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Evaluating the Volatility Forecasting Performance in the Sukuk Market

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Declaration

I declare that this dissertation is my own original work and that it has not been submitted previously, in whole or in part, for any academic degree at any institution. All sources and references used have been properly acknowledged.

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ABSTRACT:

This study investigates the predictability of sukuk market volatility and evaluates the forecasting performance of six models (ARIMA, GARCH(1,1), HAR, PROPHET, ANN, and LSTM) across short-, medium-, and long-term horizons using a panel of 31 sukuk instruments based on realized volatility. The dataset covers 1,086 daily observations for each instrument over the period from January 2019 to April 2023, including sukuk funds issued across diverse regions such as the GCC, Southeast Asia, and selected global markets. The first stage of analysis confirms that sukuk returns exhibit volatility clustering, stationarity, non-normality, and time-varying conditional variance, indicating strong potential for accurate forecasting. The second stage assesses in-sample forecast performance through a two-step evaluation process: visual comparison of predicted versus actual volatility and quantitative measurement using MAE, MSE, and RMSE.

Results indicate that forecast performance varies depending on the evaluation method used, showing that model rankings are not consistent across criteria. Based on quantitative error metrics, HAR is ranked first across all forecasting horizons, confirming its strong numerical accuracy. In the short term, ARIMA and Prophet are ranked second, while LSTM and ANN show moderate accuracy. In the medium and long term, LSTM is ranked second, followed by Prophet, and then ANN.

However, from a visual evaluation perspective, the ranking changes. LSTM is ranked first in the short and medium terms, while ANN is ranked first in the long term, as both models track the realized volatility more closely. In contrast, GARCH performs worst across all horizons under both evaluation methods, producing forecasts that fail to capture market dynamics.

Overall, these findings demonstrate that sukuk volatility exhibits structured and predictable behaviour, which directly supports your dissertation's central research question on market predictability. At the same time, the results show that the optimal model depends on the evaluation criterion: HAR is the most reliable model for numerical accuracy, LSTM and ANN are the most effective models for visual tracking, whereas GARCH is consistently the weakest performer and should be avoided.

These findings provide practical guidance for financial institutions, regulators, and investors in managing volatility and selecting forecasting tools in Islamic finance.

Keywords: Islamic Finance, Sukuk, Volatility, Forecasting, LSTM, Realized Volatility

الملخص:

تهدف هذه الدراسة إلى تحليل مدى إمكانية التنبؤ بتقلبات سوق الصكوك وتقيّم أداء ستة نماذج تنبؤية والمتمثلة في: (ARIMA، (1,1) GARCH، HAR، PROPHET، ANN، وLSTM) عبر آفاق زمنية قصيرة، ومتوسطة، وطويلة الاجل، وذلك باستخدام عينة مكونة من 31 أداة صكوك استنادًا إلى التقلبات المحققة (Realized Volatility). يغطي هذا التحليل 1,086 ملاحظة يومية لكل أداة خلال الفترة الممتدة من يناير 2019 إلى أبريل 2023، ويشمل صكوكًا صادرة من مناطق جغرافية متنوعة مثل دول مجلس التعاون الخليجي، وجنوب شرق آسيا، وأسواق عالمية مختارة. تؤكد المرحلة الأولى من التحليل أن عوائد الصكوك تُظهر تكتلات في التقلبات، واستقرارية، وانحرافًا عن التوزيع الطبيعي، وتباينًا شرطيًا متغيرًا مع الزمن، مما يشير إلى وجود قابلية قوية للتنبؤ الدقيق. أما المرحلة الثانية فتقيّم أداء النماذج داخل العينة من خلال عمليتين: مقارنة بيانية بين القيم المتوقعة والفعلية للتقلبات، وقياس كمي باستخدام مؤشرات الخطأ MAE، MSE، وRMSE.

تشير النتائج إلى أن أداء التنبؤ يختلف باختلاف طريقة التقييم المستخدمة، مما يدل على أن ترتيب النماذج ليس ثابتًا عبر المعايير المختلفة. استنادًا إلى مقاييس الاخطاء الكمية، يحتل نموذج HAR المرتبة الأولى عبر جميع آفاق التنبؤ، مما يؤكد دقته العددية العالية. في الأجل القصير، يحتل نموذجا ARIMA و Prophet المرتبة الثانية، بينما يظهر نموذجا LSTM و ANN دقة متوسطة .في الأجلين المتوسط والطويل، يحتل نموذج LSTM المرتبة الثانية، يليه Prophet ثم ANN.

لكن من منظور بياني، يتغير ترتيب النماذج. حيث يحتل نموذج LSTM المرتبة الأولى في الأجلين القصير والمتوسط، بينما يحتل نموذج ANN المرتبة الأولى في الأجل الطويل، وذلك نظرًا لقدرة هذين النموذجين على تتبع التقلبات المحققة بشكل أدق. في المقابل، يُظهر نموذج GARCH أسوأ أداء عبر جميع الآفاق الزمنية ووفق كلتا طريقتي التقييم، إذ يفشل في التقاط ديناميكيات السوق.

بشكل عام، تُظهر هذه النتائج أن تقلبات الصكوك تتسم ببنية منظمة وقابلية للتنبؤ، وهو ما يدعم مباشرة سؤال البحث الرئيسي حول قابلية التنبؤ في سوق الصكوك. كما توضح النتائج أن اختيار النموذج الأمثل يعتمد على معيار التقييم المستخدم: حيث يُعد HAR الأفضل من حيث الدقة العددية، بينما يُعد LSTM و ANN الأفضل من حيث التتبع البياني، في حين يظل GARCH الأضعف أداءً ويُفضَّل تجنبه.

وتوفر هذه النتائج إرشادات عملية للمؤسسات المالية، والهيئات التنظيمية، والمستثمرين في إدارة التقلبات واختيار أدوات التنبؤ ضمن قطاع التمويل الإسلامي.

الكلمات المفتاحية: التمويل الإسلامي، الصكوك، التقلبات المحققة، التنبؤ، LSTM

RÉSUMÉ :

Cette étude examine la prévisibilité de la volatilité du marché des sukuk et évalue les performances de prévision de six modèles (ARIMA, GARCH (1,1), HAR, PROPHET, ANN et LSTM) sur des horizons de court, moyen et long terme, en utilisant un panel de 31 instruments sukuk basé sur la volatilité réalisée. L'ensemble de données couvre 1 086 observations quotidiennes pour chaque instrument sur la période allant de janvier 2019 à avril 2023, incluant des fonds sukuk émis dans diverses régions telles que le CCG, l'Asie du Sud-Est et certains marchés mondiaux sélectionnés. La première phase de l'analyse confirme que les rendements des sukuk présentent un regroupement de la volatilité, une stationnarité, une non-normalité et une variance conditionnelle variable dans le temps, ce qui indique un fort potentiel de prévision précise. La deuxième phase évalue les performances de prévision dans l'échantillon à travers un processus en deux étapes : une comparaison visuelle entre la volatilité prévue et la volatilité réelle, et une mesure quantitative à l'aide des indicateurs MAE, MSE et RMSE.

Les résultats indiquent que la performance de prévision varie selon la méthode d'évaluation utilisée, ce qui montre que le classement des modèles n'est pas constant selon les critères. Sur la base des mesures quantitatives d'erreur, le modèle HAR est classé premier pour tous les horizons de prévision, ce qui confirme sa forte précision numérique. À court terme, les modèles ARIMA et Prophet sont classés deuxièmes, tandis que les modèles LSTM et ANN présentent une précision modérée. À moyen et à long terme, le modèle LSTM est classé deuxième, suivi de Prophet, puis de ANN.

Cependant, du point de vue de l'évaluation visuelle, le classement change. Le modèle LSTM est classé premier à court et à moyen terme, tandis que le modèle ANN est classé premier à long terme, car ces deux modèles suivent plus fidèlement la volatilité réalisée. En revanche, le modèle GARCH présente les plus faibles performances pour tous les horizons et selon les deux méthodes d'évaluation, produisant des prévisions qui ne parviennent pas à capter la dynamique du marché.

Dans l'ensemble, ces résultats montrent que la volatilité du marché des sukuk présente une structure organisée et prévisible, ce qui soutient directement la question principale de recherche sur la prévisibilité du marché des sukuk. Ils montrent également que le choix du modèle optimal dépend du critère d'évaluation utilisé : le modèle HAR est le plus performant en termes de précision numérique, les modèles LSTM et ANN sont les plus efficaces pour le suivi visuel, tandis que le modèle GARCH reste le moins performant et devrait être évité.

Ces résultats fournissent des orientations pratiques aux institutions financières, aux régulateurs et aux investisseurs pour la gestion de la volatilité et le choix des outils de prévision dans la finance islamique.

Mots-clés : Finance islamique, Sukuk, Volatilité, Prévision, LSTM, Volatilité réalisée

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List of Abbreviations:

ACF:	Autocorrelation Function
ADF:	Augmented Dickey-Fuller Test
ANN:	Artificial Neural Network
ARIMA:	Autoregressive Integrated Moving Average
DCC:	Dynamic Conditional Correlation
GARCH:	Generalized Autoregressive Conditional Heteroskedasticity
HAR:	Heterogeneous Autoregressive
LSTM:	Long Short-Term Memory
MAE:	Mean Absolute Error
ML:	Machine Learning
MSE:	Mean Squared Error
PACF:	Partial Autocorrelation Function
Prophet:	Forecasting model developed by Meta (Facebook)
RMSE:	Root Mean Squared Error
RV:	Realized Volatility
VAR:	Value at Risk

Chapter 1 : Introduction

1 Research Background and Gap:

Sukuk, often termed Islamic bonds, are certificates representing proportional ownership in tangible assets or ventures, in line with Shariah principles (IFSB, 2009). Since its first sovereign issuance in 2002 by the Malaysian government, which raised $600 million, the sukuk market has expanded significantly, reaching an outstanding value of $1.21 trillion in 2024. By 2033, the sukuk market is expected to reach USD 3.99 trillion (LTD Research and Markets, 2025)

This growth reflects rising demand for asset-backed and Shariah-compliant financing across both developed and emerging markets. Governments and multinational institutions actively issue sukuk to finance various infrastructure projects. The sukuk market spans over 27 currencies, with investor participation from Asia, the Middle East, the United States, and Europe (Abad & Connor, 2024).

Sukuk have been widely recognized for their potential to serve as safe-haven assets and contribute to effective portfolio diversification. Their structure, based on tangible assets and Shariah-compliant principles, reduces exposure to interest rate risk and speculative behaviour. Empirical studies have shown distinct return behaviour of sukuk compared to traditional bonds, which highlights their unique role in investment strategies. This difference is reflected in the fact that sukuk display lower volatility and weak correlation with conventional financial instruments, especially during periods of market turbulence (Pirgaip et al., 2021; Shahzad et al., 2019). Sukuk also show different behaviour regarding other traditional stock market instruments, with distinct responses to global shocks and market cycles, making them valuable for multi-asset diversification strategies (Asl & Rashidi, 2021; Saiti et al., 2014). Incorporating sukuk into investment portfolios has been found to enhance risk-adjusted returns and reduce overall risk exposure (Danila, 2023; Öner, 2022).

Despite the growing attention to sukuk as alternative financial instruments, and the evidence from numerous studies indicating that sukuk exhibit distinct behaviour compared to traditional stock market instruments, most existing research has focused on comparing their risk-return profiles, diversification benefits, or co-movements with conventional stock and bond markets (Asl & Rashidi, 2021; C. J. Godlewski et al., 2013; Pirgaip et al., 2021; Raei & Cakir, 2007; Rusgianto & Ahmad, 2013; Sclip et al., 2016; Selmi et al., 2015…). However, a critical research gap remains unaddressed, namely the question of whether sukuk markets are predictable. While volatility forecasting has been extensively explored in conventional and Islamic equity markets , sukuk markets have received limited empirical investigation in this context.

Volatility a fundamental concept in financial markets. It is measured by the standard deviation or variance of returns over a specific time period. (Hull, 2000). Accurate volatility estimates are essential for asset pricing, risk management, portfolio allocation, and regulatory supervision (Danielsson, 2011). Moreover, in high-frequency trading and algorithmic strategies, precise short-term volatility predictions can be the difference between profit and loss (Hansen & Lunde, 2011). The rising complexity and interdependence of global markets have further heightened the demand for reliable volatility forecasts, making it a core pillar of modern financial research and practice.

In response, a wide range of volatility forecasting models has been developed, including time-varying econometric models such as ARCH, GARCH, EGARCH, and multivariate extensions, as well as machine learning-based approaches like Random Forests, Support Vector Machines, and deep learning architectures such as LSTM and GRU networks.

The comparative performance of traditional econometric models and machine learning (ML) techniques in forecasting stock market volatility has been extensively studied, yielding mixed results. Traditional models, such as GARCH and its variants, have long been favoured for their ability to capture volatility clustering and leverage effects. For instance, Shaik & Sejpal (2020) compared the performance of Artificial Neural Networks (ANNs) and GARCH (1.1) models in forecasting the volatility of Indian stock market indices, including NIFTY 50, NIFTY Bank, and NIFTY FMCG. Their findings provide evidence that the GARCH (1.1) model outperforms ANNs for all the indices analysed. Mallikarjuna & Rao (2019) also evaluated the forecasting performance of various models, including linear, nonlinear, artificial intelligence, frequency domain, and hybrid approaches, to identify the most suitable model for predicting stock returns in developed, emerging, and frontier markets. They analysed daily stock returns from selected indices across these markets for the period from 2000 to 2018. The results indicated that no single model consistently performed well across all markets. However, traditional linear and nonlinear models were found to deliver more accurate forecasts compared to artificial intelligence and frequency domain models. In addition Nybo (2021) conducted a comparison between the volatility prediction capabilities of ANN and GARCH models for stocks with varying volatility profiles, categorized into low, medium, and high. The study analysed five sectors, which together encompass all stocks in the U.S. stock market from 2005 to 2020. The findings suggest that GARCH models have an advantage over the ANN model for medium and high-volatility assets. Conversely, other studies have demonstrated the superiority of ML models in capturing complex, nonlinear patterns in financial data, for instance Sahiner et al (2023) conducted a comparison between ANN models and traditional econometric models for ten Asian markets, using daily data from 12 September 1994 to 5 March 2018. The findings show that, across the various countries, machine learning algorithms are more effective at capturing dependencies than traditional benchmark models. Liu (2019) also evaluated two modeling techniques for regression tasks and compared them with the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model for financial volatility forecasting. The results showed that machine learning models, specifically LSTM RNNs and v-SVR, outperformed the GARCH model in forecasting volatility for the S&P 500 and AAPL indices. Moreover Bucci (2020) compared the performance of neural networks against conventional econometric models to forecast realized volatility in financial markets, using monthly data of the Standard & Poors (S&P) index, From February1950 up to December 2017. The results revealed that neural networks, particularly recurrent ones like LSTM and NARX, outperform traditional econometric methods, especially during highly volatile periods. Despite the growing body of research comparing forecasting models in conventional financial markets, their predictive performance remains largely unexplored in the context of sukuk. Existing studies have primarily focused on equities, bonds, or broad market indices, overlooking the distinct structural and behavioural characteristics of sukuk instruments. Given that sukuk differ from conventional assets in terms of risk-sharing principles, asset-backing, and compliance with Shariah law, their volatility patterns may not follow the same dynamics.

Importantly, many comparative forecasting studies have concluded that there is no universally superior model, performance often varies by market type, data frequency, and asset structure

(Mallikarjuna & Rao, 2019; Nybo, 2021; Shahzad et al., 2019). This further supports the need to assess forecasting models specifically within sukuk markets, where no such evaluation currently exists. Addressing this gap is essential to determine whether traditional econometric models, machine learning techniques, or hybrid approaches offer more reliable volatility predictions in this unique financial segment.

2 Research questions:

Based on what is discussed in the research context and gap, we ask the following research question:

To what extent are sukuk markets predictable, and how effectively can forecasting models capture their volatility across short, medium, and long-term horizons?

To address the main research question more precisely, the study considers the following sub-questions:

1. Do sukuk markets exhibit volatility patterns that suggest predictability?
2. How do traditional econometric models perform in forecasting sukuk volatility across different time horizons?
3. How do machine learning models perform in forecasting sukuk volatility across the same horizons?
4. Which forecasting models provide the most accurate predictions for sukuk volatility over short, medium, and long-term periods?

Based on the research question and the issues outlined earlier, the following hypotheses are formulated to guide the empirical investigation:

- **H1:** Sukuk market volatility exhibits predictable patterns that can be effectively captured by forecasting models across short, medium, and long-term horizons.
- **H2**: Econometric models (such as GARCH) provide more accurate short-term volatility forecasts for sukuk markets compared to machine learning models.
- **H3:** Machine learning models (such as LSTM and ANN) outperform traditional econometric models in forecasting sukuk market volatility over medium and long-term horizons.
- **H4**: No single forecasting model consistently outperforms others across all forecasting horizons.

3 Research Objectives:

This research objectives involves:

- Contribute to theoretical knowledge by analyzing the key features of sukuk instruments, exploring traditional and machine learning models used in stock market volatility forecasting, and reviewing existing literature on sukuk volatility and model performance in conventional, Islamic, and sukuk financial markets.
- Examine the volatility behaviour of sukuk markets.
- Assess the predictability of sukuk markets.
- Apply a set of suitable traditional and machine learning models to forecast sukuk market volatility.
- Evaluate the forecasting performance of each model across short, medium, and long-term horizons.
- Compare forecasting performance within and between traditional and machine learning models across the three-time horizons.

Analyze the strengths and limitations of each forecasting model in the context of sukuk markets over short-, medium-, and long-term periods.

4 Research Significance:

The significance of this research lies in its academic contribution, practical financial application, regulatory importance, and its relevance to the development of Algeria’s Islamic finance sector.

- **Academic significance:**

This research addresses two key gaps in the academic literature. First, while sukuk have been studied in terms of their risk-return profiles, diversification benefits, and co-movements with other markets, their volatility behaviour and predictability have not been empirically investigated. Second, there is a lack of studies that examine the forecasting performance of traditional and machine learning models specifically within the sukuk context. This study responds to both gaps by applying a range of forecasting models across short, medium, and long-term horizons, offering one of the first studies focusing on the empirical assessments of sukuk market volatility and model performance. As a result, the findings will contribute to the development of financial forecasting research within Islamic finance and provide a reference point for future theoretical and applied studies.

- **Financial relevance:**

This study could be beneficial to financial institutions, investment managers, financial engineers, and analysts involved in Islamic finance by providing practical insights into which forecasting models perform more effectively across different time horizons, thus supporting informed decision-making in portfolio construction, asset allocation, risk management, and return forecasting within sukuk-based investment strategies.

- **Regulatory relevance:**

Such research offers critical insights for regulatory authorities and supervisory bodies engaged in shaping the future of Islamic financial markets. Insights into the predictability of sukuk markets and the effectiveness of forecasting models are essential for designing reliable monitoring systems, anticipating market risks, and implementing regulatory frameworks that uphold Shariah principles while ensuring financial stability.

- **Local relevance (Algeria):**

This research is particularly relevant for Algeria, where the 2025 Finance Law authorizes the issuance of sovereign sukuk on the Algiers Stock Exchange. As the government moves toward implementation with the support of the Islamic Development Bank (Algérie Presse service, 2024), there is a critical need for research that can inform policy, guide institutional decisions, and support market development. This study provides timely and practical insights into sukuk volatility behaviour and forecasting model performance. Its findings can assist Algerian policymakers, financial institutions, and institutional investors in managing volatility risk, selecting appropriate forecasting tools, and building market credibility and investor confidence as the country establishes its domestic sukuk market.

5 Research Motivation:

Firstly, we conducted this study in response to the rapid expansion of Islamic finance, with sukuk becoming a key instrument in global financial markets. Sukuk, as Shariah-compliant financial products, have garnered significant attention due to their unique structure, offering diversification and stability for investors. Many studies have suggested that sukuk markets remain largely unaffected by financial crises, which positions them as a stable and attractive alternative to traditional financial instruments, especially in emerging markets.

Additionally, we observed in 2022 that Algeria is preparing to launch its own sukuk issuance (kimouche, 2022), further emphasizing the relevance and potential of these instruments within the region. Given this context, we recognize the importance of understanding sukuk market volatility, particularly through effective forecasting. Accurate volatility forecasts are critical for informed decision-making in financial markets, as they directly impact asset pricing, risk management, and investment strategies.

However, we have identified two important gaps in the existing literature. First, the predictability of sukuk market volatility has not been sufficiently explored, especially in comparison to conventional financial markets. Second, while various forecasting models have been evaluated in other financial contexts, there is limited research on the effectiveness of these models in forecasting sukuk market volatility.

We believe that both traditional econometric models and emerging machine learning techniques may offer varying levels of forecasting accuracy, and it is crucial to assess their performance within the specific context of sukuk markets.

This study seeks to address these gaps by investigating the predictability of sukuk market volatility and evaluating the forecasting accuracy of different models across multiple time horizons. By doing so, we aim to provide valuable insights for academics, practitioners, and policymakers, supporting more informed strategies in sukuk investment and contributing to the ongoing development of Islamic financial markets.

6 Research structure:

This thesis involves both empirical and theoretical chapters. The theoretical chapters lay the groundwork by explaining key concepts, models, and background information that are essential for understanding the empirical analysis that follows. These chapters help the reader become familiar with the fundamental ideas and frameworks before diving into the empirical study. The structure of this dissertation is organized as follows:

Chapter 2: Literature Review

this chapter provides a comprehensive review of the existing body of knowledge on volatility forecasting in financial markets. This chapter is divided into four key sections. The first section, Review on the Volatility of Sukuk Markets, explores the unique volatility behaviour of sukuk markets, comparing it to both conventional and Islamic equity markets. The second section, Assessment of Forecasting Models for Conventional Stock Market Volatility, examines traditional econometric models like GARCH, as well as machine learning approaches such as ANN and LSTM, and their application to conventional stock markets. The third section, Assessment of Forecasting Models for Islamic Stock Markets, investigates how forecasting models have performed within Islamic equity markets, providing context for their application to sukuk. Finally, the Assessment of Forecasting Models for Sukuk Markets discusses the limited research available on sukuk volatility forecasting and identifies the gaps this study aims to address.

Chapter 3: Overview of Sukuk (Islamic Bonds)

here we introduce the concept of sukuk, beginning with a clear definition and historical context to establish a foundation for understanding these instruments. The chapter traces the origins of sukuk, detailing its evolution and key features, and contrasts sukuk with conventional bonds. It also covers various sukuk structures, their underlying principles, and their benefits to both investors and issuers. Additionally, this chapter examines the challenges and opportunities facing the sukuk market, including regulatory frameworks, standardization, and the need for greater investor education.

Chapter 4: Univariate Time Series Forecasting for Stock Market Volatility (Models and Performance Evaluation):

this chapter explores the models used for univariate time series forecasting, focusing specifically on their application to stock market volatility and their potential relevance to sukuk markets.

The chapter is divided into three sections. The first, Traditional Statistical Models for Univariate Time Series Forecasting, reviews econometric models such as ARIMA and GARCH. The second, Machine Learning Models for Univariate Time Series Forecasting, delves into advanced machine learning approaches like ANN, Random Forests, and LSTM, highlighting their strengths and limitations. The third section, Performance Evaluation of Univariate Time Series Forecasting Models, compares the forecasting performance of these models using statistical metrics.

Chapter 5: Methodology

This chapter outlines the research process and methodology used in this study, it begins by detailing the steps taken to conduct the research, including model selection, data collection, and analysis methods. The second section, Data Description, provides a comprehensive overview of the data sources, including the sukuk market data, stock market data, and any relevant financial indices used in the study.

Chapter 6: Empirical Results and Analysis

This chapter presents the empirical findings from the forecasting models applied to sukuk market volatility. The first stage focused on examining the predictability of volatility through statistical tests and preliminary analysis. The second stage reports the results of the forecasting models, organized by short-, medium-, and long-term horizons. Each model's performance is evaluated using multiple accuracy metrics. The chapter offers a detailed analysis of forecasting accuracy and highlights which models demonstrate the strongest predictive capabilities for sukuk market volatility.

Chapter 7: Discussion

The discussion chapter groups the empirical results to provide an overall assessment of the sukuk market's predictability and to evaluate the relative performance of each forecasting model. It examines how models performed across all horizons, identifies consistent trends, and explains the factors driving model effectiveness or failure. This chapter brings together the key findings to offer a clear judgment on which models are most suitable for forecasting sukuk volatility.

Chapter 8: Conclusion

This chapter summarizes the key findings of the study and addresses the main research questions and hypotheses. It provides a final assessment of the predictability of sukuk market volatility, and the effectiveness of the forecasting models used. The chapter concludes with recommendations for stakeholders in the sukuk market, including investors, financial institutions, and regulators, and proposes suggestions for future research to further explore sukuk market dynamics.

Chapter 2 : Literature Review

This chapter provides a structured review of volatility forecasting research relevant to sukuk markets by critically examining the methodologies, findings, and limitations of existing studies. It begins by exploring the volatility behaviour and risk characteristics of sukuk instruments, highlighting their structural differences from conventional bonds.

To provide a basis for choosing appropriate models, the chapter then assesses forecasting models applied in conventional stock markets. Given the similarities between sukuk and Islamic equity instruments in terms of Shariah compliance and volatility patterns, it also evaluates forecasting models applied in Islamic stock markets. This broader perspective helps identify techniques that may be transferable to sukuk forecasting. The final section focuses specifically on forecasting models used in sukuk markets, summarizing the performance of both statistical and machine learning approaches. Together, these sections provide the empirical and methodological basis for selecting and evaluating models tailored to the unique dynamics of sukuk market volatility.

