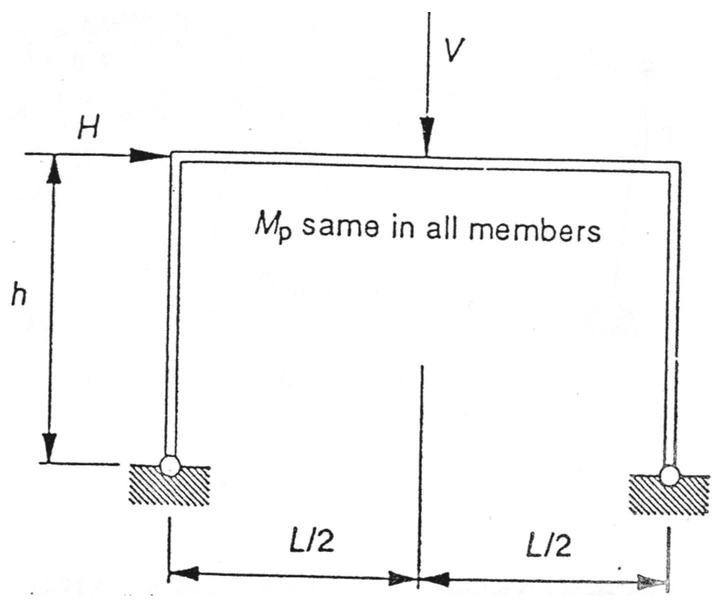


Figure 3.3

c.1 Ruin of the beam

See Figure 3.3.1

$$V_c.\theta.\frac{L}{2} = M_p.(\theta) + M_p.(2\theta) + M_p.(\theta)$$

$$\frac{V_c L}{M_p} = 8$$

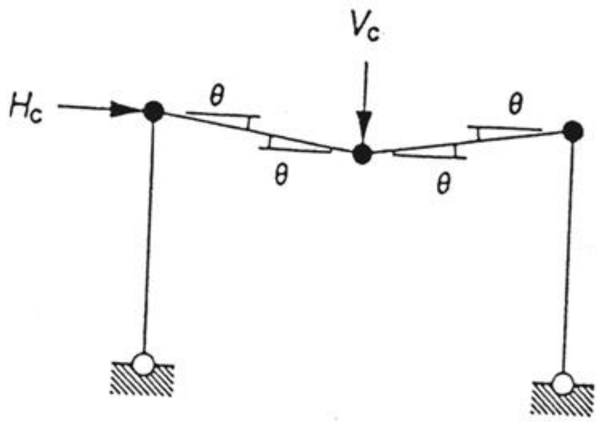


Figure 3.3.1

i) Ruin by offset

See Figure 3.3.2

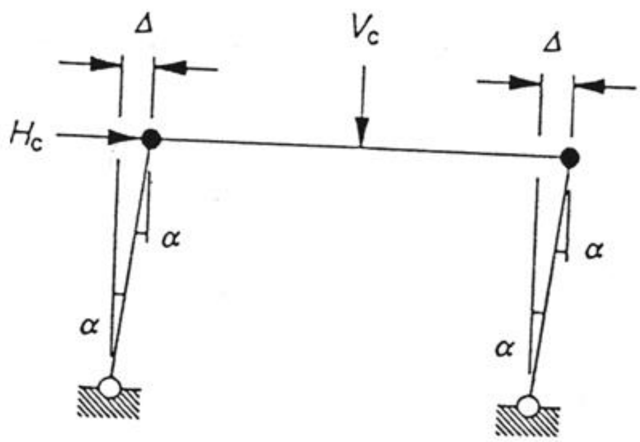


Figure 3.3.2

External Work $= H_c.\Delta$
$= H_c.h.\alpha$

Internal Work $= 2M_p.\alpha$

$\Delta = \alpha.h$

$\alpha = \theta$ (nodes are rigid and M_p is the same for all elements)

By equilibrium:

$H_c.h.\alpha = 2M_p.\alpha$

So to ruin:

$$\frac{H_c.h}{M_p} = 2$$

ii) Combination of beam failure and offset failure

See figure 3.3.3

$$\delta = \frac{L}{2}.\theta$$
$$\Delta = \alpha.h$$
$$\frac{V_c.L.\theta}{2} + H_c.h.\theta$$
$$= 4M_p.\theta + 2M_p.\theta - \underbrace{M_p.\theta}_{Poutre} - \underbrace{M_p.\theta}_{décalage}$$
$$\frac{V_c.L.\theta}{2} + H_c.h.\theta = 4.M_p.\theta$$
$$\frac{1}{2}\frac{V_c.L}{M_p} + \frac{H_c.h}{M_p} = 4$$

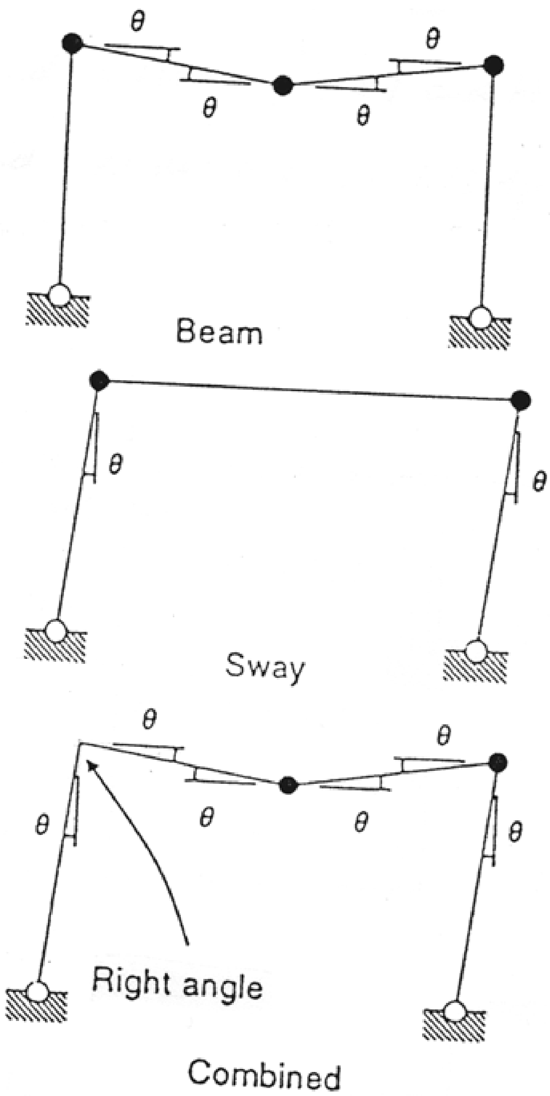


Figure 3.3.3

iii) What's the preferred mechanism?

The actual mechanism depends on the relative values of the H and V forces.

Tracing the previous equation, we get the “interaction diagram” shown in figure 3.3.4.

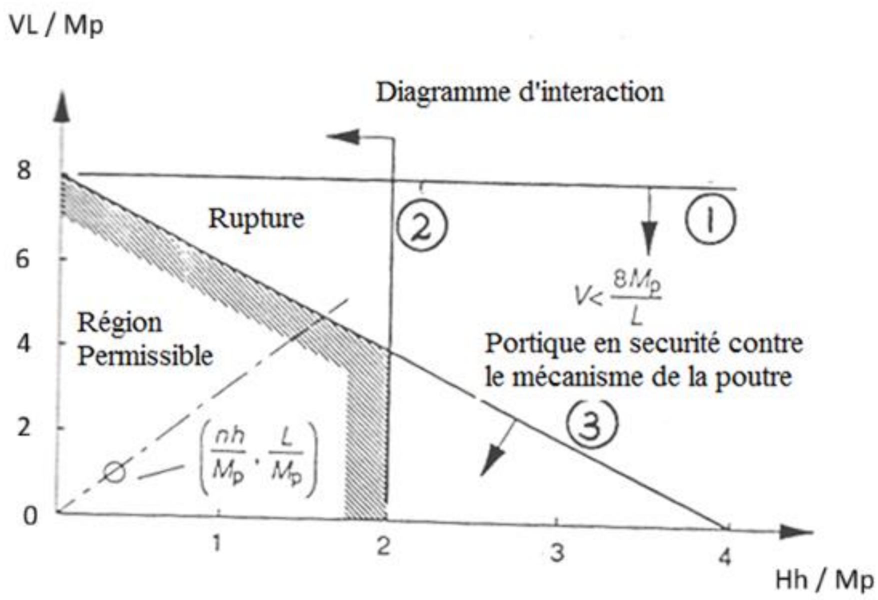


Figure 3.3.4

The vectors shown in figure 3.3.4 indicate safety. The permissible region indicates the combinations of V and H that verify safety against failure by any mechanism.

Example :

For the value of $V/H = 1/n$ and assuming that $V = 1$ and $H = n$ the rupture mechanism is combined and the values are: $V = 4.75\,M_p\,/\,L$ and $H = 1.625 M_p/n$ to la rupture.

Conclusion

In this chapter, the principles and applications of **limit analysis** were presented to determining the ultimate load capacity of structures beyond the elastic range. The fundamental assumptions of **plasticity theory** were established, emphasizing the ductile behavior of steel and the conditions under which reinforced concrete may also be analyzed plastically.

The two main **limit theorems** — the **static (lower-bound)** and **kinematic (upper-bound)** theorems — were discussed in detail, demonstrating how they complement each other to ensure both **safety** and **efficiency** in design. The concept of **limit load** and the use of both the **static** and **kinematic methods** in calculating structural failure loads were illustrated through examples involving beams and frames.

Furthermore, the chapter addressed different **collapse mechanisms**, including **partial collapse**, **over-collapse**, and **ruin by offset**, highlighting how **premature beam failure** can affect the overall ductile performance of a frame. These discussions underline the importance of controlling plastic hinge formation to ensure predictable and safe structural behavior at collapse.

The next chapter will be an introduction to the **theory of damage mechanics**, which extends beyond collapse load prediction to describe the **progressive deterioration of materials and structures** under cyclic, dynamic, and environmental effects.

Chapter 4: Damage

4.1 Introduction

The purpose of structural design is to determine a structure's operating limits, so as not to exceed its load-bearing capacity. This approach is based on the theory of elasticity and strength of materials, with the assumption of a homogeneous, isotropic elastic medium.

In general, when a material is deformed from an initial state to a pre-deformed state, its deformation capacity or residual ductility until failure decreases. During deformation, the material therefore undergoes progressive damage, leading to failure. Damage can be considered as all the phenomena associated with the cavities that form in the material during deformation [3].

Damage is thus reflected in the formation (initiation phase) and development (growth and coalescence phases) of **micro-voids** in the material [3].

4.2 Damage measurement

Damage to a deformed material, or a material undergoing deformation, can be measured using two main families of measurement methods. The first is based on microscopic observation, while the second involves indirect measurements using a physical parameter [3].

4.2.1 Damage of concrete and reinforced concrete structures

Numerous models of damage associated with other non-linear phenomena have been developed to deal with the various phenomena associated with the complex behavior of concrete, the following models can be cited [4]:

4.2.1.1 Isotropic models:

a. Mazars model:

The MAZARS model (1984) is a simple, robust model based on damage mechanics, which describes the reduction in material stiffness caused by the creation of micro-cracks in concrete. It is based on a single scalar internal variable D, describing damage isotropically, but distinguishing between tensile and compressive damage [4].