1 Review on the volatility of Sukuk markets:

Sukuk are defined by the Accounting and Auditing Organization for Islamic Financial Institutions as “Certificates of equal value representing undivided shares in ownership of tangible assets, usufructs, and services, or in the ownership of the assets of projects or special investment activities” AAOIFI (2003). Unlike conventional bonds, which represent debt obligations. This structure ensures that returns come from profit-sharing or rental income, adhering to Islamic principles that prohibit interest (Riba). The sukuk market has witnessed significant growth and has become a viable investment option. Moreover, Sukuk are likely to continue getting interest from major institutional investors across the U.S., Europe, and Asia Jason Giordano (2022).

Many studies suggest that Sukuk exhibit distinctive characteristics in comparison to traditional bonds and shares, particularly during financial crisis. For instance, Raei & Cakir (2007) evaluated the impact of Sukuk on investment portfolios using the Value-at-Risk (VaR) framework, The study focuses on sovereign issuances from Malaysia, Pakistan, Qatar and Bahrain, comparing the secondary market behaviours of Sukuk and conventional bonds issued by these governments, The dataset varies, as each instrument covers a different time span from its issuance date up to 2007. The finding proven that Sukuk are fundamentally different from conventional bonds, as indicated by their distinct price behaviours in secondary markets, consequently adding Sukuk in an investment portfolio can significantly diminish the portfolio's Value-at-Risk (VaR) in comparison to a strategy that solely relies on traditional bonds.

Rahman et al (2013) examined the volatility of Malaysian sukuk spreads using GARCH-type models and daily data spanning from August 1, 2005, to December 31, 2011, comprising 1,675 observations. Sukuk spreads were derived from the difference between sukuk yields and corresponding government bond yields. The study incorporated a crisis-related dummy variable to capture the potential impact of the 2007–2008 global financial crisis. The results showed that this variable was not statistically significant in any of the GARCH model specifications, suggesting that the financial crisis did not have a direct effect on sukuk spread volatility during the sample period. However, stock market volatility was found to have a significant effect on lower-rated sukuk spreads, indicating that these instruments are more responsive to broader market fluctuations, while higher-rated sukuk remained comparatively stable.

Rusgianto & Ahmad (2013) analysed sukuk market volatility using the EGARCH model and consider the effect of structural breaks. Their study is based on daily data from the Dow Jones Citigroup Sukuk Index (DJCSI) covering the period from 2007 to 2011. The results show that positive shocks cause greater volatility than negative ones, indicating asymmetric volatility behaviour in the sukuk market.

Selmi et al. (2015) analysed the impact of asymmetric shocks on volatility in the global sukuk market using EGARCH model . the study relies on daily data from the Dow Jones Sukuk Index (IDJS) covering the period June 9, 2009, to December 31, 2013, with a total of 1,184 observations. The study finds that structural changes due to stock market shocks significantly influence volatility patterns. the results also confirm that volatility reactions are stronger for positive shocks than negative ones.

Paltrinieri et al. (2015) investigated the role of sukuk in portfolio diversification and evaluated their performance alongside with other asset classes. Using daily data from Bloomberg Professional Service indexes for the period January 1, 2010, to December 31, 2013, the study employed asset pricing measures combined with advanced econometric methods, including ARMA, VAR, VECM, and cointegration analysis. The findings show that sukuk returns are positively skewed and leptokurtic, with a risk-return profile similar to high-yield bonds. Importantly, the results indicate that sukuk provide meaningful diversification benefits, particularly during periods of financial instability.

Sclip et al. (2016) explored the volatility and co-movements between sukuk (Islamic bonds) and international stock indexes, particularly during financial crises. The study employs symmetric multivariate GARCH models with dynamic conditional correlations (DCC) to analyse data from 2010-2014, using daily data for 68 highly liquid sukuk listed in major Islamic bond markets, for instance Bursa Malaysia, Nasdaq Dubai, and the London Stock Exchange. The results suggest that sukuk can provide diversification benefits in equity portfolios due to their reduced volatility in contrast to equities.

Balcilar et al. (2016) explored diversification advantages of Sukuk for investors in conventional stock markets, by analysing the correlations between sukuk and stock markets. The methodology includes a Markov regime-switching GARCH model to analyse risk spillover effects. The study utilises weekly return series for various indices, including sukuk stocks and bond indices, for the period 2nd January 2006 up to 19th December 2014. The study finds that Sukuk exhibit a distinct pattern in the spread of global market shocks compared to emerging market bonds, with low correlations between sukuk and global stock markets, suggesting sukuk's potential as a safe haven.

Aldhaheri. (2017) examined market risk differences between sukuk and conventional bonds using the Value-at-Risk (VaR) approach. The analysis was based on daily prices and returns from seven corporate issuers (BLME, MAF, Petronas, Rasmala, Tamweel Dubai, and DP World) and two benchmarks indices (Dow Jones Sukuk Index and the Dow Jones Corporate Bond Index). The instruments covered varied time periods. The results indicate that sukuk returns are statistically non-normal, characterized by heavier tails and tighter clustering around the mean. Additionally, the study finds that sukuk instruments exhibit less VarR values in most cases, implying that sukuk are generally less exposed to market risk. The findings further highlight sukuk's potential contribution to portfolio diversification.

Roukiane & Marzouki. (2018) compared the volatility patterns of Sukuk and conventional bonds across different maturities using various statistical tests, including the Student, GARCH, EGARCH and Granger causality test. The study employed daily closing price data from five Sukuk indices and five conventional indices within the Dow Jones index family, covering the timeframe from 1st January 2014 up to 25th April 2017.Findings indicate that Sukuk are perceived as being less risky and exhibiting lower volatility compared to traditional bonds, especially sukuk with maturities exceeding five years.

Driss & Saïd (2020) compared the volatility of Sukuk indices with that of conventional bonds, particularly in a post-crisis context, employing various statistical tests. The dataset enters Thomsons Reuters government Sukuk and Bond indices as proxies to represent the Malaysian capital market, from January 6, 2010, to March 20, 2017, with a total of 1774 observations. The research concludes that the governmental Sukuk index exhibits slightly lower volatility in comparison to the governmental bond index in the long term, eventually the results revealed that negative news has a more pronounced effect on volatility in the Sukuk market compared to positive news.

Aslam et al. (2022) examined the volatility behaviour of sukuk and conventional bonds in the Pakistani market using ARCH and GARCH models. The analysis relied on daily return data for 10-year maturity WAPDA Sukuk and Pakistan Investment Bonds, covering the period from November 12, 2009, to November 12, 2014. The findings indicate that sukuk exhibit lower volatility and greater stability compared to conventional bonds, supporting their potential as effective instruments for investment diversification.

Danila (2023) examined the impact of the COVID-19 pandemic on the bonds and sukuk market indices. using GJR-GARCH model and Expected Shortfall (ES) method to analyze the associated volatility, the dataset comprises the daily data from of five emerging markets (Malaysia, Indonesia, Saudi Arabia, Qatar, and Turkey) from January 2020 to June 2021. The findings considered that Sukuk are generally considered to carry lower levels of risk compared to traditional bonds, with no significant difference in risk before and during the pandemic.

(Nur, 2024) investigated the behaviour of sukuk investments within the context of a volatile global economy by analyzing findings from previous studies. The analysis suggests that sukuk markets tend to exhibit greater resilience than conventional bonds, particularly during periods of economic instability. However, the study also highlights that sukuk are still exposed to various sources of volatility, including global economic developments, geopolitical uncertainty, interest rate movements, and changes in regulatory frameworks.

Overall, these studies confirm that sukuk behave differently from conventional bonds in terms of volatility, particularly during periods of financial stress. The empirical evidence consistently indicates that sukuk exhibit lower or more stable volatility, respond asymmetrically to market shocks, and provide meaningful diversification benefits. These features support the inclusion of sukuk in multi-asset portfolios and underline their importance for investors seeking to manage risk more effectively.

2 Assessment of Forecasting Models for conventional Stock Market Volatility:

Volatility refers to the degree of fluctuations in the price of a financial instrument over time De Silva et al (2017). In finance, volatility is commonly interpreted as a measure of risk, as it indicates the degree of uncertainty or unpredictability in an asset's price movements Hull (2018). often measured as standard deviation, which quantifies the extent to which data points deviate from the mean Bodie et al (2017). Volatility forecasting a crucial aspect in financial market analysis has been an active area of research in recent decades Andersen et al (2010). To enhance the accuracy and reliability of volatility forecasting, Different methodologies have been employed, including traditional statistical models and modern machine learning techniques.

For instance, Brailsford and Faff (1996) examined the effectiveness of various models for predicting stock market volatility, including the Random Walk model, Historical Mean model, Moving Average models, Exponential Smoothing model, Exponentially Weighted Moving Average model, Simple Regression model, standard GARCH models, and GJR-GARCH models, using four metrics (ME, MAE, RMSE, MAPE). The study analysed the daily data from the State-Actuaries Accumulation Australian Index, spanning the time period from 1st January 1974 through 30th June 1993. The outcomes indicate that both the ARCH models and a simple regression model performed well in terms of volatility prediction. Although the accuracy of these forecasts was highly sensitive to the error statistic employed to assess them. The paper concluded that forecasting volatility is inherently challenging and suggested that no single method is consistently the best in every situation.

Yu. (2002) also assessed the performance of nine different models for predicting stock markets, including ARCH, GARCH, and a stochastic volatility (SV) model using four metrics (RMSE, MAE, Theil-U, LINEX). The dataset comprises 4741 daily returns data of the New Zealand Stock Market Exchange (NZSE), from 1 January 1980 to 31 December 1998. The study found that the SV model is the best performing model, followed by the GARCH (3,2) model. whereas simple models like random walk and exponential smoothing present low accuracy. The study emphasized the significance of selecting appropriate models based on market-specific characteristics and evaluation criteria. It also highlighted the inherent variability in model performance based on the chosen model form and evaluation measures.

Brooks & Persand. (2003) investigated the accuracy of a range of statistical models for predicting the daily volatility of several key UK stock market indices, by comparing the out-of-sample forecasting efficacy of linear and GARCH-type models, alongside a multivariate approach. The accuracy performance is assessed using a set of traditional metrics, such as mean squared error. The study concludes that while no single model is universally superior, combining multiple approaches can enhance forecasting accuracy and improve risk management strategies.

Gabriel. (2012) assessed the accuracy of various GARCH-type models for predicting the volatility of the Romanian stock market (BET index), from 2001 up to 2012. The study employed multiple models, including GARCH, IGARCH, TGARCH, EGARCH, and PGARCH, under Various error distribution form (normal, Student-t, and GED). The results suggested that the TGARCH model excelled in predicting volatility, particularly when using the GED and Student-t distributions.

Kosapattarapim C. (2012) additionally compared the capability of different GARCH models for predicting stock market volatility in Thailand (SET), Malaysia (KLCI), and Singapore (STI). The study employed six GARCH (p, q) models with normal, skewed normal, Student-t, skewed Student-t, generalized error distribution (GED), and skewed GED error distributions. The optimal model was determined utilizing the Akaike Information Criterion (AIC), and its out-of-sample volatility forecasting performance was compared with other models. The findings revealed that models exhibiting error distributions that deviate from the normal distribution generally provided better volatility forecasts, and the skewed error distributions performed particularly well. Ultimately, although the optimal model did not consistently produce most accurate forecast, the deviations were negligible, indicating that the optimal model remains a dependable choice for forecasting volatility.

Lee S. K. (2017) examined the potency of four stock markets volatility forecasting models (Exponential Weighted Moving Average (EWMA), Autoregressive Integrated Moving Average (ARIMA), and Generalized Auto-Regressive Conditional Heteroscedastic (GARCH)), across four stock markets: Indonesia, Malaysia, Japan and Hong Kong. Using monthly closing index prices from 1st January 1998 up to 31 st December 2015, the findings challenge the common assumption that developed markets always exhibit lower volatility compared to emerging markets. Out of the three models assessed, the GARCH (1,1) model performed best for the stock markets in Malaysia, Indonesia, and Japan, while the EWMA model showed the highest accuracy for the Hong Kong stock market. The study emphasized the necessity of understanding market-specific dynamics to choose the most appropriate forecasting model.

Mallikarjuna M. (2019) evaluated five different forecasting models across developed, emerging, and frontier markets over the period 2000 up to 2018, using five different models: (ARIMA, SETAR, ANN, SSA and hybrid model). The findings revealed that no singular model demonstrated consistent performance across all markets. However, non-linear models, particularly SETAR, demonstrated better forecasting performance in 10 out of 24 markets studied. Traditional linear models like ARIMA were effective in 7 markets, while hybrid models were optimal for 5 markets. The study concluded that despite progression in AI and frequency domain models, traditional statistical models still provide reliable forecasts for stock market returns. suggests a need for hybrid approaches to leveraging the strengths of both traditional and modern models, emphasizing the importance of model selection based on specific market conditions and data characteristics.

Islam M. R. (2020) conducted a study comparing the predictive accuracy of ARIMA, ANN, and the Geometric Brownian Motion (GBM) models for predicting stock prices. The analysis used historical data for the S&P 500 index, sourced from Yahoo Finance. The results revealed that traditional statistical models, such as ARIMA and GBM, outperformed the neural network model ANN for predicting stock prices on a next-day basis. While the authors acknowledged ANN's potential, they advised caution in relying on it for short-term forecasts, suggesting instead that ARIMA and GBM provide more stable predictions. They also recommended exploring further ways to enhance ANN's performance in this context.

Bucci. (2020) attempted to explore the performance of neural networks to forecast realized volatility in financial markets, by utilizing various neural network architectures including feed-forward and recurrent neural networks (RNN), with a particular focus on the recently developed Long short-term memory (LSTM) network and NARX network, then contrasting their performance against conventional econometric models. Subsequently contrasting their performance with conventional econometric models. The dataset utilized in this research comprises monthly data of the Standard & Poors (S&P) index, From February1950 up to December 2017, encompassing 815 observations. The relative accuracy of performance forecasting was evaluated using mean squared error (MSE), quasi-likelihood (QLIKE), Model Confidence Set (MCS), and Diebold-Mariano (DM) Test. The results revealed that neural networks, particularly recurrent ones like LSTM and NARX, outperform traditional econometric methods in predicting realized volatility, especially during highly volatile periods.

Patil et al. (2024) compared traditional time series forecasting methods (ARIMA, SARIMA and SARIMAX) with advanced deep learning models (DF-RNN, DSSM, and Deep AR) in predicting stock indices, using the daily closing price of NIFTY-50 index from Jan 2011 to Feb 2022. Metrics such as Mean Squared Error (MSE), Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE), Percentage of Correctly Identified Data (POCID), and Theil's U were used for evaluation. The findings indicated that Deep AR surpassed all other models in performance with the lowest MAPE of 0.01 and RMSE of 189. The performance of Deep AR and GRU models did not degrade significantly with reduced training data, indicating their robustness. This study highlighted the strength of deep learning models in handling complex datasets and suggested their potential applications across various domains like weather forecasting and other real-world scenarios.

Lu (2024) evaluated the performance of linear models, random forest and long short-term memory networks (LSTM) in predicting stock prices. using daily IBM stock data from 2010 till 2018. The findings revealed that the LSTM model, which captures long-term dependencies, showed reasonable consistency but some overfitting. The linear model performed well on training data but less so on test data, indicating limitations in handling non-linear stock price movements. Concerning the random forest model exhibited severe overfitting, performing well on training data but performing inadequately on tested data. The study concluded that while each model has its strengths and weaknesses, no single model guarantees optimal performance. Combining multiple models or enhancing feature selection may improve prediction accuracy.

Ajiga et al. (2024) provided a comprehensive review of models for stock market forecasting. First the review highlighted the limitations of traditional time-series forecasting models like

ARIMA and MACD in capturing complex financial data patterns. Subsequently they explored advanced ML models (strengths and weaknesses), including support vector machines (SVM), artificial neural networks (ANN), ensemble models like random forests and gradient boosting. The study underscores the crucial significance of feature curation and manipulation in amplifying the predictive efficacy of ML models, incorporating financial metrics, macroeconomic factors, sentiment assessment from news and social platforms, and technical indicators. The review concluded that ML models hold significant potential for enhancing forecasting accuracy, though challenges such as overfitting persist. The incorporation of alternative data sources and feature sets is crucial for developing robust and accurate predictive models.

In conclusion, the literature on the Assessment of Forecasting Models for conventional Stock Market Volatility reveals a diverse range of methodologies and models, each with their own strengths and limitations. Traditional models like ARCH, GARCH, and ARIMA continue to provide reliable forecasts in various contexts, though their performance is extremely sensitive to market-specific conditions and evaluation criteria. Advanced machine learning models, including deep learning and ensemble methods, offer significant potential for enhancing forecasting accuracy, although challenges like overfitting and the need for extensive feature engineering persist. Comparative studies emphasise the importance of context-specific model selection and the potential benefits of hybrid approaches that integrate traditional statistical methods with contemporary machine learning techniques.

3 Assessment of Forecasting Models for Islamic Stock Markets:

Islamic finance, governed by Shariah principles, has gained considerable global traction due to its ethical framework, which prohibits activities involving excessive risk (gharar), interest (Riba) and speculative trading. This growing popularity emphasises the importance of developing specialized volatility models able to deal with the distinct characteristics of Islamic financial instruments. Indeed, Islamic finance, unlike conventional finance, emphasises risk-sharing, asset-backed transactions, and adherence to ethical investment principles, which necessitates a unique approach to volatility forecasting, providing accurate predictions while aligning with the ethical mandates of Islamic finance.

In this context Omer G. S. (2014) investigated the conditional volatility and correlations of the Dow Jones Islamic Market Index focusing on Multivariate approach GARCH-DCC model. The dataset involved daily data from January 2003 to March 2013. The results reveal that the t-distribution model is considered more suitable for capturing the non-normal distribution nature of stock returns. The findings also suggest that the conditional volatilities of all indices display a high degree of correlation, particularly during periods of financial crises such as the 2008 US financial crisis. Additionally, the conditional correlations among the indices exhibit temporal variability rather than remaining constant, highlighting the importance of monitoring these correlations for effective portfolio diversification and risk mitigation. Eventually, the study concludes that Islamic indices, such as the Dow Jones Islamic UK and Dow Jones Islamic Market, exhibit lower volatility in comparison to their conventional counterparts, making them attractive options for risk-averse investors.

Ben Nasr et al (2016) employed advanced statistical models including GARCH, FIGARCH, FITVGARCH and MSM to predict the movements of the Dow Jones Islamic Market World Index (DJIM). Their analysis reveals that the DJIM shares similar statistical properties with traditional asset classes, displaying characteristics like volatility clustering and long memory. Among the models tested, the MSM model demonstrates superior forecasting performance across most time horizons. Ultimately, the study finds that incorporating long memory and regime-switching dynamics significantly improves volatility forecasts, with the MSM model outperforming GARCH-type models, especially over longer forecast horizons.

Rizwan A. (2018) applied ARIMA and GARCH models to predict the volatility of Pakistan's Karachi Meezan Index (KMI-30), an Islamic stock index, using daily data from September 2008 up May 2012. The study demonstrates the potency of ARIMA model in predicting the volatility of the KMI-30, which is crucial for investors looking to align their investments with Shariah principles. The findings indicate also that proper modelling of Islamic stock indices is essential for accurate forecasting and effective risk management.

Aridi N. A. (2018) focused on estimating Value-at-Risk (VaR) for Malaysia's conventional and Islamic stock markets using GARCH, TGARCH and CGARCH models. Their findings indicate that the TGARCH (1,1) model with a normal distribution offers the best fit. The study emphasizes the significance of choosing appropriate GARCH volatility models for effective risk management in Islamic financial markets.

Rahahleh et al. (2018) assessed the forecasting performance of various GARCH-class models including the APARCH model for tow Saudi stock market indices, the Tadawul All Share Index (TASI) and the Tadawul Industrial Petrochemical Industries Share Index (TIPISI). Using daily price data from September 2007 up to February 2015. The results suggest that the APARCH model is the optimal model for predicting volatility in the TASI and TIPISI indices. Moreover, the findings suggest that the APARCH model demonstrates superior accuracy in predicting volatility in the TASI and TIPISI indices.

Burhanuddin (2020) investigated the volatility behaviour of four major Islamic stock indices using various GARCH-type models, including GARCH, GARCH-M, TGARCH, EGARCH, and PGARCH. Applying the maximum likelihood Technique to estimate the coefficients of the GARCH models under the assumption of a Gaussian distribution, using forecasting errors – root mean square errors (RMSE) and mean absolute errors (MAE) as metrics to assess the forecasting performance. The findings indicate that asymmetric Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models surpass symmetric models in predicting volatility, though no differences are minor.

Chkili & Hamdi (2021) developed a new hybrid GARCH-ANN model to improve the forecasting accuracy of the Islamic stock market. The study employs a dataset containing daily prices from the Dow Jones Islamic Market World Index from June 1999 to December 2016, the study reveals that the proposed hybrid model, the Fractionally Integrated Asymmetric Power ARCH Artificial Neural Network (FIAPARCH-ANN), outperforms all other models in terms of forecasting performance.

Aslam et al. (2021) developed a model to forecast Islamic securities indices based on artificial neural networks. Using daily closing price data for the KMI-30 index from August 2009 to October 2019, with a total of 2520 observations. The results demonstrate that the model excels in terms of trend detection, while it is weak in terms of fluctuation detection.

Alenezy et al. (2023) presented a hybrid model to forecast the Saudi Arabian stock market, using the MODWT spectral model combined with FS.HGD and HyFIS learning approaches, where the MODWT decomposes stock market data into different scales and time frames, the FS.HGD updates model parameters iteratively, while HyFIS integrates neural networks and fuzzy logic for robust learning. Subsequently, a comparative examination was conducted between the best-performing models (MODWT-HyFIS, MODWT-FS.HGD) and alternative functions as well as traditional models (ARIMA, HyFIS, and FS.HGD). The dataset comprises the daily closing values of the Tadawul stock market during the period spanning from August of 2011 to December of 2019, with oil prices and repo rates as input variables. Using three evaluations metrics to assess the forecasting accuracy of the models: Mean Squared Error (MSE), Mean Average Error (MAE) & Mean Absolute Percentage Error (MAPE). The findings suggest that the MODWT-LA8-FS.HGD model surpasses traditional models, showing reduced RMSE, MAE, and MAPE, indicating better forecasting ability.

Ledhem and Moussaoui (2024) investigated the use of data mining techniques such as AdaBoost, K-nearest neighbors, random forest, and artificial neural networks to enhance daily precision forecasting of the Jakarta Islamic Index (JKII). The study employs big data mining techniques to forecast daily precision improvement, using big data with symmetric volatility as inputs and closing prices of JKII as target outputs. The research evaluates prediction performance based on four metrics: the mean absolute error, the mean squared error, the root mean squared error, and R-squared. The findings indicate that the AdaBoost technique provides the optimal predictive accuracy with the minimal margin of forecasting errors, followed by the random forest technique.

In conclusion, these studies collectively underscore the importance of employing advanced volatility models tailored to the unique features of Islamic finance. The integration of advanced statistical models and machine learning techniques significantly improves forecasting accuracy, providing a solid foundation for risk management and investment strategies in Islamic finance. Finally, continued research in this area is vital to refine these models and adapt to the evolving landscape of global capital markets.

4 Assessment of Forecasting Models for sukuk Markets:

Accurate forecasting of sukuk prices and volatility is essential for informed decision-making in various financial contexts. By providing insights into future price movements and risk levels, these forecasts enable investors to optimize portfolio allocation, risk managers to implement effective hedging strategies, and regulators to formulate policies that promote market stability and investor protection. This literature review synthesizes findings from numerous research that have evaluated the performance of various statistical and machine learning models to forecast volatility and prices in the sukuk market.

On this basis, Arundina & Omar (2009) investigated factors that influence sukuk ratings with multivariate approach focus, using a Multinomial Logit regression model to examine various variables such as: firm size, leverage ratio, profitability margin, fixed payment coverage, reputation and presence of a warranties to classify sukuk ratings effectively. The study's dataset covers sukuk listed in Malysia stock market. The model demonstrated 80% accuracy in classifying sukuk ratings.

Rahman et al. (2013) examined the volatility of sukuk spreads using the GARCH modelling approach. The dataset includes daily data of the listed investment grade and non-investment grade sukuk in the Malaysian stock market, spanning the time frame from August 1, 2005 up to December 31,2011, a total of 1675 observations. The study reports that the parameters of ARCH terms are statistically significant. Thus, the GARCH model captures the volatility clustering present in the sukuk data, indicating that the model is well-suited to the data.

Rahim S. A. (2014) examined the effectiveness of a hybrid Bootstrap Moving Centreline Exponential Weighted Moving Average (BMCEWMA) model in forecasting volatility in the sukuk market. The dataset involves the daily closing prices of two indices: the FTSE Bursa Malaysia Emas Shari'ah Index and the FTSE Bursa Malaysia Hijrah Shari'ah Index, focusing on the period between 2008 up to 2011. Their study found that the Bootstrap Moving Centreline Exponential Weighted Moving Average model (BMCEWMA) outperform traditional Moving Centreline Exponential Weighted Moving Average model (MCEWMA). which indicates by incorporating bootstrapping, the model could better handle the intricacies and volatility of sukuk price movements, thus providing a more accurate volatility forecasts with lower error values.