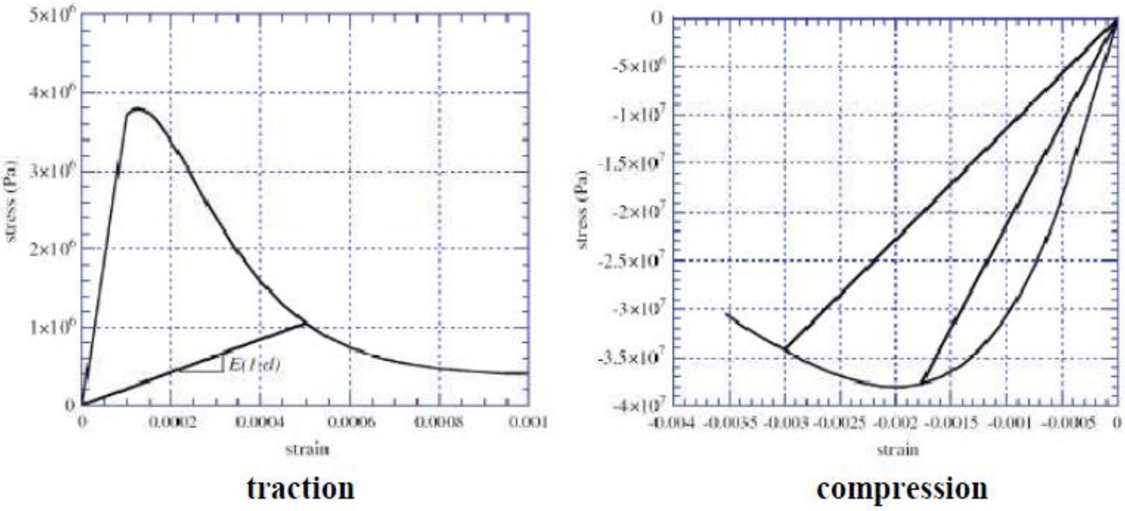


Figure 4.1: Mazars damage behavior model [5]

While this model correctly describes the evolution of concrete behavior under monotonic loading, it does not take into account anelastic deformations. We therefore have a lower dissipation than in reality [4].

b. De Vree" model:

This model retains the same methodology as MAZARS, but defines a new equivalent deformation called the modified Von Mises equivalent deformation [4].

4.2.1.2 Anisotropic models:

From a microscopic point of view, cracks and voids develop in directions that depend on the loading imposed on the concrete. Generally speaking, a preferred direction is perpendicular to the direction of greatest tensile stress. These preferred directions lead to a macroscopic anisotropy of the material, which is a function of the material's history. It is induced by damage [4].

a. Dragon and Halm model:

Dragon & Halm propose an anisotropic damage model in which damage is the only dissipative phenomenon considered; it consists of the creation and propagation of decohesive meso-surfaces within a representative volume. The model uses a second-order tensor internal damage variable describing the orientation and extent of meso-cracks [4].

4.2.1.3 Unilateral models :

Observation of the tests reveals a restoration of the material's stiffness in compression, previously damaged in tension. This unilateral effect first manifests itself as a non-linear transient phase due to the reclosure of cracks when the sign of the stress changes, followed by the restoration of compressive stiffness [4].

a. Laborderie model (1991):

Within the framework of scalar damage modeling, one solution to describe this phenomenon is to introduce several damage variables that can translate damage states. The minimum required is two variables to separate the mechanical effects of microcrack opening and closing [4].

b. Ramtani model (1990):

In this model, Ramtani uses two second-order tensors to describe tensile and compressive damage, and a scalar for volume damage [4].

4.3 Structural damage (Performance levels)

Before starting the non-linear static analysis of the structure, an essential step is to determine the level of performance required of this structure when subjected to a specific seismic hazard, and to characterize the permissible damage for structural and non-structural elements at this level [6].

The performance level is defined by the state of the structure studied after being hit by a specific level of earthquake. In other words, the performance level is the upper limit of permissible damage to a structure after it has been subjected to a specific level of seismic hazard. The codes have established two classifications of performance levels, one at the SP structural performance level and the other at the NP non-structural performance level [6].

Structural performance levels are defined as follows:

4.3.1 Level IO (start of operation)

Indicates that the damage caused by the earthquake is very limited, with the building's horizontal and vertical force-resisting systems retaining much of their pre-earthquake strength and rigidity. The risk to life from structural damage is very low, but some simple structural repairs must be carried out. [6].

4.3.2 Level LS (safe operating condition)

Indicates that the state of damage to the structure after the earthquake is significant, but there is a margin of resistance to collapse. Some structural elements and components are extensively damaged, with significant debris falling from both inside and outside the building. Damage is not significantly life-critical during the earthquake. Use of the building may be prohibited until repaired [6].

4.3.3. CP level (damage status)

It indicates that the construction is at risk of partial or total collapse, as it indicates that the great damage suffered by structural and non-structural elements with the probability of very great degradation in the rigidity of lateral loading resistance systems with the presence of a small margin of resistance to collapse, At this level, and in the presence of major degradation of the lateral load resistance systems, it is imperative for the main elements of the lateral load resistance systems to continue to resist the forces of gravity. There can be great danger from falling structural debris, and it is technically impractical to repair the structure safely - it is unusable because of the existence of replicas, which can lead to the collapse of the building [6].

Performance levels are represented graphically on the capacity curve as shown in the following figure (figure 2.2).

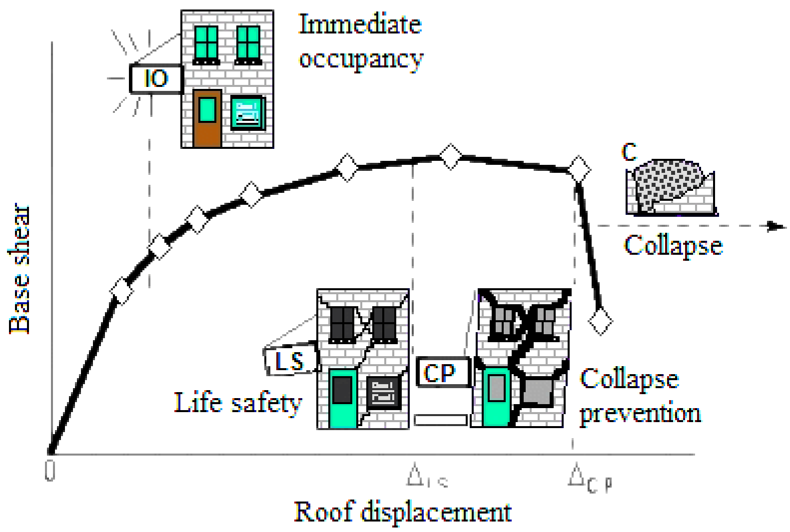


Figure 4.2: Capacity curve and structural performance levels [6].

4.4 Performance point determination

The performance point represents the equilibrium condition between a structure's capacity and the earthquake demand.

It is the point on the capacity curve (obtained from pushover analysis) where the structural capacity in terms of displacement equals the seismic demand imposed by the earthquake.

In simpler terms, it defines how far the structure will actually displace under a specific earthquake level.

- To the left of this point, the structure can safely resist the seismic forces (elastic or slightly inelastic behavior).
- Beyond this point, failure or excessive damage begins to occur.

Thus, the performance point helps engineers evaluate whether the structure will meet its performance objectives, such as:

- Immediate Occupancy (IO)
- Life Safety (LS)
- Collapse Prevention (CP)

4.5 Earthquake Demand

The earthquake demand refers to the seismic forces or displacements that an earthquake imposes on a structure.
It represents what the earthquake requires the structure to endure, and it is generally defined in terms of:

- Spectral acceleration or displacement (from the design response spectrum), or
- Base shear and roof displacement (in pushover analysis).

In performance-based design, the earthquake demand is expressed through demand spectra corresponding to different seismic intensity levels (e.g., frequent, design, or maximum considered earthquake).

4.6 Relation Between the Two

The performance point is found at the intersection of:

- the capacity curve (what the structure can resist), and
- the demand curve (what the earthquake demands).

At this intersection, both the internal resisting forces and external seismic forces are in balance — defining the expected seismic performance of the structure.

From the capacity curve, it then becomes interesting to compare it with the earthquake demand. To consider the demand of an earthquake, acceleration or displacement response spectra are generally used. The axes of the capacity curve must therefore be transformed to have the same units [7]:

- ✓ Base reaction / mass → acceleration.
- ✓ Displacement / Modal participation factor → displacement.