Hafezian et al. (2015) explored the use of symmetric and asymmetric Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models to estimate Value at Risk (VaR) for the sukuk market. The data in this research consisted of three sukuk instruments: (petrol) issued in Malaysia, (RAK) issued in the United Arab Emirate and (IDB) issued by the Islamic Development bank. The results suggest that asymmetric GARCH models, particularly those incorporating student-t distributions, performed better in capturing the volatility dynamics of sukuk prices. The study also underscores the importance of selecting appropriate models for accurate risk assessment in the sukuk market. By identifying the advantages of asymmetric models, the researchers ultimately highlight the necessity of considering non-linear behaviours and fat tails in financial data, which are critical in accurately predicting risk in the sukuk market.

Arundina et al. (2015) compared two models: the Multinomial Logistic Regression and the Neural Network models to forecast sukuk ratings based on multivariate analysis. The study employed a dataset of 317 Sukuk samples, which were selected based on specific criteria regarding their ratings and characteristics. The independent variables include metrics related to size, leverage ratio, liquidity, coverage, earnings, market activity, and specific variables like guaranteed status and Sukuk structure. The study found that the Neural Network model with 96.18 accuracy rate is better at predicting Sukuk ratings than the Multinomial Logistic regression model with 91.72 accuracy rate.

Bousalam and Hamzaoui (2017) explored the existence of long memory in the yield spreads of four Dow Jones Sukuk indices via an Autoregressive Fractionally Integrated Moving-Average (ARFIMA) model. The dataset covered five years, from 1st March 2011 to 1st March 2016, comprising 1,035 observations in total. The findings demonstrate that the ARFIMA models outperform traditional ARIMA models, highlighting the importance of considering long memory in sukuk volatility forecasting.

Alam et al. (2018) employed value at risk with non-parametric and Monte Carlo simulations to estimate potential defaults on Sukuk investments. The dataset comprises Sukuk issued by corporate firms in Malaysia across nine economic sectors, over a period of 16 years from 2000 to 2015. The study demonstrates that Sukuk are a safer option compared to conventional bonds in the Malaysian context. Furthermore, the results indicate that the value-at-risk (VaR) predictions align well with credit rating agencies, suggesting the model's reliability in assessing Sukuk risk.

Hila et al. (2019) combined Moving Average (MA) and Artificial Neural Network (ANN) algorithm in context of sukuk market, then they compare the performance of this hybrid model with traditional MA model. The study sample involve the listed sukuk in in Bursa Saham Malaysia covering the period from 2010 until 2013, with total of 400 observations. Their hybrid MA-ANN model significantly outperformed traditional MA models in terms of Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE), making it a reliable tool for medium-term forecasting. Indeed, this study demonstrated how combining statistical techniques with machine learning algorithms can yield more accurate and robust forecasting models.

Ramadhani & Kaluge (2019) tried to analyze how financial performance indicators can effectively predict sukuk ratings. It uses multiple discriminant analysis (MDA) to evaluate 37 companies with sukuk ratings by PT PEFINDO from 2011-2018. The study finds that financial performance metrics such as leverage (long-term liabilities/total assets), liquidity (current assets/current liabilities), profitability (operating income/sales) and coverage (operating income/total liabilities) form an accurate predictive model with 89.2% accuracy.

Driss & Saïd (2020) compared the volatility between Sukuk and conventional bond indices in a post-crisis context using GARCH models. The dataset uses Thomsons Reuters government Sukuk and Bond indices as proxies to represent the Malaysian capital market. The dataset covers a period from 6th January 2010 to 20th March 2017, resulting in a sum of 1,774 observations. The study reveals that the Government Sukuk index is slightly less volatile than the government bond index in the long run, while the EGARCH (1,1) model is considered the best model to predict the Sukuk markets, as this model captures the asymmetries (leverage) presented in the sukuk time series.

Nurhakim et al. (2020) compares the predictive accuracy of two models: Multinomial Logistic Regression (MLR) and Artificial Neural Networks (ANN). The researchers collected data on Sukuk rated by PEFINDO (Indonesian rating agency) including 63 samples. the finding suggests that the key factors influencing Sukuk ratings are debt levels, profitability and firm size. The study also indicates that ANN outperforms MLR in predictive accuracy, suggesting that ANN may be a more effective tool for forecasting Sukuk ratings in the Indonesian context.

Metlek, S (2022) proposed RNN-based model to forecast Turkish sovereign sukuk prices, subsequently he compared this model performance with that of three artificial neural networks models: Long Short- Term Memory (LSTM), Recurrent Neural Network (RNN) and Radial Basis Function Neural Network (RBFNN). The study dataset comprises 1880 daily observations collected between 30th August 2013 and 25th February 2021, focusing on two Dow Jones Sukuk indices (Dow Jones MENA & Dow Jones GCC). The study demonstrates that the proposed RNN-based model outperforms all other models, followed by the RNN model, then the LSTM model, while the lowest model performance is RBFNN.

Siddiqui T. A. (2023) tried to improve the forecasting accuracy of the MENA Sukuk price using the Prophet model. dataset includes daily data for Standard and Poor’s (S&P) MENA and Standard and Poor’s (S&P) GCC Sukuk price indices, from1st January 2017 to 31st December 2021. To assess the model performance, the study employs three evaluation metrics: RMSE, MAPE and R^2. The lowest RMSE and MAPE values of the model are 0.0191 and 0.0001 respectively, while the highest R^2 score is 0.9999, which demonstrates the Prophet model's capability in providing reliable forecasts for sukuk prices.

In conclusion, the reviewed studies reveal the use of a varied range of models for forecasting volatility and prices in the sukuk market. Traditional statistical models such as GARCH and ARFIMA have proved to be effective, especially when tailored to asymmetries and long memory. However, the integration of artificial intelligence models, such as ANN and Prophet, has significantly enhanced forecasting accuracy. Indeed, the progression from traditional to AI-based models represents a significant jump in the accuracy and reliability of financial forecasts, reflecting the continuous evolution and improvement in forecasting methodologies.

These advancements provide valuable insights for investors, policymakers and financial institutions in mitigating risks and formulating well-informed investment choices within the sukuk market. Nevertheless, the field of sukuk market forecasting remains relatively new and underdeveloped, with a limited number of studies available that specifically address the evaluation of the best model for this purpose. As most existing research tends to focus on the broader Islamic finance sector or on sukuk's legal and economic implications, rather than on sophisticated forecasting methodologies. While there is increasing interest for understanding and predicting the behaviour of sukuk markets, the specific characteristics of these Islamic financial instruments and the complexities of the global economic landscape present significant challenges to stakeholders. This gap highlights the need for targeted studies that explore the best model to forecast sukuk market volatility.

Chapter 3 : Overview of Sukuk (Islamic Bonds)

Sukuk, the second-largest asset class in Islamic finance after Islamic banking, according to the Islamic Financial Service Board Development 2024 report. (Board, 2024)

Indeed, Sukuk markets witnessed an increase of 16% or USD 212 billion. maintaining their position as a major growth driver in the Islamic Financial Services Industry. Sukuk account for 24.94% of the total Islamic Financial Services Industry assets, which are valued at USD 0.842 trillion out of the industry's total worth of USD 3.38 trillion. ((IIFM), 2024)

The central feature of Sukuk is their adherence to Islamic principles, particularly the prohibition of Riba (interest), which is considered exploitative and unjust under Islamic law.(M. T. Usmani, 2002).

In this chapter, we will explore the concept of Sukuk, starting with a clear definition to set the foundation. Following this, we will discuss the historical background of Sukuk, tracing its origins and evolution over time. Subsequently, we will delve into the key features of Sukuk, highlighting how they differ from conventional bonds. We will also discuss the various types of Sukuk structures, their underlying principles, and the benefits they offer to both investors and issuers. Additionally, we will examine the challenges and opportunities facing the Sukuk market, considering factors such as regulatory frameworks, standardization, and investor education.

1 Definition of Sukuk:

Sukuk have become a pivotal tool in Islamic finance, offering ethical and asset-backed investment opportunities for individuals, corporations and governments. To provide a clear definition of Sukuk, it is important to consider various perspectives, below are the predominant definitions from reputable professional bodies and academics that elucidate the core and structure of sukuk:

1.1 Organizational definitions (professional):

Key Sukuk Definitions from Leading Professional Organizations and Authoritative Bodies are as follows:

- **Accounting and Auditing Organization for Islamic Financial Institutions (AAOIFI):**

"Sukuk are certificates of equal value representing undivided shares in ownership of tangible assets, usufruct, and services or (in the ownership of) the assets of particular projects or special investment activity." (Accounting and Auditing Organization for Islamic Financial Institutions (AAOIFI), 2015)

- **World Bank Group:**

"Sukuk are a form of investment in which there must be permissible assets or transactions for which the investment is made. Depending on the structure, sukuk's risk and return characteristics are often similar to bonds or debt securities, hence the common reference to sukuk as a form of debt securities." (Kusuma & Silva, 2014)

- **Islamic Financial Services Board (IFSB):**

Sukuk, often called Islamic bonds, are certificates representing proportional ownership in tangible assets or ventures (such as a muḍārabah) in line with Sharī`ah principles. Key differences from conventional bonds include:

(a) Funds from Sukuk issuance must be invested in specified assets, not general purposes.

(b) Income from Sukuk is tied to the investment's purpose.

(c) Sukuk certificates grant ownership rights over the invested assets, transferred from the originator to Sukuk holders for a fixed period ending at maturity. (ISLAMIC FINANCIAL SERVICES BOARD (IFSB), 2009)

- **Association of Corporate Treasurers (ACT):**

Sukuk are financial products that follow Islamic laws (Sharia) to generate returns similar to traditional bonds. Unlike regular bonds, which represent a loan, sukuk represent an ownership share in an investment. This means holders earn a share of the returns from the investment and get their money back at a set date in the future. (Debashis, 2014)

- **The Securities Commission Malaysia:**

"Sukuk refers to certificates of equal value which evidence undivided ownership or investment in the assets using Shariah principles and concepts endorsed by the Shariah Advisory Council (SAC)." (The Securities Commission Malaysia, 2014)

- **Corporate Finance Institute:**

Sukuk, also known as Islamic bonds or Sharia-compliant bonds, are financial certificates that signify partial ownership in a portfolio of eligible existing or future assets. They can be viewed as the Islamic equivalent of conventional bonds. Unlike traditional bonds, sukuk do not represent a debt obligation. Instead, when sukuk are issued, the issuer sells certificates to investors. The issuer then uses the proceeds to purchase assets, granting investors partial ownership of those assets. Additionally, investors are entitled to a share of the profits generated by the asset. (Corporate Finance Institute, 2024)

- **The Association of Eurasian Central Securities Depositories (AECSD):**

Sukuk, which literally means contracts or promissory notes, are Islamic financial certificates that share many attributes with conventional bonds but adhere to Sharia principles. The issuer of Sukuk sells certificates to a group of investors and uses the proceeds to purchase an asset, granting the investors partial ownership of the asset. Additionally, the issuer makes a contractual promise to buy back the Sukuk at a future date at par value. (Farhad, 2021)

1.2 Academic Definitions of Sukuk:

Based on an analysis of various academic definitions, the following are considered the most accurate descriptions of Sukuk:

(Vishwanath & Azmi, 2009) define sukuk as an Islamic financial certificate similar to a Western bond but structured to comply with Shari'ah (Islamic law). Unlike traditional bonds that generate interest and are not compliant with Shari'ah principles, a sukuk involves the issuer selling a certificate to a group of investors. These investors then lease the underlying asset back to the issuer in exchange for a rental fee that was agreed upon in advance. meanwhile the issuer also commits to repurchasing the sukuk at face value on a specified future date. A key requirement of sukuk is that their returns and cash flows must be directly linked to the performance of specific assets. This is because Shari'ah prohibits trading in debt, therefore financing can only be raised for identifiable assets.

(S. M. T. Usmani, 1998) defines sukuk as an old Islamic jurisprudence concept, that gives this certificate holders proportional ownership of tangible assets within a fund. Unlike instruments representing debt or cash, sukuk certificates are fully tradeable in secondary markets. Indeed, the buyers step into the seller's position, acquiring both the proportional ownership and all rights and duties of the original owner. The sukuks' market value will be influenced by market dynamics basically by their profitability.

(Iqbal & Mirakhor, 2011) define sukuk as participation certificates representing partial ownership of an asset for a specified period, during which the profit and risk from the asset are shared among sukuk holders (investors). Similarly, sukuk are security instruments that provide a predictable level of return as bonds. However, the key distinction is that while Bonds represent pure debt of the issuer, sukuk represent an ownership stake in an asset or project in addition to the risk on the issuer's creditworthiness. subsequently, bonds establish a lender/borrower relationship, whereas the nature of the relationship in sukuk depends on the underlying contract.

(R. Wilson, 2004) defines sukuk as financial instruments often called Islamic bonds, are more accurately described as Islamic investment certificates. This distinction is both significant and essential, since sukuk should not be seen simply as an alternative to conventional interest-based securities. The goal is not to imitate financial products like fixed-rate bonds or floating-rate notes commonly used in the West, but rather to design innovative assets that fully comply with Sharia law.

(Thomas & Thofeek, 2005) define sukuk as Islamic investment certificates, differ from traditional bonds. While bonds are debt obligations requiring the issuer to pay interest and principal, sukuk holders have undivided ownership in the underlying Sharia-compliant assets. They share in the revenues and proceeds generated by these assets. Sukuk certificates are similar to US trust certificates, can be listed and rated, Sukuk certificates can be listed, rated, and their payments often use the same pricing methods as conventional Eurobonds.

(Paltrinieri et al., 2023) define sukuk as investment securities that grant holders ownership rights in specific asset classes, along with their associated risks. These instruments can provide either fixed returns (as in Ijarah or Murabaha sukuk) or variable returns (as in Mudharabah or Musharakah sukuk). All sukuk must meet four essential criteria:

1. Complete absence of interest payments, as required by Islamic Law.
2. Freedom from uncertainty and speculation.
3. No connection to prohibited industries (such as alcohol or pork).

4. Backing by underlying assets that support real economic activity.

(C. Godlewski et al., 2013) define sukuk as securities with features similar to both bonds and stocks, issued by sovereign and corporate entities to finance trade or tangible assets. They provide a regular income stream and a final payment at maturity. Unlike bonds, sukuk are asset-based and must be Sharia-compliant. Sukuk holders might bear asset-related expenses, and their value depends on the creditworthiness of the issuer and the market value of the underlying asset. While sukuk and stocks represent ownership claims, sukuk are linked to specific assets, services, or projects for a defined period, whereas equity shares represent ownership of the entire company with no maturity date.

Based on a synthesis of professional and academic definitions, sukuk can be understood as a unique class of Sharia-compliant financial instruments that combine elements of both bonds and equities. These certificates give investors proportional ownership in underlying assets, services, or specific projects. Each sukuk must be structured according to Islamic principles, which prohibit interest (Riba), excessive uncertainty (Gharar), and investments in prohibited industries. This fundamental requirement distinguishes them from conventional debt-based securities.

Similar to bonds, sukuk come with predetermined maturity dates. However, they operate on a profit-and-loss sharing model rather than paying interest. Returns for sukuk holders are directly tied to the performance of the underlying assets.

Governments, corporations, and individuals utilize sukuk for ethical financing needs. This financing method promotes real economic activity while maintaining compliance with Islamic law, making it an important tool in both Islamic and conventional financial markets.

2 Sharia Principles Guiding Sukuk:

Financing and investment through Sukuk must adhere to Islamic law and uphold ethical standards known as Shariah principles, The key principles include:

- **Beneficial Projects:** Sukuk should fund initiatives that provide societal benefits and avoid harmful activities, such as those involving alcohol.
- **Valid Asset Transfers:** Any transfer of assets or property must be executed through a legitimate and enforceable contract.
- **Permissible Activities and Mutual Consent:** The underlying activity of the transaction must comply with Shariah law and be based on the mutual agreement of all parties involved.
- **Transparency and Integrity:** Transactions must be free from unlawful conditions, including deceit, fraud, uncertainty, or ambiguity that could lead to loss or destruction.
- **Prohibition of Speculation and Gambling:** Sukuk transactions should avoid elements of speculation (Gharar) and gambling (Maysir).
- **Interest-Free Structure:** The charging or payment of interest (Riba) is strictly prohibited in Sukuk arrangements.

- **Asset-Backed Security:** Sukuk must be supported by tangible or intangible underlying assets, ensuring that the investment is backed by real economic activity.
- **Defined and Commercial Assets:** The assets backing Sukuk should be well-defined with clear specifications, physically existing (including their location), and possess commercial value that generates revenue, such as land, buildings, or machinery. These assets must be owned by the issuer.
- **Revenue-Generated Returns:** Returns to Sukuk holders should primarily come from the revenue or cash flows generated by the underlying assets, such as rent, profits, or proceeds from disposal.
- **Tangible or Intangible Assets:** The underlying assets can be either tangible (e.g., real estate, equipment) or intangible (e.g., marketing rights, concession agreements).
- **Tradability Requirements:** For Sukuk to be tradable in the secondary market, at least one-third of their underlying assets must be tangible.

By adhering to these Shariah principles, Sukuk ensure that financing and investment activities are conducted ethically and in compliance with Islamic law. (Securities & Exchange Commission, Nigeria., 2024)

3 Key Differences Between Sukuk and Conventional Financial Instruments:

Table 1 provides an overview of key characteristics of Sukuk, conventional bonds, and shares across various aspects such as compliance, ownership, returns, and regulatory oversight. These aspects define the unique features and structures of each financial instrument, offering insights into their underlying principles, legal frameworks, and market dynamics.

Table 3-1 Sukuk vs. Conventional Bonds vs. Equities

Aspect	Sukuk (Islamic Bonds)	Conventional Bonds	Equities
Compliance	Must comply with Shariah law, prohibiting Riba (interest), Gharar (uncertainty), and Maysir (speculation).	No religious law compliance required.	Subject to company law and stock exchange regulations.
Ownership	Represents ownership in tangible assets or services.	Represents a debt obligation of the issuer.	Represents ownership in a company.
Returns	Profit and loss sharing based on asset performance.	Fixed interest payments regardless of issuer's performance.	Dividends based on company profits (not guaranteed).
Asset-Backed	Typically backed by tangible assets, providing security to investors.	May or may not be backed by specific assets.	Not backed by specific assets; value depends on company performance.
Risk	Shared risk between issuer and investors; linked to asset performance.	Primarily issuer's risk: interest payments are obligatory.	Risk borne by shareholders; dependent on company's success.

Aspect	Sukuk (Islamic Bonds)	Conventional Bonds	Equities
Transparency	High transparency; detailed disclosure of assets and use of funds.	Varies; may not always specify use of proceeds.	Financial statements and disclosures as per regulations.
Legal Structure	Based on Islamic contracts such as Ijarah, Mudarabah, Musharakah, and Murabaha.	Based on conventional debt agreements.	Governed by company law and shareholder agreements.
Market Liquidity	Generally, less liquid; specialized market participants.	More liquid; widely traded in public markets.	High liquidity; traded on stock exchanges.
Ethical Investment	Aligned with ethical investment principles; avoids prohibited sectors.	No specific ethical guidelines required.	Can be ethical depending on company practices.
Regulatory Oversight	Dual oversight by financial regulators and Shariah boards.	Oversight by financial regulators.	Oversight by financial regulators and stock exchanges.

Source. Adapted from (Din, 2016), (Yesuf, 2016), (y N. M. Naveed, 2014).

4 Historical Background:

The evolution of Sukuk is firmly rooted in early Islamic commercial practices and has progressed significantly over time. Understanding this historical context is essential to recognizing the foundational principles and modern advancements that have shaped the Sukuk market.

4.1 Early Islamic Financial Concepts:

The roots of Sukuk can be traced to early Islamic commercial practices, which emphasized principles of fairness, transparency, and risk-sharing. "Sukuk," the plural of the Arabic word "Sakk," translates to "certificate" or "order of payment" and was commonly used during the early Islamic period. Additionally, the term "sakk" is the linguistic root of the British word "cheque", which entered the English language during the midst of the 18th century and eventually evolved into the American term "check". (Essia et al., 2022)

The concept of Sukuk, as an instrument representing ownership and investment, is deeply embedded in Islamic jurisprudence, which mandates that financial transactions must involve tangible assets and real economic activity. This historical context set the stage for the development of modern Islamic finance, where Sukuk plays a crucial role. (El-Gamal, 2006)

Caliph Omar ibn al-Khattab was the first to introduce the use of sukuk as stamped checks for payroll and distribution of goods. According to Rodoni and Hamid (2008), as cited in (Musari, 2019), these sukuk were also used for people to collect wheat from the bait al-maal.

Afterward, in the Middle Ages of Islam, Sukuk were widely used for transferring financial assets from trade and business activities as a promise to pay for goods upon delivery, designed to eliminate the need to transport money across dangerous terrain. As evidenced by Genizah documents, which are documents that were stored in Middle Eastern mosques and synagogues. Indeed, fragments found in the Cairo Genizah reveal evidence of the sakk's existence as early as the 12th century CE. This order of payment closely resembled the modern check in its form and function. It included the amount to be paid, the payee's name, the date, and the issuer's name. (Saeed & Salah, 2012)

Consequently, these Sukuk were transported across numerous countries and gradually spread around the world. Jewish merchants from the Muslim world brought both the concept and the term "sakk" to Europe. (Braudel, 1995)

Notably the trade and use of these Sukuk inspired the creation of modern-day checks. (Essia et al., 2022), For instance Jewish merchants from the Muslim world frequently used sukuk and saftaja for trade between Muslim and Christian territories. (Thomas, 2021)

4.2 Modern Development:

In the contemporary financial landscape, sukuk are recognized as key instruments in Islamic capital markets. The validation of sukuk as a Sharia-compliant financial instrument took place in February 1988 during the fourth conference of the Council of the Islamic Fiqh Academy in Saudi Arabia. This event marked an important milestone in the development of sukuk. (Essia et al., 2022)

Subsequently, in 1990, Bank Islam Malaysia Berhad structured the first RM125 million sukuk, which was issued by Shell MDS, a foreign-owned company (N. W. Ahmad et al., 2015)

After the first sukuk was issued in 1990, there were no other issuances until 2001. The Government of Bahrain issued the first international ijarah sovereign sukuk in September 2001. Not long after, Kumpulan Guthrie, a Malaysian company, issued a US$150 million ijarah sukuk. In 2002, the Malaysian government issued US$600 million ijarah sukuk, which were listed on the Luxembourg Stock Exchange and rated by Standard & Poor's and Moody's. This issue achieved remarkable success, being oversubscribed twice (Mat Radzi, 2018).

In response to the rapid growth of the sukuk market, the AAOIFI introduced "Shari'ah Standard No. 17" on 'Investment Sukuk' in May 2003. This standard, which took effect in early 2004, helped streamline sukuk practices and ensured they followed common principles. (Ramuthi, 2023)

Following this, there were several successful sukuk issues, including Qatar's US$700 million ijarah sukuk and Bahrain's US$250 million ijarah sukuk in 2004 (Mat Radzi, 2018). Afterward, Sukuk issues are consistently oversubscribed, indicating strong investor demand. The volume of sukuk issued has increased significantly, from $7.2 billion in 2004 to nearly $50 billion outstanding by the end of 2007. (Visser, 2009)

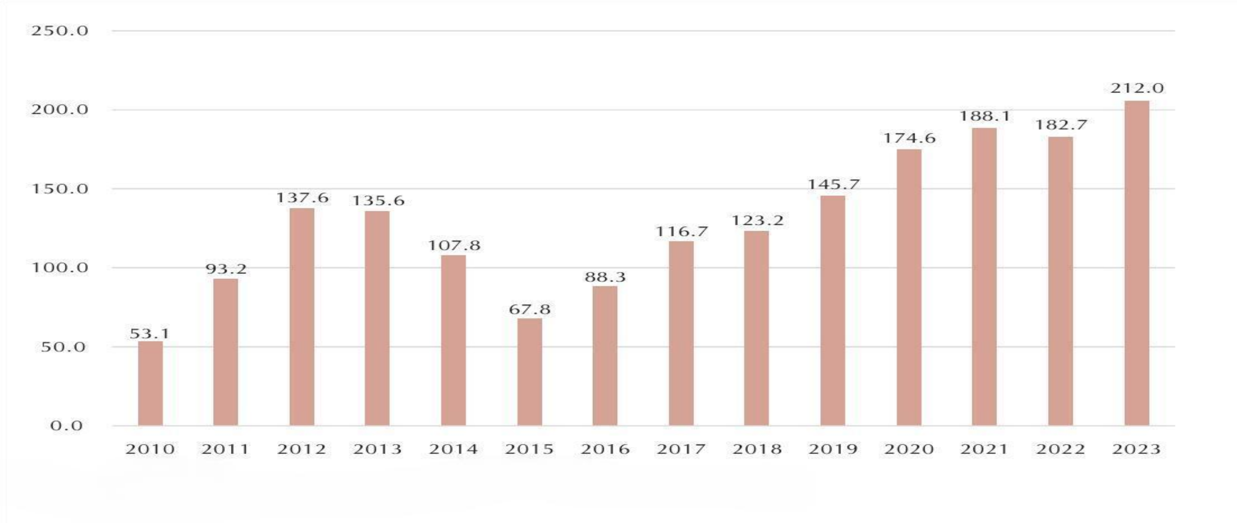
The 2008 financial crisis had a significant impact on the sukuk market. Initially, the market experienced a sharp decline in issuances due to the global credit crunch, which led investors to step back from fixed-income markets, including Islamic finance. For instance, Sukuk issuances dropped by more than 50% by the end of 2008 (W. Ahmad & Mat Radzi, 2011). Nonetheless, as 2009 started, the sukuk market staged a strong recovery due to their asset-backed nature, proving its robustness and attracting investors back. This recovery highlighted the potential of sukuk as a stable investment option even during economic downturns. (Rahim & Ahmad, 2023)

Consequently, Sukuk began attracting non-Muslim-majority countries seeking diversification of funding and ethical investment opportunities. For instance, the UK, Luxembourg, and Hong Kong expressed interest in sukuk markets to tap into Middle Eastern and Southeast Asian capital.