Several methods are available for joining the two curves. EC 8, for example, finds a performance point based on the rule of equal displacements. American standards, on the other hand, call for iterations with several spectra representing different viscous damping coefficients. In both cases, we find what is known as a “performance point”, which allows us to make several considerations concerning the structure's behavior in the face of an earthquake. According to EC 8, the elastic displacement of an equivalent structure is found by extending the elastic part of the capacity curve up to the intersection with the spectrum (point A), (see Figure 4.5 and Figure 4.6). The inelastic displacement of the real structure is that which corresponds to it on the capacity curve at point B. With this construction, we can for example, to define whether the structure requires an increase in its deformation capacity or an increase in its rigidity [7].

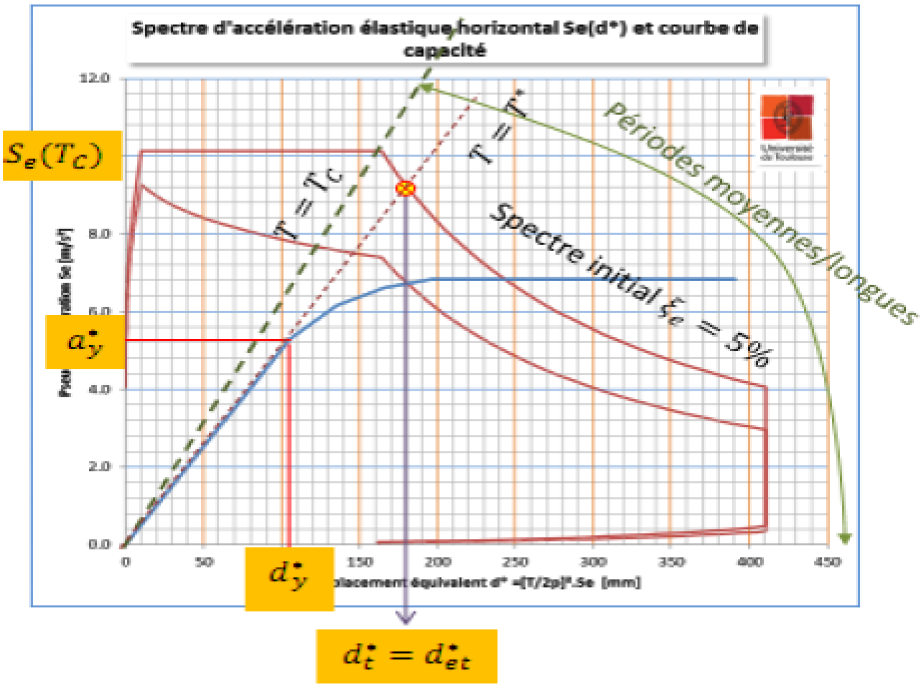


Figure 4.5: Performance point [7].

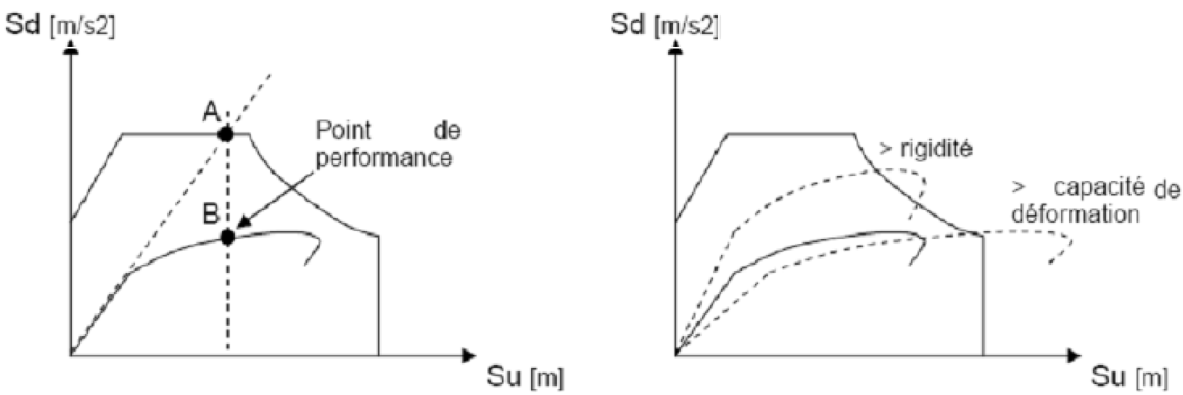


Figure 4.6: Performance point according to EC 8 and principle of capacity curve evaluation [7].

4.7 Notion of damage index

Damage Index

The damage index (DI) is a quantitative measure used to evaluate the extent of structural or material degradation under loading, especially during seismic events.
It provides a normalized value, typically ranging from 0 (no damage) to 1 (total collapse), representing the degree of damage sustained by a structure or structural element.

The damage index combines both deformation (e.g., displacement or ductility) and energy dissipation (e.g., hysteretic energy) parameters to describe how close the structure is to failure.

Mathematically, several formulations exist, the widely used model is:

$$D_i = \frac{\delta_m - \delta_y}{\delta_u - \delta_y}$$

δ_m : is the maximum displacement in the non-linear zone (performance point);

δ_u : is the ultimate displacement (total ruin);

δ_y : is the displacement corresponding to the plastification threshold.

4.8 Local-global damage relationship

Based on the degree of structural damage, an equivalence between the previously defined damage index DI and the state of deterioration is given in Table 9.1.

In order to calibrate the parameters of the fragility functions, it is necessary to establish a correlation between the previously defined damage index D_I and the interstage displacement Δ_i .

Damage index	**Damage status**
$D_I \leq 0.1$	No damage
$0.1 \prec D_I \leq 0.25$	minor damage
$0.25 \prec D_I \leq 0.4$	Moderate
$0.4 \prec D_I \leq 1$	Significant

$D_I \succ 1$	Ruin

Table 9.1: Equivalence between damage index and damage status [7].

Conclusion:

Conclusion

In this chapter, the concept of **damage** was explored from both the **material** and **structural** perspectives. Different **damage models**—isotropic, anisotropic, and unilateral—were presented to capture the complexity of concrete behavior under various loading conditions.

At the structural scale, the notion of **performance levels** was introduced to evaluate the expected condition of a structure after seismic loading. The relationship between **structural capacity** and **earthquake demand** was explained through the **performance point**, illustrating how seismic performance can be assessed within the framework of performance-based design. Furthermore, the **damage index** was defined as a quantitative measure of degradation, linking local material behavior to global structural performance.

Through these developments, this chapter achieved its objectives of:

- Presenting the fundamental principles governing damage initiation and evolution in structural materials.
- Describing key analytical models used to represent damage in concrete and reinforced concrete.
- Introducing performance-based assessment tools, including the performance point and damage index, for evaluating seismic response.

These foundations provide a comprehensive understanding of how damage influences structural behavior, paving the way for advanced studies on **fragility analysis** and **resilience assessment** in the following works.

Conclusion

The *"Plasticity and Damage"* handout provides a comprehensive introduction to the fundamental principles governing the nonlinear behavior of structures beyond their elastic limits. It is intended as an essential learning aid for Master 2 Civil Engineering students specializing in structural analysis and design.

Through the four chapters, the document progressively guides students from the **concepts of anelastic and plastic behavior** to the **advanced notions of limit analysis and damage mechanics**. The first chapter emphasizes the limitations of purely elastic design and introduces the need for plastic calculations as a more realistic representation of structural behavior at failure. The second chapter develops the basis of **plastic design**, highlighting concepts such as **plastic hinges**, **moment-curvature relationships**, and **capacity curves**, which are crucial for understanding how structures redistribute internal forces and maintain stability beyond the yield point.

The third chapter focuses on **limit analysis**, providing a theoretical and practical framework to determine the **ultimate load and collapse mechanisms** of structures. Finally, the fourth chapter introduces the **damage mechanics** concept, exploring the initiation, propagation, and accumulation of material degradation—an essential step toward understanding real-world structural behavior under repeated or extreme loading.

Overall, this handout aims to strengthen students' analytical skills and conceptual understanding of structural nonlinearity. It encourages a transition from classical elastic analysis toward a more advanced, **realistic, and performance-oriented design philosophy**. By integrating the principles of plasticity, limit analysis, and damage modeling, future engineers will be better equipped to assess, predict, and optimize the performance and safety of civil engineering structures.

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