Since the global Sukuk market has demonstrated remarkable growth and resilience, as evidenced by Chart 1.1 Starting from a modest base of $53.1 billion in 2010, the market experienced rapid expansion in the early 2010s, more than doubling to reach $137.6 billion by 2012. Then, the market faced some volatility in the mid-2010 decade, with a notable contraction to $67.8 billion in 2015, marking the lowest point in the decade. This downturn can be attributed to various factors, including global economic uncertainties and oil price fluctuations that affected major Sukuk-issuing countries. However, the market demonstrated its fundamental strength by rebounding steadily from 2016 onwards, with issuances climbing to $116.7 billion in 2017 and continuing their upward trajectory.

The period from 2019 to 2023 marks the most significant phase of growth in the Sukuk market. Despite the global economic challenges posed by the COVID-19 pandemic, Sukuk issuances showed remarkable resilience, reaching $174.6 billion in 2020. This growth continued into 2021, hitting $188.1 billion, before a slight dip to $182.7 billion in 2022. The market reached a historic peak in 2023 with $212.0 billion in issuances, underscoring the increasing mainstream acceptance of Islamic financial instruments. ((IIFM), 2024)

Figure 3.1 Total Global Sukuk Issuances - All Tenors, All Currencies, in USD Billion.



Source: IIFM database 2024.

5 Global Sukuk Issuance:

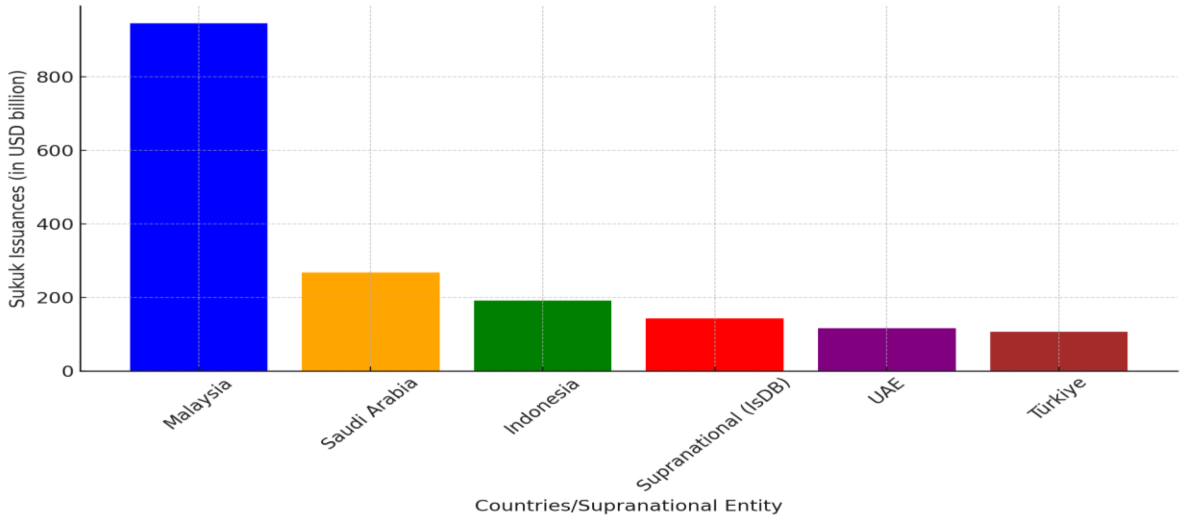
The global Sukuk market has experienced significant growth over the past decade, emerging as a crucial component of Islamic finance. This Shariah-compliant financial instrument has gained popularity among both Muslim-majority and non-Muslim countries seeking to diversify their funding sources and attract Islamic investors. The market has shown remarkable resilience despite global economic challenges, with total issuances reaching substantial volumes across various jurisdictions. Both sovereign and corporate entities have increasingly turned to Sukuk as a viable funding alternative, particularly for infrastructure development and sustainable projects. (M. Hussain et al., 2015)

5.1 Key Leaders:

The landscape of global Sukuk issuances from 2001 to 2023 as shown in Chart 1.2, reveals a clear hierarchy among issuing nations, with Malaysia emerging as the undisputed leader in the Islamic finance market. Malaysia's remarkable achievement of USD 945 billion in cumulative domestic and international issuances demonstrates its sophisticated Islamic financial infrastructure and robust regulatory framework. This dominance represents nearly half of the total global Sukuk issuances during this period, highlighting Malaysia's role as the primary driver of Islamic capital market development. (IIFM, 2024)

The second tier of major issuers is led by Saudi Arabia, which has accumulated USD 267.92 billion in issuances, followed by Indonesia with USD 191.72 billion. The significant contribution from Supranational entities, including the Islamic Development Bank (ISDB), amounting to USD 142.58 billion, underscores the importance of multilateral institutions in developing the Sukuk market. The United Arab Emirates and Türkiye have also established substantial market presence with USD 116.27 billion and USD 106.79 billion in issuances respectively, rounding out the top tier of Sukuk issuers. (IIFM, 2024)

Figure 3.2 Leading Sukuk Issuers



Source: IIFM Sukuk Report 2024.

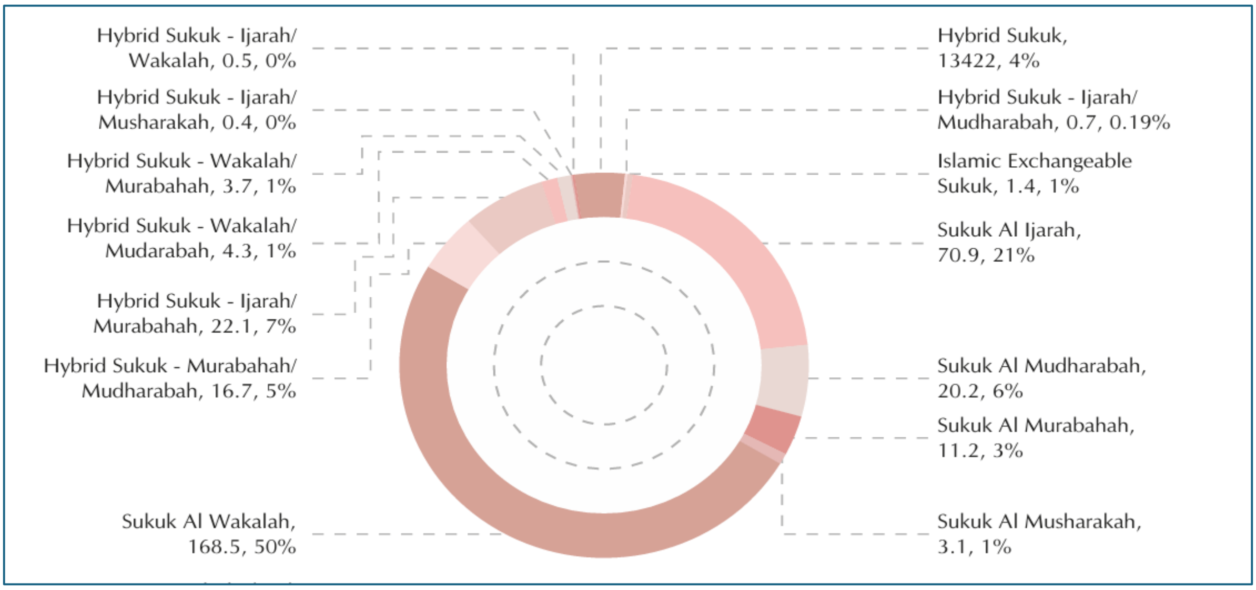
5.2 Trends in Sukuk markets:

In terms of structural preferences, the period from 2010 up to 2023 witnessed a clear preference for specific Sukuk structures in the international market, as depicted from Chart 1.3 Sukuk Alwakalah emerged as the dominant structure, with an impressive 168.5 billion USD (50%) in issuances, reflecting its flexibility in managing assets and investments which makes it highly favoured in the market. Sukuk Al Ijarah, another significant segment with 70.9 billion USD (21%) in issuances, underscores its broad acceptance and straightforward implementation in the Islamic finance industry. ((IIFM), 2024)

The chart also highlights the role of hybrid Sukuk structures, such as Hybrid Sukuk - Ijarah/Murabahah and Hybrid Sukuk - Murabahah/Mudarabah, which combine various Sharia-compliant principles to meet market needs, accounting for 22.1 billion USD (7%) and 16.7 billion USD (5%) respectively. These hybrid structures represent the industry's innovative approaches to diversifying financial instruments. (Armstrong, 2024)

Smaller segments, like Sukuk Al Mudharabah and Sukuk Al Murabahah, each contributing 20.2 billion USD (6%) and 11.2 billion USD (3%) respectively, demonstrate the variety in profit-sharing and cost-plus financing contracts available in the market. Sukuk Al Musharakah, with 3.1 billion USD (1%), and Islamic Exchangeable Sukuk, with 1.4 billion USD (1%), highlight niche yet innovative segments that offer partnership-based investments and conversion into equity options. The data clearly reflects the market's preference for Sukuk Al Wakalah and Sukuk Al Ijarah, which together constitute 71% of the total issuances, indicating their reliability and efficiency. ((IIFM), 2024)

Figure 3.3 International Sukuk Issuances JAN 2010 - DEC 2021



Source: IIFM database 2024.

6 Benefits of Sukuk investment:

Sukuk combines ethical investing with financial stability, offering unique advantages. Below are the key benefits of Sukuk investment.

- **Compliance with Shariah Principles:**

Sukuk offers an ideal investment avenue for individuals seeking alignment with Islamic law. Unlike conventional bonds, Sukuk avoids elements prohibited by Shariah, such as Riba (interest), gharar (excessive uncertainty), and speculative transactions. Each Sukuk issuance is tied to tangible assets or specific economic activities, ensuring investments are grounded in real value. This adherence to Islamic ethical standards not only attracts Muslim investors but also appeals to those prioritizing ethical financial practices. (Georgiev, 2024)

- **Asset-Backed Investments:**

A key feature of Sukuk is its asset-backed structure, which provides investors with ownership stakes in real assets or designated projects. Unlike traditional bonds, which represent debt obligations, Sukuk connects investors to productive and tangible ventures. This structure enhances investor confidence, reduces risks, and provides stability, making Sukuk a reliable alternative for those concerned about the volatility of conventional financial instruments. (Naveed, 2014)

- **Enhanced Portfolio Diversification:**

Sukuk plays a vital role in portfolio diversification, offering exposure to markets and asset classes that often behave differently from traditional investments. For global investors, Sukuk's low correlation with conventional financial instruments helps reduce overall portfolio risk, particularly during periods of market volatility. This makes Sukuk an effective tool for robust financial planning. (Armstrong, 2024)

- **Stable and Predictable Returns:**

Sukuk is structured around income-generating assets, such as leasing arrangements or joint ventures, resulting in consistent and reliable returns. In many cases, Sukuk outperforms conventional bonds in terms of predictability and stability. These steady income streams are particularly attractive to conservative investors seeking low-risk investments with dependable earnings. (Armstrong, 2024)

- **Ethical and Responsible Investing:**

Investments in Sukuk are exclusively allocated to Shariah-compliant activities, avoiding industries like alcohol, gambling, pork, and other prohibited sectors. This ethical framework aligns with the global trend towards socially responsible investing. By investing in Sukuk, individuals not only steer clear of morally questionable industries but also contribute to projects that promote social and economic welfare, such as infrastructure development and community initiatives. (Georgiev, 2024)

- **Growing Market and Increased Accessibility:**

The global Sukuk market is expanding rapidly, driven by rising demand from both institutional and individual investors. Governments and corporations increasingly utilize Sukuk to finance large-scale projects, particularly in emerging economies. This growth provides investors with opportunities to engage in high-potential sectors and benefit from international diversification, further enhancing the appeal of Sukuk as a modern investment option. (Armstrong, 2024)

7 Sukuk Market Participants:

Securitization, like any other contractual process, involves various parties with distinct roles, rights, and obligations. These participants can be categorized into two main groups: Primary Participants and Supporting Participants.

7.1 Primary Participants:

These are the main entities directly involved in the securitization process:

7.1.1 Originator:

The originator, also referred to asset owner, or initiator, is the entity seeking liquidity. It pools its assets into a securitization portfolio and transfers (sells) them to a Special Purpose Vehicle (SPV) in exchange for cash. The originator could be a company, government, financial institution, or individual from the private or public. The funds raised from this process are employed in a Sharia-compliant manner to meet financing needs. (Organisation of Islamic Cooperation / Standing Committee for Economic and Commercial Cooperation, 2018)

7.1.2 Issuer or special purpose vehicle (SPV):

The issuer, often called the securitization company or sukuk manager, is responsible for issuing and managing sukuk securities. This entity handles the sale of sukuk to investors and supervises the securitization process. It ensures compliance with the terms outlined in the prospectus, maintains transparency, and protects the interests of sukuk holders. The issuer operates independently to provide legal and financial safeguards to the investors. (خليفة المخمري, 2016)

7.1.3 Investors (Sukuk Holders):

Investors, or sukuk holders, are the entities or individuals purchasing sukuk securities. These securities represent ownership of tangible assets, services, or a combination of both. The primary goal of sukuk holders is to recover the principal value of their investment and earn returns from the securitization portfolio. Investors typically include Islamic and traditional banks, financial institutions, governments, or individuals who prefer Sharia-compliant financial instruments. (Organisation of Islamic Cooperation / Standing Committee for Economic and Commercial Cooperation, 2018)

7.2 Supporting Participants:

These entities provide specialized services that facilitate the securitization process:

7.2.1 Trustee:

The trustee is a financial intermediary responsible for preserving the interests of sukuk holders. This entity ensures that the issuer adheres to the terms outlined in the prospectus and retains documentation and guarantees. The trustee acts as the representative of sukuk holders, operating under an agency contract, and can be replaced at their discretion if needed. (Organisation of Islamic Cooperation / Standing Committee for Economic and Commercial Cooperation, 2018)

7.2.2 Sharia Supervisory Board:

The Sharia Supervisory Board plays a critical role in ensuring the sukuk issuance complies with Islamic principles. It reviews the structure, contracts, and subscription methods of the sukuk and validates all proposed activities and transactions to ensure Sharia compliance.(خليفة المخمري, 2016)

7.2.3 Credit Rating Agencies:

Global rating agencies assess the creditworthiness and financial stability of sukuk securities. They evaluate the associated guarantees and risks, providing investors with insights into the credibility of issuers and other participants. Notable agencies include Fitch, Standard & Poor's, and Moody's, as well as Islamic agencies like the Islamic International Rating Agency (IIRA) and RAM Ratings. (خليفة المخمري, 2016)

7.3 Additional Supporting Entities:

Additional participants, such as issuance managers, underwriters, guarantors, contribute to the securitization process. Their roles depend on the specific nature and structure of the securitization and the type of underlying assets. These entities ensure the smooth execution and management of sukuk issuance. (خليفة المخمري, 2016)

8 Sukuk process from initiation to maturity:

The lifecycle of Islamic sukuk is a carefully interconnected process, ensuring adherence to Shari'a principles, transparency, and efficient asset management.

8.1 Structuring Phase:

The sukuk issuance process begins with identifying the assets to be securitized. These assets are evaluated to ensure they comply with Shari'a principles, making them suitable for sukuk issuance. During this phase, a thorough analysis of legal, regulatory, and Shari'a requirements is conducted. Simultaneously, an economic feasibility study is performed to determine the viability of the proposed sukuk issuance. Based on these findings, the most appropriate issuance method is selected, whether it be a public offering to attract a broad investor base or a private placement tailored to specific investors. Each step in this phase ensures that the foundation of the sukuk issuance is both robust and compliant. (Shofni, 2012)

8.2 Establishing the Special Purpose Vehicle (SPV):

Once the structuring phase is complete, a Special Purpose Vehicle (SPV) is created to manage the sukuk process. Ownership of the identified assets is transferred from the originator to the SPV, ensuring separation of ownership for transparency and risk mitigation. The SPV is responsible for restructuring these assets into units that align with investors' needs. This restructuring process includes securitizing the assets into sukuk certificates, which represent ownership in the underlying assets. By ensuring proper asset management and compliance, the SPV plays a pivotal role in this interconnected process. (Shofni, 2012)

8.3 Offering Sukuk for Subscription:

With the SPV in place, the sukuk certificates are offered for subscription. This involves either a public offering, where the sukuk are made available to a wide range of investors, or a private placement, targeting a specific group. An issuance manager oversees this phase, ensuring the process aligns with the established framework. The funds raised during this step are intended to finance the assets represented by the sukuk, directly linking investor contributions to the underlying assets. (Shofni, 2012)

8.4 Operating Proceeds and Managing the Sukuk Portfolio:

Once funds are raised, they are deployed according to the investment strategy outlined in the sukuk prospectus. The SPV actively manages the portfolio, ensuring the assets generate periodic returns for sukuk holders. This phase is critical, as it involves the careful allocation of proceeds, ongoing management of the assets, and distribution of returns. The SPV ensures that all activities remain compliant with Shari'a principles and that services essential for portfolio maintenance are consistently delivered. (نور مطر & نيفين, 2022)

8.5 Post-Issuance Phase:

After the sukuk are issued, the first general assembly of sukuk holders is convened to elect a sukuk holders' committee. This committee represents the collective interests of the investors and oversees the management of the sukuk. Additionally, tradable sukuk are registered and listed on secondary markets to enhance liquidity and provide trading opportunities. To uphold transparency and investor confidence, the financial statements of the sukuk project are audited by an external auditor, ensuring compliance with the highest standards of accountability. (نور مطر & نيفين, 2022)

8.6 Sukuk Redemption Phase

At the end of the sukuk term, the underlying assets are liquidated, and the proceeds are distributed to sukuk holders. This final stage marks the conclusion of the sukuk lifecycle as outlined in the prospectus. The redemption phase ensures that investors receive the returns promised, completing the interconnected journey from structuring to maturity. (نور مطر & نيفين, 2022)

9 Sukuk Classifications:

Sukuk can be categorized based on several considerations, with the most common criteria focusing on the nature of the underlying asset and the contract structure that follows it:

9.1 By asset nature:

Sukuk are primarily categorized into two types based on their asset structure: asset-based and asset-backed:

9.1.1 Asset-Based Sukuk:

Asset-Based Sukuk are a financial instruments that involve securitizing receivables, which are amounts owed to an organization (issuer) , in the other hand this issuer grants beneficial ownership to investors, allowing them to benefit from the Sukuk without the right to sell the underlying asset. (Salman, 2013)

Subsequently, while asset-based Sukuk provides beneficial ownership certificate that demonstrates rights to the associated indebtedness, it's important to note that Sukuk holders' ownership of the underlying asset is not legally recognized. Consequently, in the event of default, Sukuk holders' recourse is limited to the issuing company rather than the underlying asset itself. (Lahsasna et al., 2018)

In short Asset-based sukuk function similarly to conventional bonds, but incorporate an asset into the contract to comply with Shariah law.(Ahmed, 2010)

9.1.2 Asset-Backed Sukuk:

Asset-backed sukuk represents an Islamic securitization instrument where the underlying assets are legally transferred from the originator through a true sale mechanism. This structural feature ensures complete separation of the assets from the originator's balance sheet, this separation means that in cases of default or liquidation, these assets remain separate from the originator's other assets. Thus, the underlying asset serves as direct collateral, limiting sukuk holders' recourse exclusively to the asset itself rather than the originator.in the other hand if default occurs or payments fall below expected levels, investors cannot seek compensation from the originator. Furthermore, sukuk holders face market risk during redemption, as the value is determined by prevailing market conditions and could potentially drop to zero. (Kol & Tekdogan, 2024)

Table 2 presents a summary of the key distinctions between asset-based and asset-backed sukuk:

Table 3-2 Difference Between Asset-Based and Asset Back Sukuk

	Asset-based sukuk	**Asset-backed sukuk**
Source of payment	Payments derive from the originator/obligor's cash flows	Payments come directly from the revenue generated by the underlying asset
Ownership Structure	Sukuk holders have beneficial ownership but cannot dispose of the asset	Sukuk holders possess legal ownership with full rights to dispose of the asset
Recourse	Investors have recourse only to the obligor through a purchase undertaking at par, with no direct claim on the asset	Investors' recourse is limited to the underlying asset, which serves as genuine security in default scenarios
disclosure of the asset	The underlying asset remains on the originator/obligor's balance sheet	The asset is completely removed from the originator's books

Source: adapted from (Herzi. 2016).

9.2 By underlying contracts:

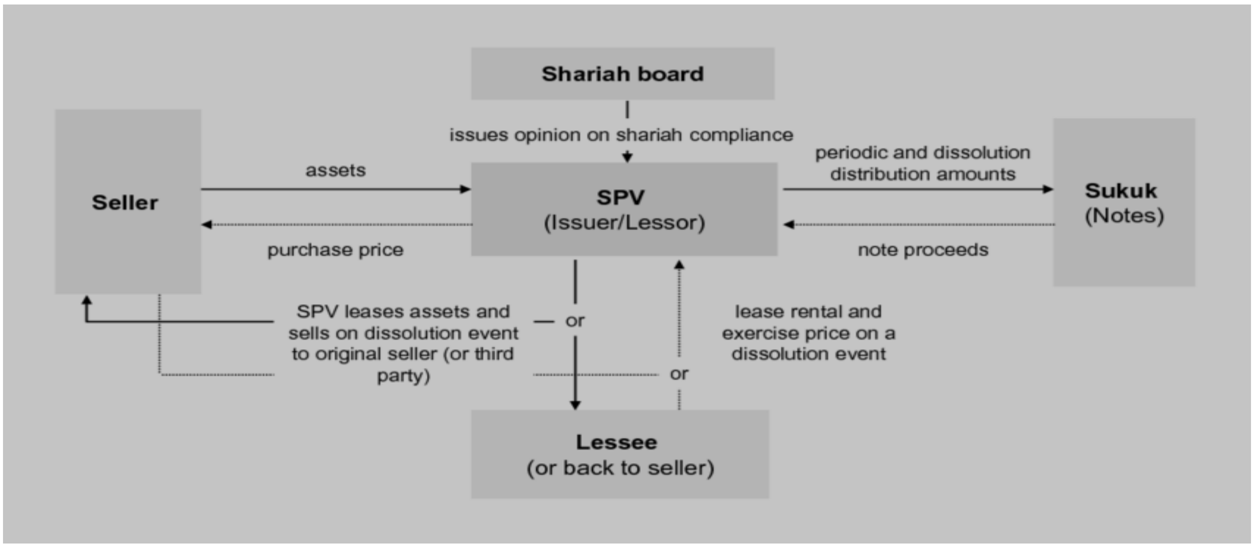
Based on the structure of the underlying contracts, Sukuk are categorized into different classes, The more common of these include:

9.2.1 Ijara (Lease) Sukuk:

Ijara Sukuk are financial certificates representing ownership in assets linked to a lease agreement, where one party (the lessor) purchases an asset needed by another party (the lessee) and then leases it to them. The rental payments from these leases provide returns for investors. Subsequently, these Sukuk can be traded on the secondary Sukuk market. (Media, 2018)

The diagram below illustrates this structure:

Figure 3.4 Ijara Sukuk structure



Source: (Jobst, 2007).

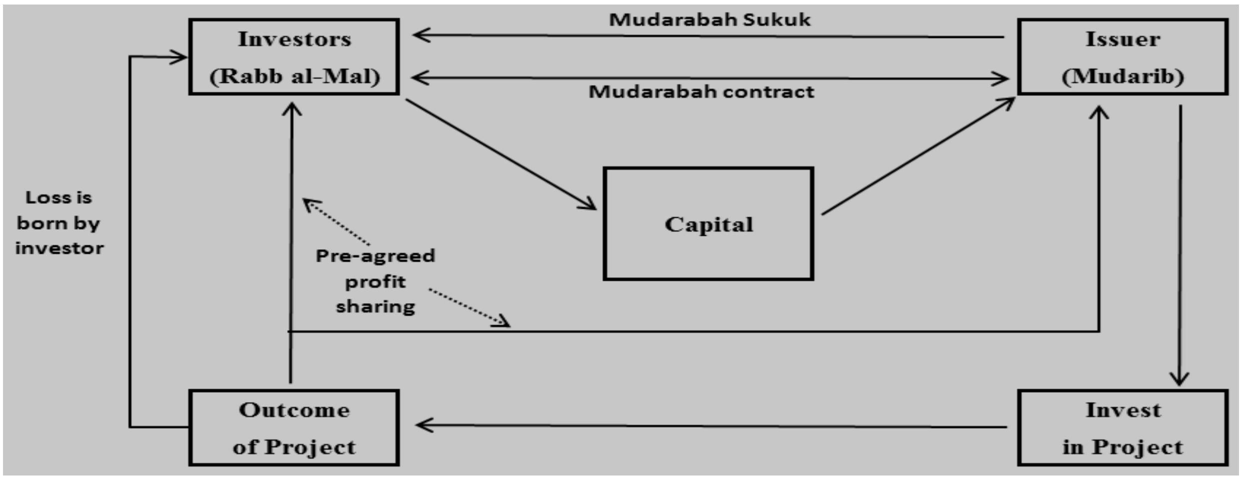
9.2.2 Mudarabah sukuk:

Mudarabah, or muqaradah, is a profit-sharing contract that establishes a partnership between a capital provider and a mudarib, who acts as the managing trustee or entrepreneur running the business. In a Mudarabah arrangement, one party contributes capital to a trustee who manages the investment as a partner.

Profits from this joint venture are distributed between the partners based on a predetermined ratio. Losses, however, are the sole responsibility of the capital provider, and the mudarib receives no compensation. Nevertheless, the mudarib can be held liable for breaches of contract, negligence, or misconduct. (Lahsasna et al., 2018)

This structure can be represented visually as the following diagram:

Figure 3.5 Mudarabah Sukuk structure

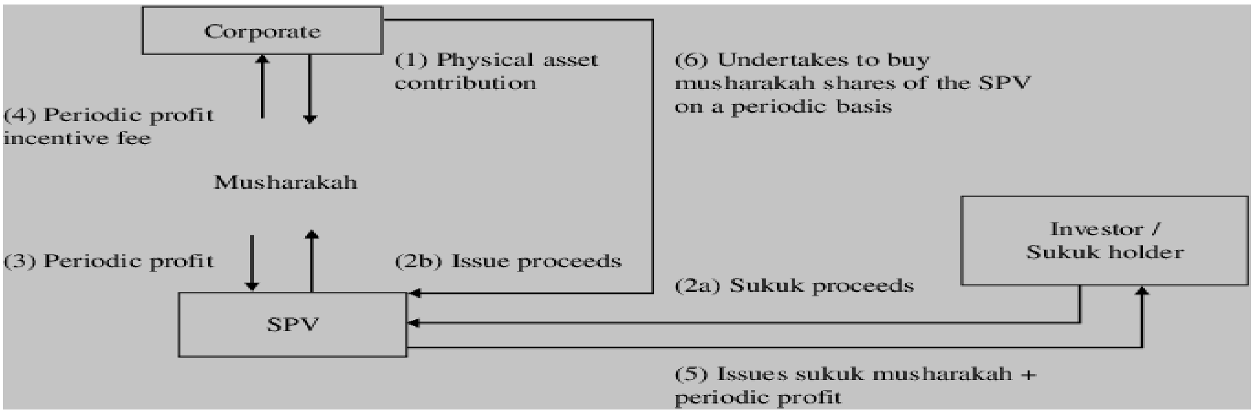


Source: (Soeleman & Lestari, 2014).

9.2.3 Musharakah (Participation) Sukuk:

Sukuk musharakah is an Islamic financial instrument that represents an undivided ownership share in a business venture based on partnership (musharakah) principles. Each sukuk holder becomes a partner according to their investment share, participating in both profits and investment decisions. These Shariah-compliant certificates can be traded in secondary markets.(Lahsasna et al., 2018)

Figure 3.6 Musharakah Sukuk structure



Source :(Hanim Kamil et al., 2010)

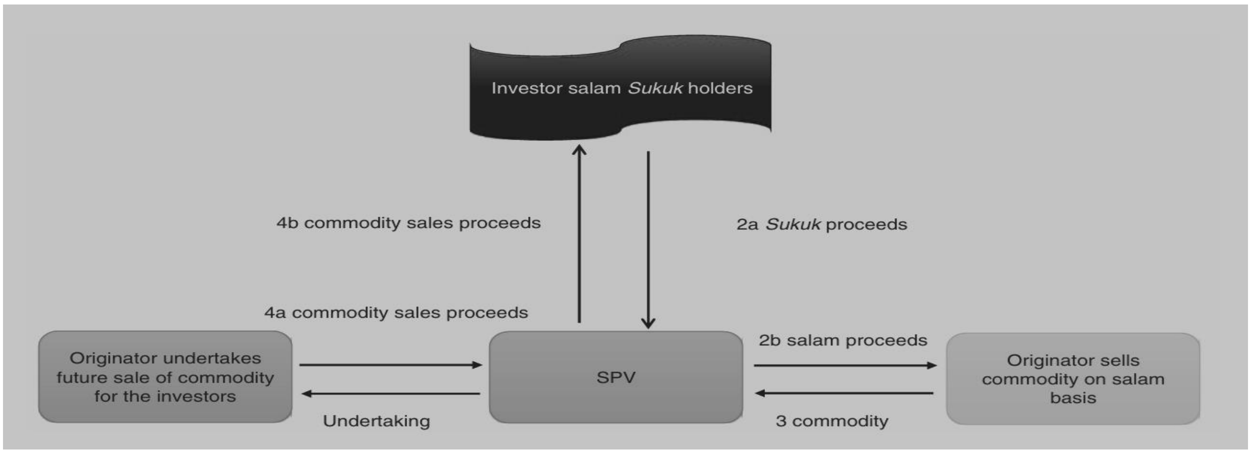
9.2.4 Salam sukuk:

Salam sukuk are certificates that represent equal-value shares, where payment is made in advance for goods that will be delivered at a future date, often used in agriculture (Hanif, 2014).

by purchasing these certificates, Investors provide immediate capital and secure proportional ownership of the promised goods. Their investment can generate returns through certificate resale or a Parallel Salam arrangement, where the goods are pre-sold, securing a profit. (Lahsasna et al., 2018)

See the diagram below:

Figure 3.7 Salam Sukuk structure

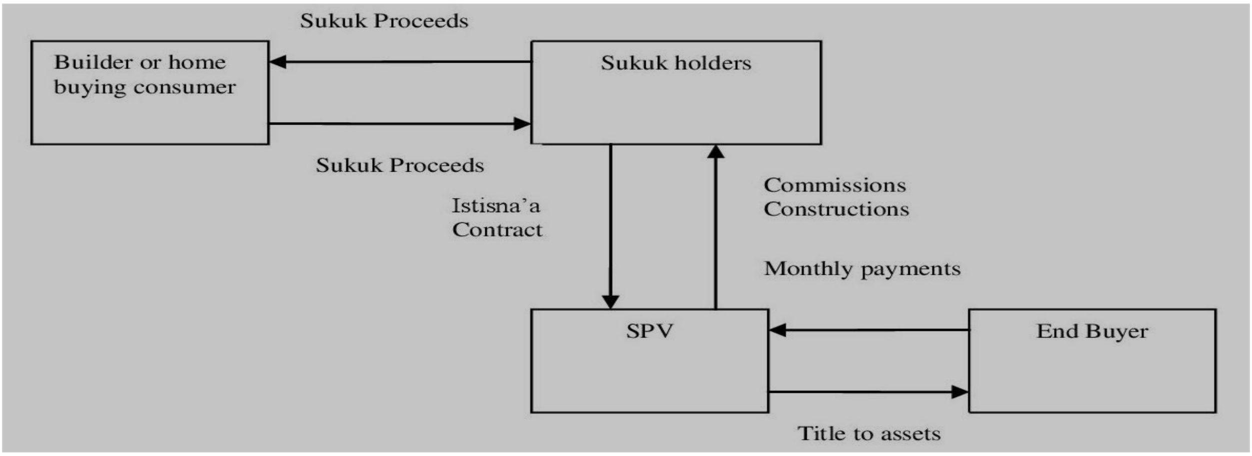


Source : (Lahsasna et al., 2018)

9.2.5 Sukuk Istisna:

Istisna Sukuk are equal-value investment certificates that fund manufacturing or construction projects. These certificates allow multiple investors to finance the development of specific assets, such as buildings, housing complexes, educational institutions, or infrastructure projects. The manufacturer or developer issues the certificates, while investors become partial owners of the project being funded. Through this arrangement, the issuer receives the required capital for production, and certificate holders gain ownership rights in the completed assets. (Alsaeed, 2012)

Figure 3.8 Istisna Sukuk structure



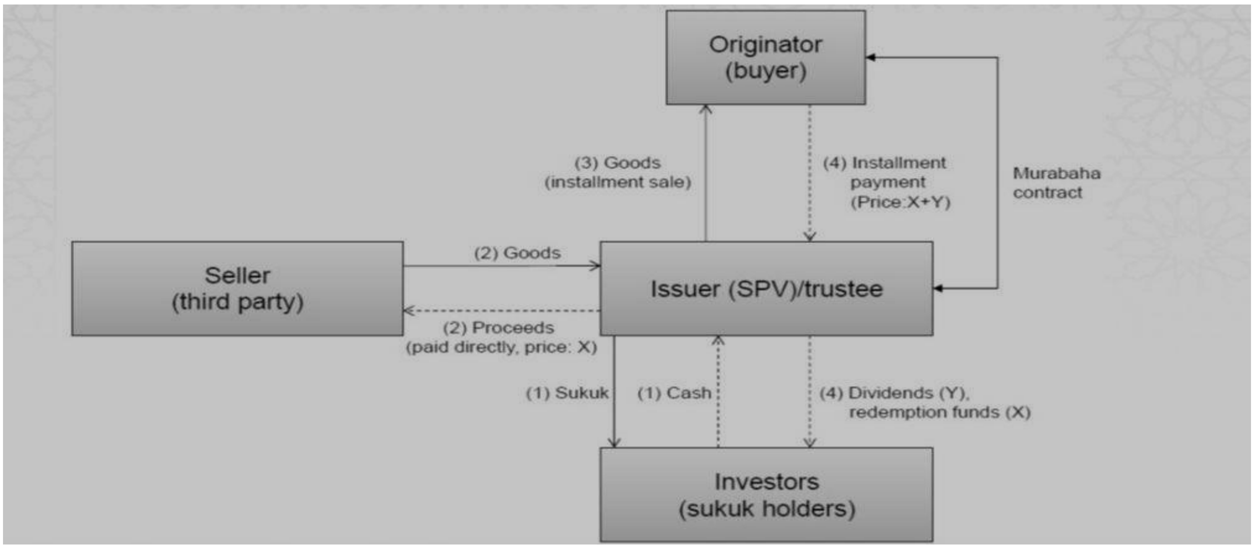
Source : (Alsaeed, 2012)

9.2.6 Sukuk Murabaha:

Sukuk Murabaha are certificates of equal value, where the issuer of the certificate is the seller of the Murabaha commodity. While the subscribers are the buyers, and the funds they provide are used to purchase the commodity. Once purchased, the certificate holders own the commodity and will receive its resale price.

Murabaha-based Sukuk can only be issued and sold in the primary market. They cannot be traded in the secondary market because, under Shariah law, the certificates represent a debt owed by the next buyer to the certificate holders. Trading debt for a deferred payment is considered Riba (interest), which is not allowed in Shariah. (Alsaeed, 2012)

Figure 3.9 Mudarabah Sukuk structure



Source : (Turkhan, 2015)

9.2.7 Hybrid sukuk:

Hybrid sukuk are financial instruments, that represents a key innovation in Islamic finance and a notable advancement in the Sukuk industry. In brief, hybrid sukuk refer to a Sukuk structure backed by a mix of underlying assets such as (Murabahah receivables and Ijarah tangibles) within a single portfolio. Developed in response to market demand. (Lahsasna et al., 2018)

10 Sukuk limitations and challenges:

The Sukuk market, though promising as a Shariah-compliant alternative to conventional bonds, faces several significant challenges and controversies. These issues include:

The sukuk market is still in its early stages but has significant potential to contribute to the growth of the Islamic financial industry. Several challenges hinder its development:

- **Asset-Linked Sukuk obstacle:**

Most sukuk, except those issued by the Islamic Development Bank, are tied to specific real assets rather than diversified pools of assets. This structure works well for large-scale borrowers but creates obstacles for smaller institutions looking to raise capital. Sukuk based on salam or murabahah contracts cannot be traded in secondary markets, reducing liquidity. Although Islamic banks hold securitizable assets, they have yet to issue sukuk due to limitations like short-term assets or lack of large-scale holdings. Developing sukuk backed by diverse asset pools with varying maturities and credit qualities is essential. Greater participation from Islamic banks can foster market growth. (Ulusoy & Ela, 2018)

- **Excessive Cost :**

Islamic financial transactions often incur higher costs compared to conventional bonds, including legal, documentation, and distribution expenses. New sukuk issues also require robust structural evaluation and credit quality assessments, increasing the overall cost. (Ulusoy & Ela, 2018)

- **Low Secondary Market Activity:**

Institutional investors, central banks, and private Islamic banks often hold sukuk until maturity, which reduces activity in secondary markets. This lack of liquidity results in higher transaction costs, reflected in wide bid-ask spreads. Increasing sukuk supply and developing a retail investor market can improve secondary market activity and reduce these costs. (Iqbal & Mirakhor, 2011)

- **Intermediation Costs:**

Issuing sukuk often involves multiple investment banks, which raises intermediation costs. Conventional banks frequently co-lead sukuk issues, taking larger shares of fees and limiting efforts to develop small-scale sukuk in local markets. Islamic investment banks must take a more active role to bridge this gap. (Iqbal & Mirakhor, 2011)

- **Monitoring and proper Diligence:**

Sukuk issuance requires ongoing monitoring to ensure borrowers meet their obligations. Borrowers may exaggerate their capabilities or commitment, posing risks for investors. To mitigate these risks, contracts should be established with credible entrepreneurs. Islamic investment banks play a key role in conducting due diligence and ensuring transparent execution of sukuk deals.(Iqbal & Mirakhor, 2011)

11 Conclusion:

This chapter explored Sukuk in depth, defining them as Shariah-compliant financial instruments backed by assets and structured around ownership. It highlighted key features such as risk-sharing, ethical standards, and returns linked to asset performance, which distinguish Sukuk from conventional financial instruments.

The chapter also traced Sukuk’s evolution from early Islamic finance to global prominence. It analysed global issuances, showing Malaysia’s market dominance and investor preference for flexible structures like Sukuk Al Wakalah and Al Ijarah.

It discussed key market participants, detailed the issuance process, and examined Sukuk classifications. Additionally, it identified major challenges, including liquidity constraints and cost inefficiencies.

In summary, Sukuk represents a dynamic and growing segment of global finance, offering Shariah-compliant investment options that balance profitability with ethical considerations. Continuous market development and addressing existing challenges will strengthen Sukuk’s position in both Islamic and conventional markets.

Chapter 4 : Univariate Time Series Forecasting for Stock Market Volatility (Models and Performance Evaluation)

Time series refers to a sequence of values measured on a uniform scale and indexed by a time-related parameter. It is defined by its data points and underlying parameters. Time series can take on countless forms, representing the vast range of functions of real numbers. (Brillinger, 2015)

Time series data is widely used across various fields, such as weather forecasting, where it tracks temperature and precipitation over time, or healthcare, where it monitors patient vitals and treatment progress. It is also crucial in economics, capturing metrics like GDP and inflation to study economic trends. In transportation, time series data helps optimize traffic flows and transit schedules. However, financial markets are one of the most significant domains for time series analysis, where data such as stock prices, trading volumes, and market indices are recorded continuously. This data is essential for analyzing market trends, forecasting price movements, managing risks, and making investment decisions, making it a cornerstone of financial research and strategy.

Stock market volatility a fundamental concept in financial markets. It refers to the extent to which a security's price fluctuates over time, providing an indication of the risk associated with its price movements.(Corporate Finance Institute, 2025)

statistically Volatility is measure of the dispersion of returns for a given security or market index, often measured by the standard deviation or variance of returns over a specific time period. (Hull, 2000)

Forecasting is the process of predicting unknown future values based on known past observations. Many different forecasting methods exist. Some of these include the time series analysis method. (Tanique, 2024)

Forecasting time series is challenging task, since time series data can exhibit complex patterns that vary over time and are difficult to understand or explain. This complexity has driven growing interest in forecasting within both academia and industry. Consequently, a variety of methods have been developed to forecast time series data ranging from traditional statistical modelling techniques to modern machine learning approaches.

1 Traditional Statistical Models for Univariate Time Series Forecasting:

Traditional time series forecasting methods rely heavily on mathematical and statistical frameworks designed to analyze patterns in sequential data. These methods are generally divided into two main categories: linear models and non-linear models:

1.1 Linear Models:

Linear time series forecasting models estimate future values by using a linear combination of past data points. They are based on the assumption that there is a linear relationship between historical observations and future outcomes, making them relatively simple to implement and interpret. The most commonly used univariate linear time series forecasting models for the stock market include:

1.1.1 Autoregressive Integrated Moving Average (ARIMA) Model:

ARIMA model proposed by Box and Jenkins (1970) and denoted as ARIMA (p, d, q). is a common technique in time series forecasting, using historical data to make predictions. (Alexander, 2016)

a) ARIMA Model components:

ARIMA model combines three essential elements: autoregression (AR), differencing (I), and moving average (MA) to capture patterns in the data.

- **Autoregressive (AR):**

The autoregressive component (AR) is based on the concept that the current value of a series can be explained by its previous values. It uses a linear combination of past observations to predict future values, with the number of past values considered being determined by the parameter p.

An autoregressive model of order p can be expressed as:

$$\boldsymbol{y_t = c + \phi 1 y_t - 1 + \phi 2 y_t - 2 + \cdots + \phi p y_t - p + \epsilon t}$$

where:

y_t: The value of the time series at time t.

C: A constant term or intercept in the model.

$\emptyset$: Coefficients of the lagged values of y_t.

ϵt: The error term or white noise at time t. Which represents the part of y_t that cannot be explained by the model. (R. J. Hyndman & Athanasopoulos, 2018)

- **Integrated (I):**

ARIMA models assume stationarity, meaning that the time series has constant mean and variance over time. To achieve stationarity, non-stationary time series are differenced, transforming the series by subtracting current observations from previous ones. This process denoted as (d) where d indicates the number of differencing steps applied.

The differencing process is formalised as:

$$\boldsymbol{y'_t = y_t - y_{t-1}}$$

If the series still exhibits non-stationary behaviour, higher-order differencing (d = 2 or d=3...) is applied until the series becomes stationary.

$$\boldsymbol{y_t'' = (y_t - y_{t-1}) - (y_{t-1} - y_{t-2})}$$

$$\boldsymbol{y_t''' = [(y_t - y_{t-1}) - (y_{t-1} - y_{t-2})] - [(y_{t-1} - y_{t-2}) - (y_{t-2} - y_{t-3})}$$

(R. J. Hyndman & Athanasopoulos, 2018)

- **Moving Average (AR):**

The Moving Average component models the dependency of the current value on the weighted sum of past forecast errors. It captures short-term fluctuations in the data caused by random shocks or noise, in this context the moving average order donated as q.

The general form of an MA(q) model is:

$$y_t = c + \varepsilon_t + \theta 1 \varepsilon_{t-1} + \theta 2 \varepsilon_{t-2} + \cdots + \theta q \varepsilon_{t-q}$$

Where:
c**:** The intercept or constant term

ε_t: white noise

θ: The coefficient for the lagged value

(R. J. Hyndman & Athanasopoulos, 2018)

b) The ARIMA Modeling Process (Box and Jenkins Methodology):

Time series analysis through ARIMA modeling represents one of the most sophisticated approaches to forecasting, combining autoregressive and moving average components with differencing procedures. As initiated by Box and Jenkins (1970) in their influential work, this methodology has become fundamental in various fields. The process involves the following steps:

- **Step 1: Time Series Decomposition**

Understanding the fundamental components of a time series is crucial before proceeding with model specification. describe how decomposition reveals the fundamental structure of the data by separating it into trend, seasonal, and irregular components. This decomposition process provides valuable insights into the appropriate transformation methods needed for subsequent analysis. Modern decomposition techniques, such as STL (Seasonal and Trend decomposition using Loess), offer robust approaches to understanding these components, as demonstrated in numerous empirical studies. (Brockwell & Davis, 2016)

- **Step 2: Achieving Stationarity**

The concept of stationarity stands as a cornerstone of time series analysis. Since non-stationary data can lead to spurious relationships and unreliable forecasts. The process of achieving stationarity typically involves differencing and transformation techniques. The Augmented Dickey-Fuller test, as explained by Dickey and Fuller (1979), provides a formal framework for assessing stationarity. Additionally, when dealing with seasonal data it is recommended considering both regular and seasonal differencing to ensure stationarity across all components of the series.. (R. J. Hyndman & Athanasopoulos, 2018)

- **Step 3: Model Identification**

The phase of identifying the correct ARIMA model involves selecting appropriate values for p, d, and q. This is often done using a combination of visual inspection of autocorrelation function (ACF) and partial autocorrelation function (PACF) plots and statistical criteria, as Box and Jenkins (1970) originally proposed. where ACF Used to determine the order q of the moving average process by checking for significant correlations at lagged intervals, while the PACF helps to identify the order p of the autoregressive process by inspecting correlations at various lags while excluding the influence of intermediate lags. Furthermore, modern information criteria, such as AIC and BIC, supplement this traditional approach by providing objective measures for model selection.(Schaffer et al., 2021)

- **Step 4: Model Estimation**

The estimation phase involves determining the optimal parameters for the identified model structure. For instance, the maximum likelihood estimation provides efficient and consistent parameter estimates. the estimation process should account also for the uncertainty in parameter estimates through appropriate standard error calculations and confidence intervals. (Brockwell & Davis, 2016)

- **Step 5: Diagnostic and model validation**

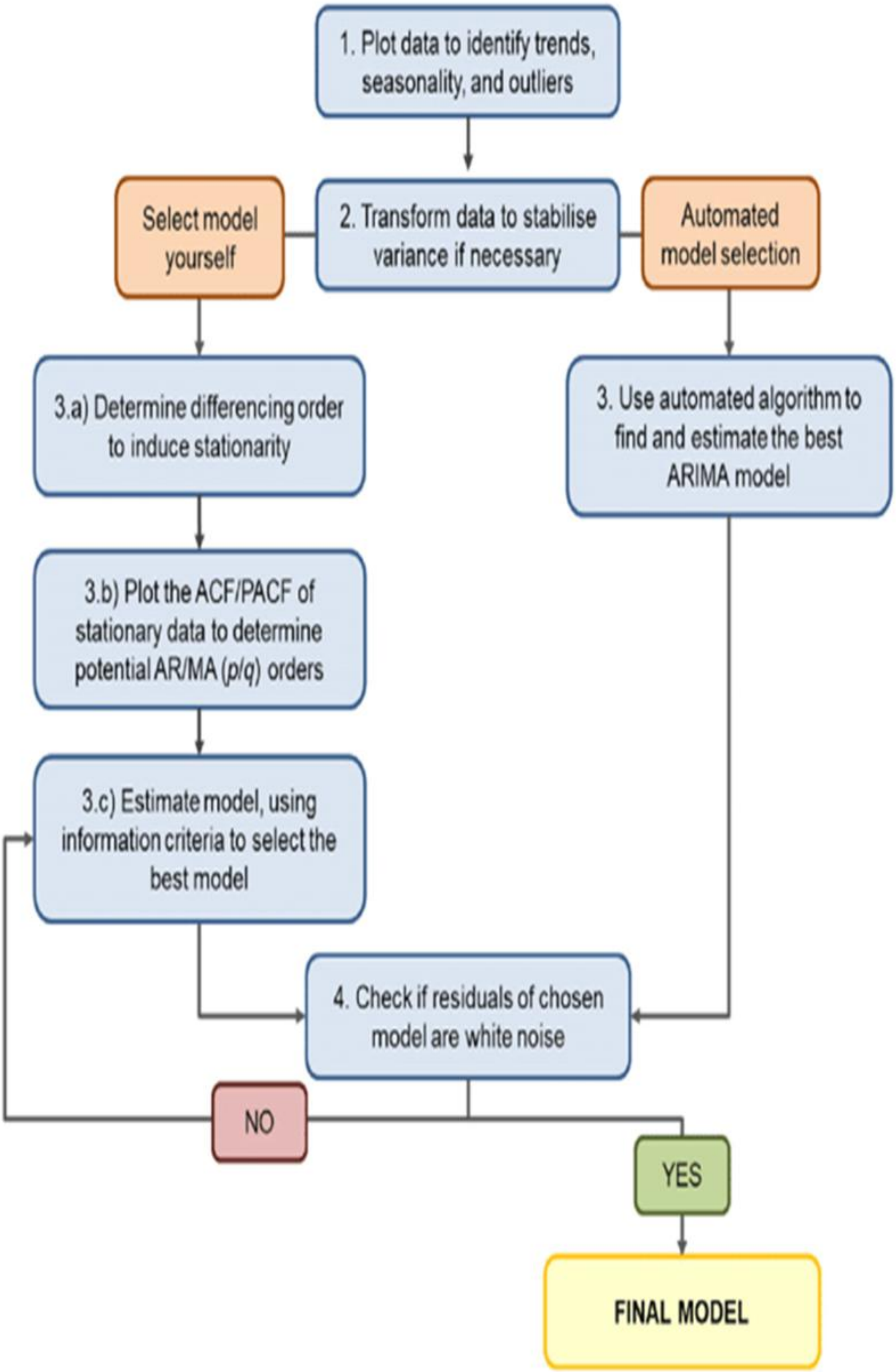
The diagnostic phase involves examining residuals for independence, normality, and homoscedasticity. Indeed, residual analysis should go beyond basic statistical tests to include careful graphical examination of residual patterns. Any systematic patterns in the residuals indicate model misspecification and require returning to earlier steps in the modeling process. Refinement should continue until the residuals exhibit white noise characteristics, such as a mean of zero and no autocorrelation. (Tsay, 2010)

- **Step 6: Forecasting and evaluation**

After selecting the fitted model, the next step is to predict future values using the trained ARIMA model. These predictions are then visualized by plotting the forecasts alongside confidence intervals to better understand their accuracy and uncertainty. Finally, to ensure the model's reliability, the predicted values are compared with the actual data using evaluation metrics such as Mean Absolute Percentage Error (MAPE) and Mean Squared Error (MSE). (Schaffer et al., 2021)

Diagram 4.1 illustrates the process of ARIMA modeling:

Figure 4.1 ARIMA Modelling Process (Box and Jenkins Methodology)



Source :(Schaffer et al., 2021)

c) Seasonal Autoregressive Integrated Moving Average (SARIMA) model:

While ARIMA models assume the seasonal component is deterministic and independent of nonseasonal components, in practice, many time series exhibit stochastic seasonality that interacts with other elements. SARIMA, which stands for Seasonal ARIMA, extends the standard ARIMA model by incorporating seasonal terms is a robust tool for forecasting in various domains, especially when dealing with seasonal data. Its ability to model complex patterns makes it a valuable method for time series forecasting. (Wei, 2006)

The SARIMA model combines standard ARIMA parameters (p, d, q) with additional seasonal terms (P, D, Q), where:

P: represents the seasonal autoregressive term

D: is the seasonal differencing term

Q: denotes the seasonal moving average term

The mathematical representation of the SARIMA model is as follows:

$$\boldsymbol{\Phi_P(B^m)\phi_p(B)(1-B^m)^D(1-B)^d yt = \Theta_Q(B^m)\theta_q(B)w_t}$$

Where:

$\boldsymbol{\Phi_P(B^m)}$: Seasonal autoregressive polynomial of order P

$\boldsymbol{\phi_p(B)}$: Non-seasonal autoregressive polynomial of order p

$\boldsymbol{(1-B^m)^D}$: Seasonal differencing operator, where D is the seasonal differencing degree and m is the seasonal period

$\boldsymbol{(1-B)^d}$: Non-seasonal differencing operator, where d is the differencing degree.

$\boldsymbol{yt}$: The time series being modelled

$\boldsymbol{\Theta_Q(B^m)}$: Seasonal moving average polynomial of order Q

$\boldsymbol{\theta_q(B)}$: Non-seasonal moving average polynomial of order q

$\boldsymbol{w_t}$: Gaussian white noise process

$\boldsymbol{B}$: Backshift operator, where $B^k y_t = y_{t-k}$ (ArunKumar et al., 2022).

1.1.2 Exponential Smoothing Models (ETS):

Exponential smoothing is a statistical approach used for time series forecasting. This model was introduced in the late 1950s by researchers such as Brown (1959), Holt (1957), and Winters (1960), and has since inspired some of the most effective forecasting techniques. These methods generate forecasts by calculating a weighted average of past observations, where the weights decrease exponentially as the data points become older. Essentially, more recent observations are given greater weight in the forecast. This approach is advantageous because it produces accurate forecasts quickly and can be applied to a wide variety of time series data.(R. J. Hyndman & Athanasopoulos, 2018)

The ETS (Error, Trend, Seasonality) model combines different trend and seasonal patterns. It also comprises a forecast equation and a smoothing equation, integrated into a state space model with either additive or multiplicative error assumptions. This setup yields an observation equation for the observed data and a state transition equation for the unobserved (level, trend, and seasonality components). Trend and seasonality can be modelled as (none, additive, additive damped, or multiplicative), allowing for various model configurations. The model is represented by a three-character string (E,T,S), indicating the error, trend, and seasonality types. the additive model is the most commonly used which is linear model, and it is the focus here. (Perone, 2022)

The general formulation of the ETS model, as described in Hyndman et al. (2008) is below:

a) Observation Equation (Forecasting Equation):

The observation equation describes how the observed value y_t is generated from the state components (level, trend, and seasonality):

$$\boldsymbol{y_t = l_{t-1} + b_{t-1} + s_{t-m} + \varepsilon_t}$$

Where:

$\boldsymbol{y_t}$: Observed value at time t

$\boldsymbol{l_{t-1}}$: Level component at time t−1

$\boldsymbol{b_{t-1}}$: Trend component at time t−1

$\boldsymbol{s_{t-m}}$: Seasonal component at time t−m

$\boldsymbol{\varepsilon_t}$: Error term at time t.

b) State Transition Equations (Smoothing Equations):

The state transition equations update the level, trend, and seasonality components over time. These equations depend on the type of ETS model (additive or multiplicative):

- **Level Equation:**

$$\boldsymbol{l_t = l_{t-1} + b_{t-1} + \alpha\varepsilon_t}$$

Where:

$\boldsymbol{l_t}$: Updated level component at time t

α: Smoothing parameter for the level

- **Trend Equation:**

$$b_t = b_{t-1} + \beta\varepsilon_t$$

Where:

$\boldsymbol{b_t}$: Updated trend component at time t.

$\boldsymbol{\beta}$: Smoothing parameter for the trend

- **Seasonality Equation:**

$$s_t = s_{t-m} + \gamma_{\varepsilon t}$$

Where:

$\boldsymbol{s_t}$ **:** Updated seasonal component at time t

$\boldsymbol{\gamma}$: Smoothing parameter for the seasonality

(R. Hyndman et al., 2008)

The ETS method utilizes information criteria such as the Akaike Information Criterion (AIC), the Corrected Akaike Information Criterion (AICc), and the Bayesian Information Criterion (BIC) for model selection, which involves first calculating the information criteria for all candidate models fitted to the data and then selecting the model with the smallest information criterion value, ensuring the chosen model achieves the best balance between fit and complexity.(Qi et al., 2023)

1.1.3 Heterogeneous Autoregressive (HAR) Model:

The HAR model, introduced by Corsi (2009), is one of the commonly used models to predict the realized volatility (RV) of financial assets using high-frequency data. This model is widely adopted due to its simplicity in estimation, ease of interpretation, and robust forecasting capabilities, which together make it a highly effective and accessible tool for volatility modeling in financial markets.

This time-varying autoregressive model leverages lagged RV values across different time scales as predictors. The standard HAR-RV (1, 5, 22) model incorporates three predictors: daily RV, weekly average RV, and monthly average RV.(Zhuo & Morimoto, 2024)

The HAR-RV model can be expressed mathematically as:

$$logRV_{t+1}^{(d)} = c + \beta^{(d)} logRV_t^{(d)} + \beta^{(w)} logRV_t^{(w)} + \beta^{(m)} logRV_t^{(m)} + \epsilon_{t+1}$$

Where:

$logRV_t^{(w)} = \frac{1}{5}\sum_{i=1}^{5} logRV_{t-i+1}^{(d)}$ (weekly average of daily log RV)

$logRV_t^{(m)} = \frac{1}{22}\sum_{i=1}^{22} logRV_{t-i+1}^{(d)}$ (monthly average of daily log RV)

$\boldsymbol{\epsilon_{t+1}}$: is a random error term with zero mean.

The model parameters ($\boldsymbol{c}$, $\boldsymbol{\beta^{(d)}}$, $\boldsymbol{\beta^{(w)}}$, $\boldsymbol{\beta^{(m)}}$) can be estimated using ordinary least squares. (Audrino & Knaus, 2016)

1.2 Non-linear Models:

While linear models are effective for forecasting time series data, they present several limitations, particularly in complex financial scenarios.

Non-linear time series forecasting models are statistical models designed to capture and predict complex, non-linear relationships in time-dependent data. Unlike linear models, which assume a straight-line relationship between variables, non-linear models can represent more intricate patterns, such as curvature, thresholds, regime shifts, and interactions between variables. (Fan & Yao, 2008)

These models are particularly useful when the underlying system exhibits behaviours like volatility clustering, asymmetry, or state-dependent dynamics, which are common in stock markets. (Tsay, 2010)

The most commonly used univariate nonlinear time series forecasting models for the stock market include:

1.2.1 Generalized Autoregressive Conditional Heteroskedasticity (GARCH):

GARCH one of the most used models to model and forecast stock markets volatility. Bollerslev (1986) introduced the model to address the Autoregressive Conditional Heteroskedasticity (ARCH) model's limitation, which often requires a high lag order (s) to accurately capture volatility dynamics over time. By incorporating past conditional variances into the forecasting process, offering a more comprehensive interpretation of volatility. (G. T. Wilson, 2016)

The GARCH(p, q) model is expressed as:

$$\boldsymbol{\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \epsilon_{t-1}^2 \alpha_i t + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2}$$

Where:

$\boldsymbol{\sigma_t^2}$: The conditional variance at time t.

$\boldsymbol{\alpha_0}$: A constant term.

$\boldsymbol{\sum_{i=1}^{q} \epsilon_{t-1}^2 \alpha_i t}$: The ARCH component, which captures the impact of past squared residuals (shocks). where larger past shocks lead to higher current volatility.

$\sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$: The GARCH component, which incorporates past conditional variances, allowing volatility to persist over time. (Shah & Thaker, 2024)

While the basic GARCH(1,1) model is sufficient for capturing volatility in financial time series, the model orders section primarily relies on the Akaike Information Criterion (AIC), which is widely used and highly significant.(Kuhe, 2019)

1.2.2 Exponential GARCH (EGARCH) Model:

To overcome the limitations of the GARCH model in financial time series analysis, Nelson (1991) introduced the exponential GARCH (EGARCH) model. He specifically considered weighted innovation to account for asymmetric effects between positive and negative asset returns. (Kuhe, 2019)

An EGARCH(p, q) model can be written as:

$$ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left(\frac{|\epsilon_{t-1}|}{\sigma_{t-1}} - E\left[\frac{|\epsilon_{t-1}|}{\sigma_{t-1}}\right] \right) \sum_{i=1}^{p} \gamma_i \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \sum_{j=1}^{q} \beta_J \, ln(\sigma_{t-1}^2)$$

Where:

α_0: Constant term.

α_i: Magnitude effect

$\frac{|\epsilon_{t-1}|}{\sigma_{t-1}}$: Standardized absolute residual, measuring volatility response.

γ_i: Asymmetry or leverage effect

$\frac{\epsilon_{t-1}}{\sigma_{t-1}}$: Standardized residual

β_J: Persistence term, capturing past volatility's effect on current volatility.

$ln(\sigma_{t-1}^2)$: Past log variance.(Tsay, 2010)

1.2.3 The Threshold GARCH (TGARCH) Model:

Also known as e GJR-GARCH model, the Threshold GARCH (TGARCH) model is another widely used volatility model designed to capture leverage effects. It was introduced by Glosten, Jagannathan, and Runkle (1993) and Zakoian (1994).

The TGARCH (p, q) model is formulated as:

$$\boldsymbol{\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{q} \gamma_i I_{t-i} \epsilon_{t-i}^2}$$

Where:

$\boldsymbol{\alpha_0}$: the constant term.

$\boldsymbol{\alpha_i}$: past squared residuals (represent the ARCH component)

$\boldsymbol{\beta_j}$: past conditional variances (represent the GARCH component)

$\boldsymbol{I_{t-i}}$: indicator function

$$\boldsymbol{I_{t-i} = \begin{cases} 1, & \text{if } \epsilon_{t-1} < 0 \text{ (negative shock)} \\ 0, & \text{if } \epsilon_{t-1} \geq 0 \text{ (positive shock)} \end{cases}}$$

$\boldsymbol{\gamma_i}$: leverage effect order. (Tsay, 2010)

1.2.4 Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) model:

the Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity model was introduced by Baillie, Bollerslev, and Mikkelsen (1996).

The main goal behind the introduction of the FIGARCH model was to create a more flexible framework for modeling conditional variance, capable of capturing and representing the observed temporal dependencies in financial market volatility. Specifically, the FIGARCH model enables a slow, hyperbolic rate of decay for the influence of past squared or absolute innovations on the conditional variance. This model is designed to account for the time-dependent nature of variance and the leptokurtic (fat-tailed) unconditional distribution of returns, while also incorporating long memory behaviour in conditional variances, making it suitable for modeling financial data with complex volatility dynamics.

The FIGARCH(p,d,q) model is defined as:

$$\boldsymbol{\left(1-\beta(L)\right)\sigma_t^2 = \omega + \left[1-\beta(L)-\phi(L)(1-L)^d\right]\epsilon_t^2}$$

Where:

$\boldsymbol{\beta(L)}$: Polynomial of order q for the GARCH terms.

$\boldsymbol{\sigma_t^2}$: Conditional variance at time t.

$\boldsymbol{\omega}$: Constant term.

$\boldsymbol{\phi(L)}$: Polynomial of order pp for the ARCH terms.

$\boldsymbol{(1-L)^d}$: Fractional differencing operator, where LL is the lag operator and dd is the fractional integration parameter (0<d<1).

$\boldsymbol{\epsilon_t^2}$: Squared residuals (innovations) at time t. (Tayefi & Ramanathan, 2012)

1.2.5 Threshold Autoregressive (TAR) Model:

Threshold Autoregressive (TAR) models were first introduced by Tong (1978) and further developed by Tong and Lim (1980). (G. T. Wilson, 2016)

This model is particularly relevant in financial time-series analysis, especially in scenarios where price deviations from equilibrium values are influenced by discrete transaction costs. Additionally, TAR techniques are applicable when market regulators implement intervention strategies based on threshold levels of control variables. A significant application of TAR models in finance is their use in modeling price differences between equivalent assets in the presence of transaction costs. (Yadav et al., 1994)

A two-regime TAR model can be expressed as:

$$\boldsymbol{Y_t} = \begin{matrix} \boldsymbol{\phi_{1,0} + \sum_{i=1}^{p} \phi_{1,i} Y_{t-i} + \epsilon_t}, & \boldsymbol{Y_{t-d} \leq r} & \text{(Regime 1)} \\ \boldsymbol{\phi_{2,0} + \sum_{i=1}^{p} \phi_{2,i} Y_{t-i} + \epsilon_t}, & \boldsymbol{Y_{t-d} > r} & \boldsymbol{(Regime\ 2)} \end{matrix}$$

Where:

$\boldsymbol{p}$: the autoregressive order

$\boldsymbol{\phi_{1,i}}$ and $\boldsymbol{\phi_{2,i}}$: the autoregressive coefficients for different regimes.

$\boldsymbol{r}$: the threshold level

$\boldsymbol{d}$: the delay parameter

$\boldsymbol{\epsilon_t}$: a white noise error term.(G. T. Wilson, 2016)

1.2.6 Smooth Transition Autoregressive (STAR) Model:

The Smooth Transition Autoregressive (STAR) model, introduced by Chan and Tong (1986) and further developed by Teräsvirta (1994). This model was designed to address the discontinuity issue inherent in the Threshold Autoregressive (TAR) model. While the TAR model exhibits sharp breaks in the conditional mean equation at the thresholds $\boldsymbol{y_t}$, the STAR model employs a smooth transition function to shift between regimes. This approach ensures the continuity of the conditional mean function $\boldsymbol{\mu_t}$, eliminating abrupt changes. By offering a more flexible and realistic framework for capturing threshold dynamics, the STAR model has become a widely used tool in nonlinear time series analysis. (Tsay, 2010)

The STAR model is typically represented by the following equation:

$$\boldsymbol{y_t = \left(\phi_{1,0} + \sum_{i=1}^{p} \phi_{1,i} y_{t-i}\right)\left(1 - G(s_t; \gamma, c)\right) + \left(\phi_{2,0} + \sum_{i=1}^{p} \phi_{2,i} y_{t-i}\right) G(s_t; \gamma, c) + \epsilon_t}$$

Where:

$\boldsymbol{\phi_{1,0}}$, $\boldsymbol{\phi_{1,i}}$,$\boldsymbol{\phi_{2,0}}$, $\boldsymbol{\phi_{2,i}}$: the autoregressive coefficients for the two regimes.

$\boldsymbol{p}$: the order of the autoregressive process.

$\boldsymbol{s_t}$: the transition variable.

$\boldsymbol{\gamma}$: the smoothness parameter.

$\boldsymbol{c}$: the threshold parameter.

the transition function $\boldsymbol{G(s_t; \gamma, c)}$ is often chosen to be a logistic function or an exponential function. (Dijk et al., 2002)

1.2.7 Stochastic Volatility Models:

The roots of Stochastic Volatility (SV) models can be traced to the foundational work of Hull and White (1987) and Heston (1993). These models are a class of financial frameworks used to describe the time-varying nature of volatility in asset prices. Unlike traditional models that assume constant volatility, SV models allow volatility to evolve randomly over time, capturing key empirical features observed in financial markets, such as volatility clustering and the leverage effect. These models are widely used in options pricing, risk management, and financial econometrics.

A basic version of a stochastic volatility model is defined as the follow:

$$\boldsymbol{ln(\sigma_t^2) = \alpha_0 + \beta_1 \, ln(\sigma_{t-1}^2) + \cdots + \beta_r \, ln(\sigma_{t-r}^2) + \nu_t}$$

Where :

$\boldsymbol{\alpha_0}$: A constant term.

$\boldsymbol{\beta_r}$: the coefficients of the lagged terms.

$\boldsymbol{v_t}$: A random shock or innovation term, typically assumed to be normally distributed with mean zero and constant variance $(v_t \sim \mathrm{N}(0, \sigma_v^2))$. (G. T. Wilson, 2016)

1.2.8 Markov Switching Model (MSM):

Markov-switching models (MSMs) were introduced into the econometric mainstream by Hamilton (1989). These probabilistic models utilize multiple sets of parameters to characterize distinct dynamic regimes that a time series may transition through over different time periods. The switching mechanism between these regimes is governed by unobserved variables that follow a first-order Markov chain. MSMs are widely employed for segmenting time series data or uncovering the hidden dynamics beneath noisy observations. (Chiappa, 2014)

Markov switching model can be expressed as:

$$\boldsymbol{y_t = \mu_{s_t} + \phi_{s_t} y_{t-1} + \sigma_{s_t} \epsilon_t}$$

Where:

$\boldsymbol{s_t}$: The unobserved state or regime at time t.

$\boldsymbol{\mu_{s_t}}$: The intercept or mean in regime $\boldsymbol{s_t}$.

$\boldsymbol{\phi_{s_t}}$: The autoregressive coefficient in regime $\boldsymbol{s_t}$.

$\boldsymbol{\sigma_{s_t}}$: The standard deviation of the error term in regime $\boldsymbol{s_t}$.

$\boldsymbol{\epsilon_t}$: The error term, typically assumed to be normally distributed with mean 0 and variance 1. (Lai, 2022)

The probability of transitioning from one regime to another is defined as:

$$\boldsymbol{P(S_t = j \mid S_{t-1} = i) = p_{ij}}$$

$\boldsymbol{p_{ij}}$: The probability of transitioning from regime i*i* at time t−1 to regime j at time t.

For a model with two regimes, the transition probabilities are often represented in a transition matrix P:

$$p = \begin{bmatrix} p11 & p12 \\ p21 & p22 \end{bmatrix}$$

where:

p11: Probability of staying in regime 1.

p12: Probability of transitioning from regime 1 to regime 2.

p21: Probability of transitioning from regime 2 to regime 1.

p22: Probability of staying in regime 2. (Aydın & Mermi, 2022)

2 Machine Learning Models for Univariate Time Series Forecasting:

Machine learning, a dynamic and fast-evolving sub-field of artificial intelligence, has seen remarkable growth in recent years, with applications expanding across industries and gaining significant public attention. In simple terms, machine learning involves using computers to perform tasks that they can learn from and improve at over time, without the need for explicit programming. (Nilsson, 1998)

For many years, traditional methods have been used to predict the stock market. However, the introduction of machine learning has revolutionized this process, making predictions more accurate and accessible. A wide range of machine learning techniques have been successfully employed in the field of stock market prediction; The most widely used techniques include the following:

2.1 Artificial Neural Networks (ANNs):

Artificial Neural Network (ANNs) modeling, an artificial intelligence technique inspired by the human brain, has been widely employed since the 1980s for forecasting system variables, particularly in computer science research. (Malekian & Chitsaz, 2021)

This approach advanced significantly with the development of the multi-layer perceptron (MLP), an advance architecture for modeling nonlinearly separable functions. (Kotu & Deshpande, 2018)

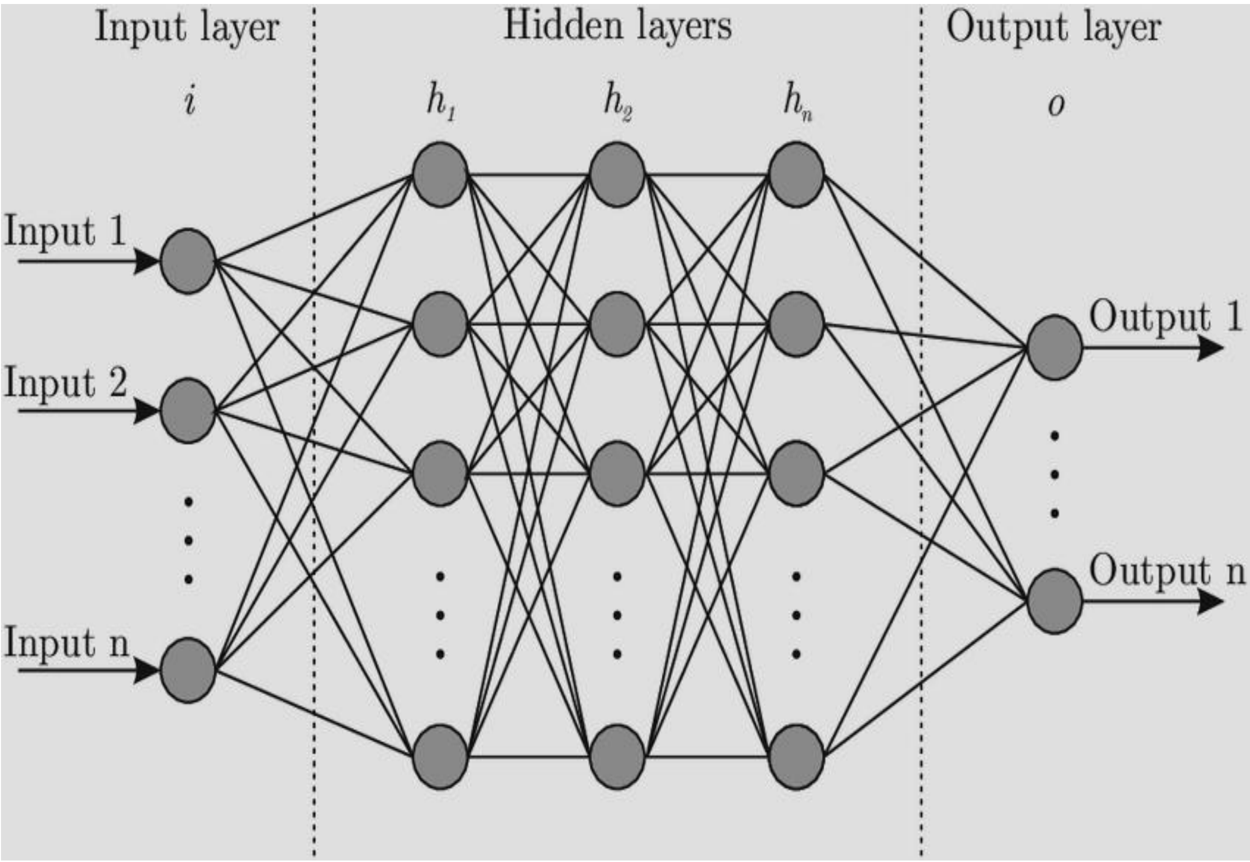
The primary objective of ANNs modeling is to enable adaptive problem-solving by learning patterns from data across diverse fields such as electrical engineering, autonomous vehicle navigation, medical diagnostics, and financial markets. In fact, ANNs mimic biological neurons, which are interconnected cells that transmit data through electrical or chemical signals. Instead of relying on predefined algorithms, ANN models acquire knowledge by identifying patterns and relationships within data, enabling them to learn from iterative training. This approach encompasses various learning mechanisms, including inductive and deductive reasoning, making ANNs particularly effective for predictive modeling and interpolation tasks. (Malekian & Chitsaz, 2021)

Artificial Neural Networks (ANNs) are composed of layered computational units (neurons) interconnected via weighted connections. The architecture is structured into three primary components: input layer, hidden layers (intermediate layers) , and output layer.

The input layer neurons correspond to independent variables (e.g., input data features), while output layer neurons represent dependent variables (e.g., predictions or classifications). During signal propagation, each neuron processes inputs from the preceding layer by computing a weighted sum, which is then transformed by an activation function into an output signal. This output is transmitted to neurons in the subsequent layer, enabling hierarchical learning and pattern extraction. (Bi et al., 2019)

The architecture of artificial neural networks (ANNs) is structured as follows:

Figure 4.2 ANNs Architecture



Source: (Bre et al., 2017)

For time series forecasting, a dynamic neural network (DNN) is recommended, since it characterized by multiple hidden layers and depends on both current and past values. Its structure can be represented as follows:

$$\boldsymbol{y_t = f(y_{t-1}, y_{t-2}, y_{t-3}, \dots, y_{t-d}) + \varepsilon_t}$$

Where:

$\boldsymbol{y_t}$: the original time series.

$\boldsymbol{f}$: a nonlinear function.

$\boldsymbol{y_{t-1}, y_{t-2}, y_{t-3}, \dots, y_{t-d}}$: feedback lags.

$\boldsymbol{\varepsilon_t}$: error terms. (Shah & Thaker, 2024)

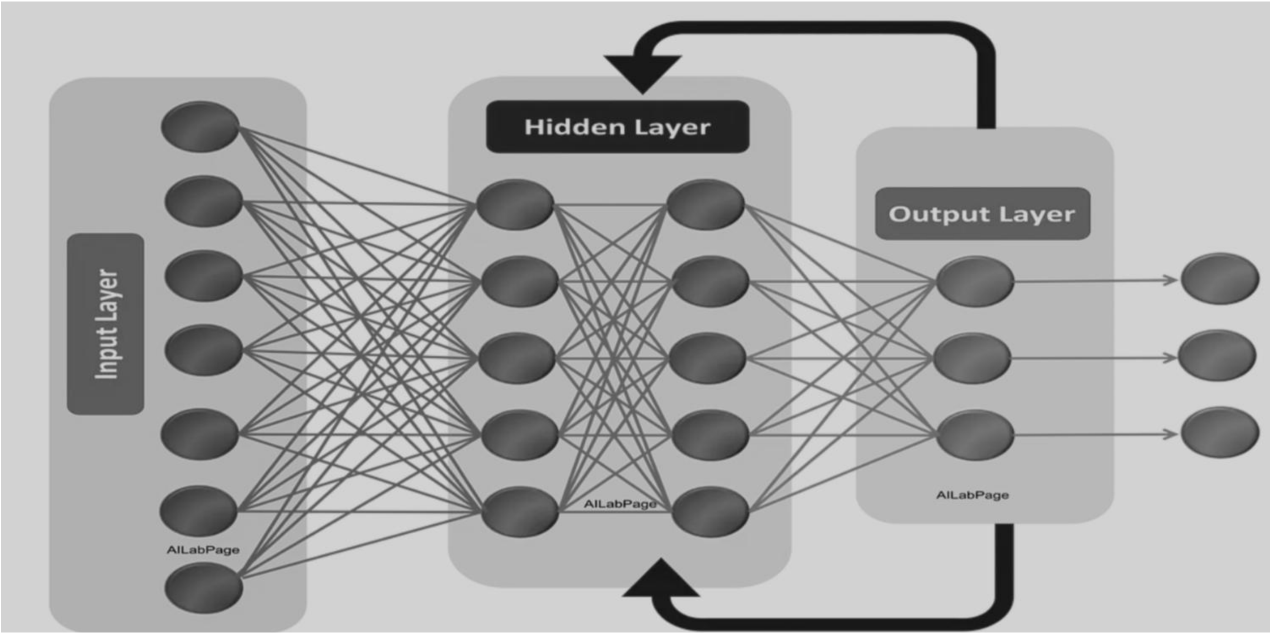
2.2 Recurrent Neural Networks (RNNs):

A Recurrent Neural Network (RNN) is a type of neural network designed for sequential data processing, addressing a critical limitation of Multilayer Perceptrons (MLPs), which lack inherent mechanisms to model temporal dependencies. Unlike MLPs, RNNs process inputs sequentially, with each step's output influenced by both the current input and previous states. This is achieved through feedback loops within hidden layers, where neurons transmit information not only to the next layer but also back to themselves. These loops enable RNNs to retain contextual information across time steps, forming a memory of past data, which gives the network its recurrent nature.

Indeed, the ability to propagate information through time allows RNNs to model dynamic temporal patterns, capturing both linear and nonlinear relationships in sequential datasets. This makes RNNs particularly effective for tasks requiring analysis of time-dependent behaviours, such as time series prediction, language modeling, and speech recognition. By integrating historical context, RNNs excel in applications where outputs depend on evolving sequences such as sequence classification tasks. Additionally, they can be adapted for sequence classification tasks and applied to static, dynamic, spatial, or temporal decision boundaries. (Malekian & Chitsaz, 2021)

The architecture of artificial neural networks (ANNs) is structured as follows:

Figure 4.3 RNNs Architecture



Source: (kondreddy, 2023)

2.3 Long Short-Term Memory Networks (LSTM):

The long short-term memory (LSTM) network, introduced by Hochreiter and Schmidhuber in 1997, is an advanced type of recurrent neural network (RNN) designed to address the vanishing gradient problem inherent in traditional RNNs. (Staudemeyer & Morris, 2019)

While standard RNNs rely on backpropagation through time (BPTT) for training, BPTT struggles with long sequences due to its reliance on gradient chains. As gradients are propagated backward across each timestep from the sequence's end to its beginning repeated multiplicative operations cause gradients to decline exponentially with sequence length. This phenomenon, known as vanishing gradients, severely limits the model's ability to learn dependencies in long sequences.

To overcome this limitation, LSTM introduces a novel architecture centred on memory cells and gating mechanisms, Unlike RNNs, which use a single hidden state *h(t)*, LSTMs maintain two states:

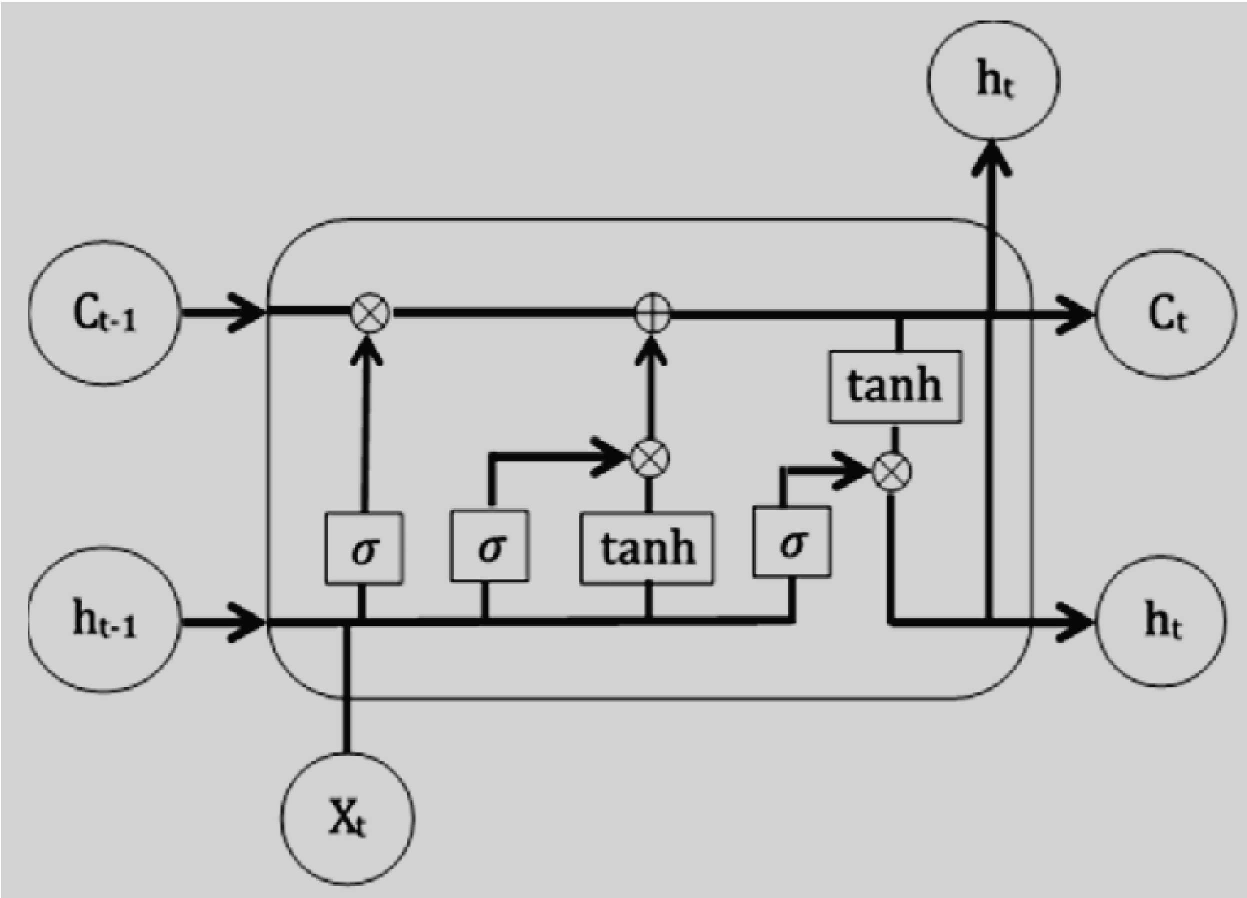
- ***h*(*t*):** The short-term memory state, analogous to traditional RNNs.
- ***c*(*t*):** A dedicated long-term memory cell state.

These components work together, regulated by three gates (input, forget, and output) that selectively update, retain, or discard information. By separating the long-term memory state ***ct*** from the short-term hidden state ***ht***, LSTMs facilitate stable gradient flow over long sequences. This structure allows the network to capture both short- and long-term dependencies effectively

This architecture makes LSTMs particularly robust for tasks requiring memory over prolonged intervals, such as time-series prediction or natural language processing. (M. M. Hussain et al., 2021)

To better understand how LSTMs function, we explore the fundamental components of an LSTM cell below:

Figure 4.4 LSTM Cell Architecture



Source: (Toma et al., 2019)

The process relies on four main components: inputs, gates, cell state, and hidden state, which work together to manage information flow and maintain long-term dependencies.

where:

Inputs:

- X_t : Current input data.
- h_{t-1} : Previous hidden state (captures past information).
- c_{t-1} : Previous cell state (carries long-term memory).

Gates:

- Forget Gate ($\boldsymbol{\sigma}$): Decides what part of the past cell state (c_{t-1}) should be kept or forgotten.
- Input Gate (σ and tanh): Decides what new information should be added to the cell state.
- Output Gate (σ and tanh): Decides what information should be passed to the next hidden state.

Cell State *c(t)*:

- This is the "memory" of the LSTM, which is updated based on the forget and input gates.

Hidden State *h(t)*:

- The output of the LSTM cell, which will be used in the next time step.

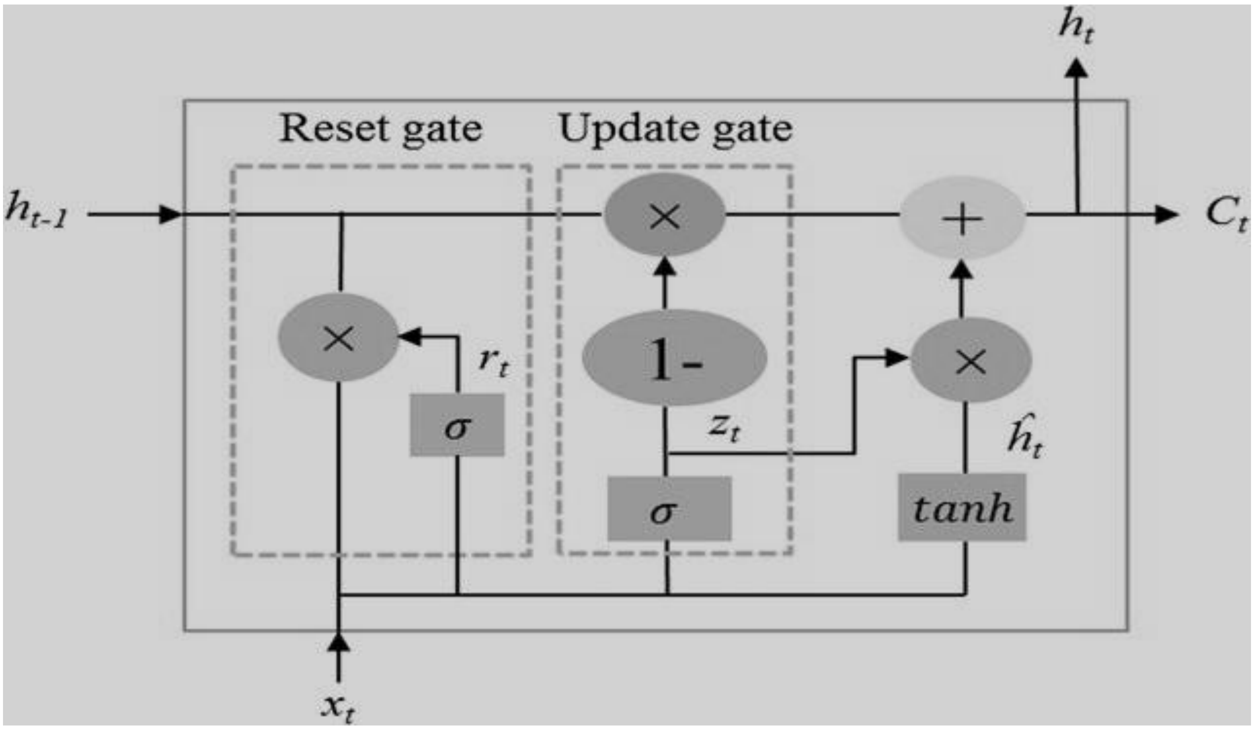
2.4 Gated Recurrent Units (GRUs):

The Gated Recurrent Unit (GRU), introduced by Cho et al. in 2014, is a streamlined variant of the Long Short-Term Memory (LSTM) architecture. (M. M. Hussain et al., 2021)

The GRUs is designed to enhance computational efficiency, particularly when processing large-scale datasets. it simplifies the internal structure of traditional LSTM blocks by reducing the number of gating mechanisms. Instead of three gates as in LSTMs, the GRU employs only two gates (update gate and reset gate). The update gate governs the retention or integration of new information across sequential time steps, while the reset gate regulates how much historical data is retained or discarded. This simplification reduces model complexity and accelerates training, making GRUs particularly advantageous for tasks requiring rapid processing of sequential data. (Oğuz & Ertuğrul, 2023)

To gain a clearer understanding of how GRUs work, we examine the core components of a GRU cell below:

Figure 4.5 GPUs Cell Architecture



Source: (Mohsen, 2023)

The process relies on five main components: inputs, Reset Gate (r_t), **Update Gate** (z_t**)**, **Candidate Hidden State (**$\hat{h}_t$), Final Hidden State (h_t):

Where:

- **Inputs:**

$\boldsymbol{X_t}$**:** Current input vector at time step *t.*

$\boldsymbol{h_{t-1}}$: Hidden state from the previous time step.

- **Reset Gate (**$\boldsymbol{r_t}$**):**

($\boldsymbol{r_t}$) controls how much of the past hidden state h_{t-1} is forgotten

$$\boldsymbol{r_t = \sigma(W_r x_t + U_r h_{t-1})}$$

$\boldsymbol{\sigma}$ (sigmoid function) squashes the values between 0 and 1.

A value close to 0 means less past information is retained.

- **Update Gate (**$\boldsymbol{z_t}$**)**:

($\boldsymbol{z_t}$) Controls how much of the past hidden state is carried forward to the next step.

$$\boldsymbol{z_t = \sigma(W_z x_t + U_z h_{t-1})}$$

If z_t is close to 1, the previous hidden state is preserved.

- **Candidate Hidden State (**$\boldsymbol{\hat{h}_t}$**):**

($\hat{h}_t$): Defines the new hidden state based on the reset gate.

$$\widehat{h_t} = tanh\big(W_h x_t + U_h(r_t \cdot h_{t-1})\big)$$

tanh function ensures values are between -1 and 1.

- **Final Hidden State (h_t):**

(h_t) decides whether to keep the old hidden state or use the new candidate state.

$$h_t = (1 - z_t) \cdot h_{t-1} + z_t \cdot \widehat{h_t}$$

If z_t is close to 1, the new state relies more on h_{t-1}; if close to 0, it updates more with $\hat{h}_t$.

2.5 Convolutional Neural Networks (CNNs):

Convolutional Neural Networks (CNNs), first conceptualized in the 1980s by Kunihiko Fukushima, are a specialized subclass of artificial neural networks (ANNs) and a cornerstone of deep learning for tasks like object recognition. (Bhatnagar, 2023)

Convolutional Neural Networks (CNNs) are composed of four core elements, each serving distinct roles:

- **Convolutional Layer**:

Uses learnable kernel filters to detect spatial patterns (edges, textures…) in input data through local feature extraction.

- **Pooling Layer**:

compress feature maps to reduce computational complexity and spatial dimensions.

- **Fully Connected Layer**:

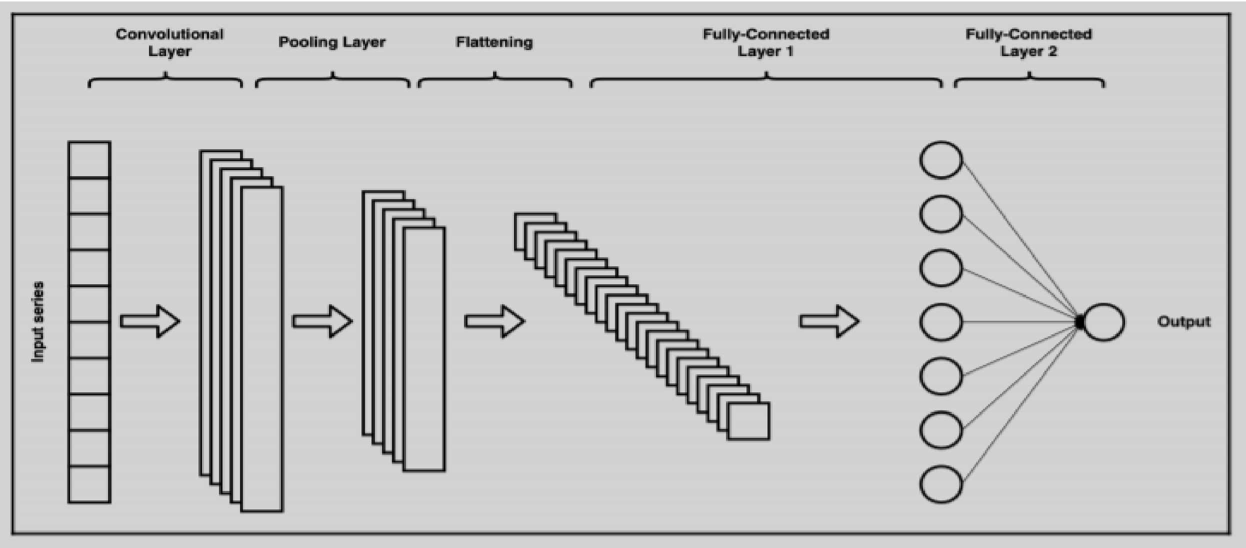
Functions as the classifier or regressor, integrating high-level features from earlier layers to produce final predictions.

- **Activation Functions**:

Introduce non-linearity to enable complex pattern learning. (Purwono et al., 2023)

Figure 4.6 illustrates the Convolutional Neural Networks (CNNs) architecture adapted for time series forecasting.

Figure 4.6 CNNs Architecture for Time Series Forecasting



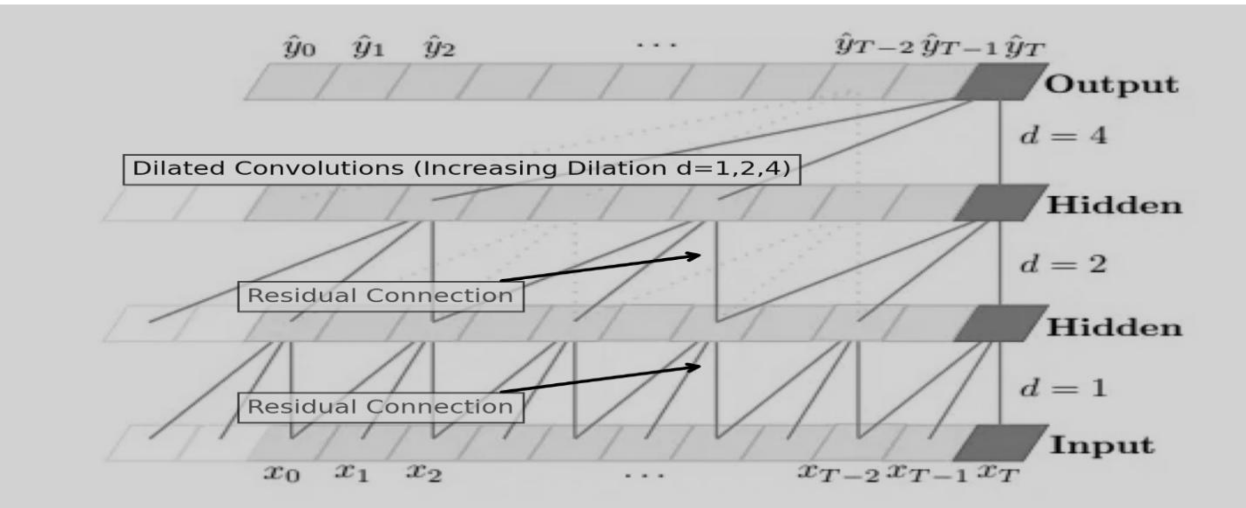
Source: (Avinash, 2021)

2.6 Temporal Convolutional Networks (TCNs):

Temporal Convolutional Networks (TCNs), a specialized architecture for sequence modeling, excel in tasks such as time series forecasting. Built hierarchically with stacked convolutional layers, TCNs leverage three core components: causal convolutions to prevent information leakage from future to past, dilated convolutions to expand the receptive field efficiently, and residual connections to stabilize gradients and accelerate training. By combining these elements, TCNs achieve long-term dependency capture, mitigate vanishing/exploding gradient issues, and enhance computational performance, making them robust for sequential data analysis. (Guerdan, 2019)

Figure 4.7 presents a clear illustration of the Temporal Convolutional Networks (TCNs) architecture:

Figure 4.7 TCNs Architecture



Source: adapted from (Roy, 2019)

2.7 Transformer:

Transformers are one of the most powerful deep learning architectures for time series forecasting. They rely entirely on attention mechanisms to process sequential data without recurrence. Its core innovation is the Scaled Dot-Product Attention, which computes contextual relevance across sequences through three steps. (Lara-Benítez et al., 2021)

- **Query-Key Alignment**:

This measures how similar each key is to the query. The result is scaled by $\frac{1}{\sqrt{d_k}}$ to prevent very large values.

- **Attention Weights:**

This converts similarity scores into probabilities (weights) that sum to 1.

- **Context Synthesis:**

This gives an output that is a combination of the values, based on the attention scores.

The Transformer equation is expressed as:

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V$$

Where:

Q **(Query)**: A matrix representing the current token or sequence trying to find relevant information.

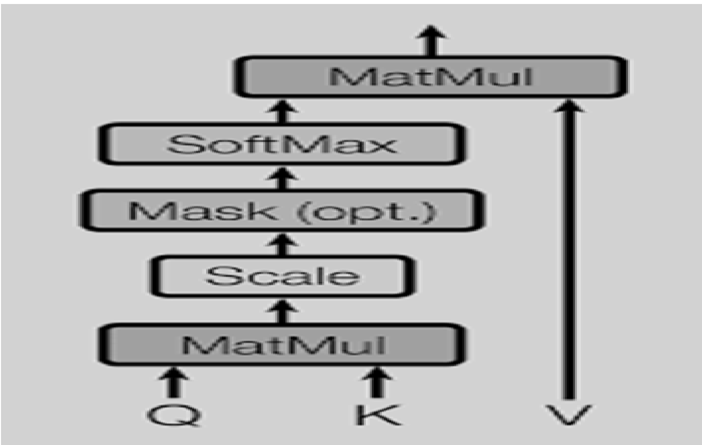
K **(Key)**: A matrix containing all potential reference points in the sequence.

V **(Value)**: A matrix containing the actual information that will be retrieved based on attention scores.

d_k: The dimension of the key vectors, used for scaling.(Lara-Benítez et al., 2021)

Figure 2.7 illustrates the attention mechanism clearly:

Figure 4.8 Scaled Dot-Product Attention mechanism



Source: (Wang et al., 2022)

Where:

- **MatMul (Q, K)**: Calculating similarity between the query and keys.
- **Scaling**: Divides by $\sqrt{d_k}$ for numerical stability.
- **Masking (optional)**: Sometimes used in autoregressive tasks.
- **Softmax**: Converting scores into probabilities.
- **MatMul (Attention Weights, V)**: Synthesizes context by weighting values.

2.8 Prophet model:

The Prophet model, created by Facebook researchers Taylor and Letham in 2018, is designed for forecasting daily data with weekly and yearly seasonality, along with holiday effects. It was later extended to handle a broader range of seasonal data. The model performs best when applied to time series with strong seasonality and multiple seasons of historical data.

The model is defined as:

$$y_t = g(t) + s(t) + h(t) + \epsilon_t$$

where:

- $g(t)$: represents A flexible, segmented trend line that automatically adjusts direction when the data's growth pattern changes.
- $s(t)$: captures seasonal patterns,
- $h(t)$: accounts for holiday effects,
- ϵ_t: is a white noise error term. (R. J. Hyndman & Athanasopoulos, 2021)

To fit the proposed model with seasonality effects and forecast based on it, Prophet uses a Fourier series, which provides a flexible approach for modeling seasonality. The seasonal effects s(t) can be represented as:

$$s(t) = \sum_{n=1}^{N} \left(a_n \cos\left(\frac{2\pi nt}{P}\right) + b_n \sin\left(\frac{2\pi nt}{P}\right) \right)$$

where P denotes the cycle length of the seasonal pattern. And N number of fourier terms.(Samal et al., 2019)

3 Performance Evaluation of Univariate Time Series Forecasting Models:

Performance evaluation is a broad research area that encompasses multiple disciplines. Performance metrics, or error measures, play a crucial role in evaluation frameworks across various fields. In forecasting, performance metrics have long been used to assess how much predictions deviate from actual observations. These deviations help determine forecast quality and guide the selection of appropriate forecasting methods. (Botchkarev, 2019)

Forecasting accuracy is measured using errors, which represent the difference between an observed value and its predicted value. Mathematically, this error can be expressed as:

$$e_{T+h} = y_{T+h} - \hat{y}_{T-h|T}$$

Where:

$\{y_1, \dots, y_T\}$**:** The training data**.**

$\{y_{T+1}, y_{T+2} \dots\}$**:** The test data. (R. J. Hyndman & Athanasopoulos, 2021)

Choosing an appropriate performance metric is essential for evaluating the quality of a forecasting model. This evaluation helps compare different forecasting techniques and supports decision-making processes. (St-Aubin & Agard, 2022)

Over time, many measures of forecast accuracy have been proposed. Researchers have also provided guidelines on selecting the most suitable metric for comparing the accuracy of different forecasting methods, particularly for univariate time series. (Koehler, 2006)

Some of the most commonly used accuracy measures include:

3.1 Absolute Errors:

Absolute error represents the difference between actual and predicted values, calculated by subtracting the predicted values from the actual values (Rama, 2021). It ensures that positive and negative errors do not cancel each other out by taking only non-negative values, making it easy to aggregate distances across a dataset. Absolute error retains the same units as the data and assigns equal weight to all errors. Its arithmetic mean represents the average error, providing a clear measure of overall prediction accuracy. (Botchkarev, 2019)

Common absolute errors metrics include:

3.1.1 Mean Absolute Error (MAE) :

The Mean Absolute Error (MAE) measures the average absolute difference between predicted and actual values. Given a test set of n observations, where $\hat{y}$ represents the predicted value and y_i represents the actual value, MAE is calculated as:

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$$

An MAE of 0 represents perfect accuracy. A lower MAE indicates better model performance, whereas a higher MAE is preferred. (Lendave, 2021)

3.1.2 Median absolute error (MedAE):

The Median Absolute Error (MedAE) is a performance metric that measures the median of the absolute differences between observed values (true values) and predicted values (model output).

The median absolute error (MDAE) is calculated as:

$$\text{MedAE} = \text{median}(|y_i - \hat{y}_i|)$$

Where:

y_i: represents the actual value.

$\hat{y}$: represents the predicted value.(Michael, 2023)

3.2 Squared Errors:

Squared error a performance metric, that prevents negative values and error cancellation, like absolute error. However, squaring makes large errors more influential while reducing the impact of small ones. squared error is widely used due to its favourable mathematical properties, such as being continuously differentiable, which simplifies optimization. (Botchkarev, 2019)

Common squared errors metrics include:

3.2.1 Mean Square Error (MSE):

Mean Squared Error (MSE) measures the average squared difference between forecasted and actual values. It prevents error cancellation by squaring deviations, ensuring all errors contribute positively. MSE penalizes large errors more than small ones, making it sensitive to extreme deviations. This highlights significant forecasting mistakes, but it also means that large errors have an extremely high impact on the overall metric. Due to its mathematical properties, MSE is widely used in model training, as it simplifies optimization and improves performance evaluation. (Adhikari & Agrawal, 2013)

Mathematically MSE can be expressed as:

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2$$

3.2.2 Root Mean Squared Error (RMSE) :

RMSE is the square root of MSE and inherits all its properties.
Mathematically, it is defined as:

$$RMSE = \sqrt{MSE}$$

3.2.3 Normalized Mean Squared Error (NMSE):

The Normalized Mean Squared Error (NMSE) is a balanced error measure that evaluates forecast accuracy by normalizing the Mean Squared Error (MSE) through division by the test variance. A smaller NMSE value indicates better forecasting performance. NMSE shares the same properties as MSE and is effective for comparing models across different datasets.(Adhikari & Agrawal, 2013)

NMSE is calculated as:

$$\mathbf{NMSE} = \frac{\sum_{t=1}^{n}\left(Y_t - \widehat{Y_t}\right)^2}{\sum_{t=1}^{n}(Y_t - \bar{Y})^2}$$

3.2.4 Sum of Squared Error (SSE):

SSE measures the total squared deviation of forecasted observations from the actual values and has the same properties as MSE. (Adhikari & Agrawal, 2013)

It is defined mathematically as:

$$\mathbf{SSE} = \sum_{t=1}^{n}\left(Y_t - \widehat{Y_t}\right)^2$$

3.2.5 Signed Mean Squared Error (SMSE):

The Signed Mean Squared Error (SMSE) is similar to the Mean Squared Error (MSE), but it retains the original sign for each individual squared error, allowing it to indicate the direction of the overall error. Unlike MSE, positive and negative errors balance each other in SMSE, which penalizes extreme forecasting errors. However, SMSE is sensitive to changes in scale and data transformations like MSE. (Adhikari & Agrawal, 2013)

SMSE Mathematically expressed as:

$$\mathbf{SMSE} = \frac{\sum_{t=1}^{n}\left(Y_t - \widehat{Y}_t\right)^2}{n \cdot \mathrm{Var}(Y)}$$

3.3 Percentage Errors:

Percent error is the relative difference between the estimated and actual values, expressed as a percentage. It is calculated by multiplying the relative error by 100. (BYJUS, 2020)

Common percentage errors metrics include:

3.3.1 Mean Absolute Percentage Error (MAPE):

MAPE measures the average absolute error as a percentage of actual values. It is scale-independent but affected by data transformation. It does not indicate error direction or penalize extreme deviations, and opposite errors don't cancel out. MAPE is calculated as the average absolute difference between predicted (Ft) and actual values (At), divided by the actual value, with n representing the total number of values in the test set. (Adhikari & Agrawal, 2013)

The MAPE is calculated as:

$$\mathrm{MAPE} = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Y_t - \widehat{Y_t}}{Y_t}\right| \times 100$$

3.3.2 Mean Percentage Error (MPE):

Mean Percentage Error (MPE) measures the average percentage error in a forecast. Like Mean Absolute Percentage Error (MAPE), it quantifies prediction accuracy but also indicates the direction of errors. Positive and negative errors balance each other, potentially leading to cancellation. A lower MPE value indicates a more accurate forecast.(Adhikari & Agrawal, 2013)

MPE is calculated as:

$$\mathrm{MPE} = \frac{1}{n}\sum_{t=1}^{n}\left(\frac{Y_t - \widehat{Y_t}}{Y_t}\right) \times 100$$

3.3.3 Symmetric Mean Absolute Percentage Error (SMAPE):

SMAPE (Symmetric Mean Absolute Percentage Error) is a common way to measure how accurate forecasting models are, especially in time series analysis.

It calculates the difference between predicted and actual values while treating both overestimations and underestimations equally. This makes it fairer than some other error measures. (Mertbayraktar, 2023)

The formula for SMAPE is:

$$\mathrm{SMAPE} = \frac{1}{n}\sum_{t=1}^{n}\frac{\left|Y_t - \widehat{Y_t}\right|}{\frac{\left|Y_t\right| + \left|\widehat{Y_t}\right|}{2}} \times 100$$

3.3.4 Weighted Absolute Percentage Error (WAPE):

The Weighted Absolute Percentage Error (WAPE) measures how much the predicted values differ from the actual values. It is calculated by comparing the total of the predicted values to the total of the actual values. A lower WAPE means the model is more accurate. (Amazon Forecast developer guide, 2024)

WAPE is calculated as:

$$\text{WAPE} = \frac{\sum_{t=1}^{n}\left|Y_t - \widehat{Y_t}\right|}{\sum_{t=1}^{n}|Y_t|} \times 100$$

3.4 Scaled Errors:

Scaled errors are a type of forecast error metric used to assess the performance of forecasting models relative to a benchmark, typically the naïve forecast. They help standardize errors, allowing for comparison across different datasets and models. A scaled error is calculated by dividing the forecast error by the average absolute error of a naïve one-step in-sample forecast. (R. Hyndman, 2006)

Commonly used scaled errors metrics include:

3.4.1 Mean Absolute Scaled Error (MASE):

Mean Absolute Scaled Error (MASE) proposed by Hyndman and Koehler (2006), it is a reliable measure of forecast accuracy, addressing issues found in other metrics. The MASE scales forecast errors using the in-sample Mean Absolute Error (MAE) from the naïve forecast method, which generates one-step-ahead forecasts for each point in the sample.

MASE offers two key advantages:

- Compare forecasting methods on a single time series.
- Assess forecast accuracy across different time series.

MASE is defined mathematically as:

$$\text{MASE} = \frac{\frac{1}{n}\sum_{t=1}^{n}|y_t - \widehat{y_t}|}{\frac{1}{n-1}\sum_{t=2}^{n}|y_t - y_{t-1}|}$$

MASE can result in three possible outcomes:

MASE < 1: Forecast performs better than the naïve forecast.

MASE = 1: Forecast performs equally to the naïve forecast.

MASE > 1: Forecast performs worse than the naïve forecast.

3.4.2 Root Mean Squared Scaled Error (RMSSE):

The Root Mean Squared Scaled Error (RMSSE) is an extension of the Mean Absolute Scaled Error (MASE) that uses the squared error instead of the absolute error. It provides a scale-independent measure of forecast accuracy and is more sensitive to large forecast errors. (R. Hyndman & Koehler, 2005)

RMSSE is defined mathematically as:

$$\text{RMSSE} = \sqrt{\frac{\frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2}{\frac{1}{n-1}\sum_{t=2}^{n}(y_t - y_{t-1})^2}}$$

3.4.3 Median Absolute Scaled Error (MdASE):

The Median Absolute Scaled Error (MdASE) is another variation of the Mean Absolute Scaled Error (MASE) that uses the median instead of the mean. It provides a more robust measure of forecast accuracy, especially in the presence of outliers, as the median is less sensitive to extreme values. (R. Hyndman & Koehler, 2005)

MdASE is defined mathematically as:

$$\text{MdASE} = \frac{\text{median}(|y_t - \hat{y}_t|)}{\text{median}(|y_t - y_{t-1}|)}$$

3.5 Relative Errors:

Relative Error refers to a way of measuring the difference between an estimated value and the actual value, expressed as a ratio the absolute error divided by the actual value. (Ford, 2015)

Commonly used relative errors metrics include:

3.5.1 The Relative Absolute Error (RAE):

The Relative Absolute Error (RAE) is the ratio of the mean absolute error (MAE) to the average absolute deviation of the actual values from their mean. (Eland, 2022)

RAE is calculated as:

$$\text{RAE} = \frac{\sum_{t=1}^{n}|y_t - \hat{y}_t|}{\sum_{t=1}^{n}|y_t - \bar{y}|}$$

If RAE < 1: The model is better than a mean prediction.

If RAE > 1: The model is worse than a mean prediction.

3.5.2 Relative Squared Error (RSE):

The Relative Squared Error (RSE) measures the performance of a forecasting model by comparing its Mean Squared Error (MSE) to the square of the difference between the actual and the mean of the data. (Padhma, 2021)

The RSE formula is:

$$\text{RSE} = \frac{\sum_{t=1}^{n}(y_t - \hat{y}_t)^2}{\sum_{t=1}^{n}(y_t - \bar{y})^2}$$

RSE can result in three possible outcomes:

RSE < 1: The forecasting model performs better than the baseline model.

RSE = 1: The forecasting model performs equally to the baseline model.

RSE > 1: The forecasting model performs worse than the baseline model.

3.5.3 Relative Root Mean Squared Error (RRMSE):

Relative Root Mean Squared Error (RRMSE) is another version of the Root Mean Squared Error (RMSE). It adjusts the mean squared residuals relative to the sum of squared predicted values, enabling fair comparison across different measurement methods and scales.

RRMSE is calculated as:

$$\text{RRMSE} = \sqrt{\frac{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(\hat{y}_i)^2}}$$

The performance of a forecasting model based on RRMSE values can be classified as follows:

- **Excellent**: RRMSE less than 10%
- **Good**: RRMSE from 10% to 20%
- **Fair**: RRMSE from 20% to 30%
- **Poor**: RRMSE greater than 30% (Rama, 2021)

3.5.4 Normalized MAE (NMAE):

Normalized MAE (NMAE) or Coefficient of Variation of MAE metric, enables the comparison of MAE across datasets with different scales. It normalizes the error by using the mean of the measured data for model performance evaluation. (Borgogno Mondino et al., 2022)

NMAE is defined mathematically as:

$$\text{NMAE} = \frac{\frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|}{\bar{y}}$$

Forecasting model performance classification based on NMAE values is as follows:

- **Excellent**: NMAE < 10%
- **Good**: 10% ≤ NMAE < 20%
- **Fair**: 20% ≤ NMAE < 30%
- **Poor**: NMAE ≥ 30%

3.5.5 Theil's U statistic:

Theil's U statistic measures relative accuracy by comparing forecasted values to those generated using minimal historical data. It squares the deviations, giving more weight to larger errors, which helps identify and eliminate models with significant forecasting inaccuracies. (oracle help centre, 2015)

The RSE formula is:

$$\mathbf{U} = \sqrt{\frac{\sum_{t=1}^{n-1}\left(\frac{\widehat{Y_{t+1}} - Y_{t+1}}{Y_t}\right)^2}{\sum_{t=1}^{n-1}\left(\frac{Y_{t+1} - Y_t}{Y_t}\right)^2}}$$

- If **U = 1:** Then the forecasting model performs the same as a naïve model
- If **U < 1:** The forecasting model performs better than the naïve model.
- If **U > 1:** The forecasting model performs worse than the naïve model.

3.6 Goodness-of-fit (GoF):

Goodness-of-fit (GoF) refers to the extent to which a model's predicted values match the observed data. It is evaluated using fit statistics or discrepancy measures that assess the accuracy, consistency, and variance explained by the model. (Kéry & Royle, 2016)

Common Goodness-of-fit metrics include:

3.6.1 R-squared (R^2):

R-squared (R^2) or the coefficient of determination, is metric that measures the proportion of variance in the dependent variable explained by the independent variable(s) in a regression model. It assesses the goodness of fit, showing how closely the model's predictions match the actual data. (Gao, 2023)

R^2 is defined mathematically as:

$$\mathbf{R^2} = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

The R-squared (R^2) value can be interpreted as follows:

- **R^2 = 0:** predictions match the mean only.

- **$R^2 = 1$:** predictions perfectly match actual data.
- **$0 < R^2 < 1$:** Predictions explain part of the variance.

3.6.2 Adjusted R²:

Adjusted R^2 is a statistical measure used to evaluate the goodness of fit of a regression model, particularly when comparing models with different numbers of predictors. Unlike the traditional R^2, which can increase with the addition of more variables regardless of their relevance, adjusted R^2 accounts for the number of predictors, providing a more accurate reflection of model performance. (Potters & EICHELER, 2024)

Adjusted R^2 Can be expressed mathematically as:

$$\mathbf{R}^2_{\text{adj}} = \mathbf{1} - \left(\frac{(\mathbf{1} - \boldsymbol{R}^2)(\boldsymbol{n} - \mathbf{1})}{\boldsymbol{n} - \boldsymbol{k} - \mathbf{1}}\right)$$

Where:

- **n**: is number of observations
- **k**: is number of predictors

The results adjusted R^2 can be interpreted as follows:

- **Higher values:** Strong model fit after adjustment.
- **Close to R^2 :** Predictors contribute meaningfully.
- **Drops with irrelevant predictors** reflects Reduced risk of overfitting.
- **Negative values:** Model performs worse than a simple mean model.

3.7 Information criterion:

An Information Criterion is a method used to select the best model from a set of models by maximizing the likelihood of the data while adjusting for the number of parameters to prevent overfitting. (Colin Haser et al., 2014)

Key information criterion metrics include:

3.7.1 The Akaike Information Criterion (AIC):

The Akaike Information Criterion (AIC), introduced by Hirotugu Akaike in 1971 and formally proposed in 1974, measures how well a statistical model fits the data. It shows how much information is lost when using a model to represent a real-world process. AIC helps compare different models, with lower values indicating a better fit.

Akaike linked the prediction of the Kullback–Leibler information measure to the Maximum likelihood function. He also showed that the bias of the support function is equal to the number of model parameters p when the sample size is large.

The AIC formula is:

$$AIC = -2 \log L(\hat{\theta}) + 2p$$

Where:

$L(\hat{\theta})$**:** is the maximum likelihood value

p: is the number of parameters in the model. (Emiliano et al., 2014)

3.7.2 Bayesian information criterion (BIC):

The Bayesian Information Criterion (BIC), also known as the Schwarz Criterion, was introduced by Gideon Schwarz in 1978. It serves as a criterion for model selection based on the posterior probability of the models under comparison. Schwarz derived this criterion using Bayesian arguments and approximations of Laplace integrals to address the bias in the support function.

The BIC formula is:

$$\text{BIC} = k \ln(n) - 2 \ln(\hat{L})$$

Where:

- k: is the number of parameters in the model.
- n: is the sample size.
- $\hat{L}$: is the maximized value of the likelihood function. (Emiliano et al., 2014)

3.7.3 Corrected Akaike information criterion(AICc):

The Corrected Akaike Information Criterion (AICc), introduced by Hurvich and Tsai (1989), is an extension of the Akaike Information Criterion (AIC). It is specifically designed to enhance model selection performance, especially when dealing with small sample sizes. The AICc modifies the AIC by adding a correction term that accounts for finite sample sizes, consequently reducing the bias associated with the standard AIC.

AICc is defined mathematically as:

$$\text{AICc} = \text{AIC} + \frac{2k(k+1)}{n-k-1}$$

Here, k is the number of parameters, and n is the sample size. (HURVICH & TSAI, 1989)

Conclusion:

This chapter first examined the univariate time series forecasting models for stock market volatility. It began by defining traditional statistical models like ARIMA, SARIMA, and ETS, which rely on linear relationships and structured mathematical frameworks.

Next, it introduced non-linear models, including GARCH family models, TAR, STAR, and stochastic volatility models. These models add value by capturing complex market behaviours such as volatility clustering and asymmetry.

Subsequently, the chapter explored machine learning models such as (ANNs, RNNs, LSTMs, GRUs, CNNs, TCNs, and transformers). In addition, it highlighted their ability to capture long-term dependencies and non-linear patterns for superior forecasting performance. Moreover, the discussion included the Prophet model, emphasizing its flexibility in handling seasonality and trend shifts.

Finally, it detailed key performance evaluation metrics, including MAE, RMSE, MAPE, MASE, R^2, and information criteria like AIC and BIC, which are essential for assessing forecasting accuracy and guiding model selection.

In summary, this analysis establishes a foundation for evaluating univariate volatility forecasting models for sukuk markets, focusing on enhancing volatility prediction and improving forecasting accuracy in these markets.

Chapter 5 : Methodology

This chapter presents the methodological approach used to examine sukuk market predictability and evaluate the forecasting performance of selected time series models. It covers the research process, data description, model selection, and accuracy assessment.

1 research process:

This study adopts a quantitative, empirical, univariate forecasting approach based on realized volatility to examine whether sukuk markets exhibit predictable patterns and determine which forecasting models best capture their volatility across short, medium, and long-term horizons. Given these objectives the use of a quantitative empirical approach ensures objectivity and replicability, relying directly on observed market data rather than theoretical assumptions. Employing realized volatility as the forecasting target provides a direct measure of actual market fluctuations, thus enhancing transparency and reliability. Additionally, the univariate modeling framework is purposely chosen to capture each sukuk instrument's unique volatility characteristics without interference from potentially misleading interdependencies among instruments.

All analysis is conducted using Python due to its flexibility, efficiency, and wide adoption in data science and financial modeling. The study relies on several of its scientific libraries to handle data processing, statistical analysis, and machine learning tasks. Pandas is used for data manipulation and time series handling. Statsmodels supports the implementation of econometric and statistical models. Keras enables the design and training of deep learning models, particularly neural networks. Sklearn is employed for a range of machine learning algorithms, model evaluation, and preprocessing. This integrated toolset ensures reproducibility, scalability, and compatibility with current research standards.

The process begins with the calculation of log returns for each sukuk instrument. Logarithmic transformation is used because it produces returns that are time-additive, stabilizes variance, and improves statistical properties required for most forecasting models. The resulting return series is then plotted to visually inspect patterns such as mean shifts, volatility clustering, and other structural changes. These visual plots provide early indications of whether market behaviour is purely random or follows repeatable dynamics.

Descriptive statistics are calculated next to summarize the main features of each sukuk return series. The analysis includes the mean, standard deviation (std), minimum (min), median, and maximum (max), describing the average returns, variability, and range. Skewness and kurtosis are also computed to measure the asymmetry and fat-tailed behaviour of returns. After examining basic distributional features, the next step is to test for normality, using the Jarque-Bera statistic and its p-value. These measures provide essential insights into the distributional properties of the data and support the selection of appropriate forecasting models.

To further investigate volatility dynamics, autocorrelation (ACF) and partial autocorrelation (PACF) analyses are employed. These provide crucial insights into the memory and lag structures inherent in sukuk returns

The Augmented Dickey-Fuller (ADF) test is then used to determine whether the return series is stationary. Stationarity, meaning stable mean and variance over time, is essential for applying traditional time series models and achieving reliable forecasts. A stationary series implies that the relationship between past and future values is consistent, which strengthens the predictability of the model.

To test for time-varying volatility, the ARCH Lagrange Multiplier (LM) test is applied. It examines whether past squared residuals contain information about future variance, signalling volatility clustering. Evidence of strong ARCH effects in most return series confirms that volatility changes over time and is predictable to some extent, which justifies the use of conditional variance models like GARCH, designed to accurately capture and forecast such patterns.

We then estimate a GARCH(1,1) model for each sukuk return series. This model reveals how volatility responds to both recent shocks and past volatility levels. This provides a clearer view of the underlying volatility structure and helps to identify whether shocks have short-lived or lasting effects on sukuk markets.

Volatility co-movements between sukuk instruments are also explored using the Dynamic Conditional Correlation (DCC-GARCH) model. This test identifies whether volatilities evolve together across funds, adding a market-wide dimension to volatility understanding.

Following the preliminary test results, six forecasting models are selected for implementation: ARIMA, GARCH(1,1), HAR, Prophet, ANN, and LSTM. These models cover both statistical and deep learning approaches.

Each model is chosen based on its suitability to address the behaviours observed in the return series:

Among the traditional models, the Autoregressive Integrated Moving Average (ARIMA) model is selected due to the stationary properties exhibited by most sukuk return series, as confirmed by the Augmented Dickey-Fuller (ADF) test. ARIMA models are well-suited for capturing linear patterns and trends in stationary data, making them indispensable for modeling the central tendencies and variability inherent in sukuk returns.

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH(1,1)) model is chosen in response to the significant ARCH effects detected across most sukuk series. Since GARCH models are specifically designed to model time-varying volatility and volatility clustering, they are highly appropriate for accurately capturing the observed dynamics of sukuk market volatility.

The Heterogeneous Autoregressive (HAR) model is also included, motivated by the volatility variations observed across different time horizons in the sukuk dataset. HAR models incorporate volatility components at multiple time scales (daily, weekly, monthly) and are effective at capturing long-memory effects and multi-scale volatility behavior, making them a strong candidate for analyzing sukuk return dynamics.

In addition to traditional models, several machine learning models are implemented to address the non-linear and sequential patterns evident in the data. Artificial Neural Networks (ANN) are selected for their ability to model complex non-linear relationships. The diverse performance profiles and non-linear dependencies observed across sukuk instruments necessitate a flexible modeling approach, and ANN models are recognized for their capacity to capture hidden patterns that linear models may overlook.

The Long Short-Term Memory (LSTM) model is incorporated to handle the sequential dependencies and long-term memory identified in certain sukuk series. LSTM networks are designed to retain information across long sequences, making them highly effective for modeling the evolving structure of financial volatility over time.

The Prophet model is selected based on the clear seasonal and trend components observed in the sukuk data, likely driven by economic cycles and religious events. Its robust structure makes it well-suited for capturing these patterns, which enhances the reliability of forecasting sukuk market volatility.

By combining these six models, the study ensures a comprehensive evaluation across different modeling paradigms, capturing both linear and non-linear, short-term and long-term, and trend-based dynamics present in the sukuk market.

After selecting this set of models, each is trained on historical sukuk return data using rolling windows. Once training is complete, the models are used to forecast realized volatility over three fixed horizons: one week (short-term), one month (medium-term), and 200 days (long-term). The use of multiple horizons is intentional, as model behaviour often varies depending on the forecast period. Some models are better at responding to short-term noise, while others are designed to capture long-run trends. Comparing performance across timeframes supports a more complete understanding of each model’s practical strengths and limitations.

Model evaluation in this study is based on in-sample forecasting, meaning that each model is tested on the same historical data it was trained on. Forecasting in this context refers to the model’s ability to predict future volatility values based on past returns, even though the forecasts are made within the known data period. The goal is to see how well each model can reproduce the patterns and behaviour of the sukuk market.

To evaluate the models' performance clearly, two complementary approaches are used. First, visual inspection compares the predicted and actual realized volatility series to assess how closely the models track market movements. Second, quantitative evaluation is conducted using three error metrics: Mean Squared Error (MSE), Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE). This combination of metrics provides objective measures of forecasting accuracy, where MSE penalizes large forecast errors more heavily, revealing model sensitivity to tail risk and market shocks. RMSE, as the square root of MSE, expresses error in the same units as the original data, aiding direct interpretation. MAE, by treating all errors equally, offers a stable and robust measure of average deviation. Together, MSE, RMSE, and MAE capture both the magnitude and consistency of forecast errors, offering a comprehensive basis for comparing forecasting models.

The final step involves comparing model performance across the three-time horizons. This comparison identifies models that are horizon-specific versus those that perform well under multiple time scales. It also helps determine whether model accuracy degrades over time or remains stable, offering practical guidance for real-world forecasting applications in sukuk risk management and investment decision-making.

By integrating rigorous statistical testing, focused univariate modeling, multi-horizon forecasting, and dual-method performance evaluation, this research process provides a transparent, replicable, and well-justified framework for answering the central questions of sukuk market predictability and forecasting model effectiveness.

2 Data Description:

The dataset comprises daily log returns for 31 sukuk-related variables, including 26 sukuk investment funds and five sukuk market indices. The funds are legally registered and domiciled in the United Kingdom, Luxembourg, the United Arab Emirates, and Malaysia, while their portfolios are diversified across global sukuk markets. The indices included are the S&P Global Sukuk Index, the S&P MENA Sukuk Index, the Dow Jones Sukuk Total Return Index (excluding reinvestment), the S&P GCC Sukuk Index, and the Tadawul Sukuk & Bonds Index. This combination of funds and indices data provides a comprehensive basis for analyzing sukuk market volatility and forecasting model performance across different regions and market structures.

The selection of this dataset is based primarily on data availability and accessibility. Due to the limited public access to detailed sukuk market data compared to conventional bond markets, the sample was constructed from all reliable and complete series that could be obtained from open financial sources, to ensure that the analysis is based on the most comprehensive dataset available within these constraints.

The sample period covers daily observations from April 4, 2019, to June 30, 2023, yielding 1,082 observations for each fund and index. The data were sourced from Investing.com and S&P Global (spglobal.com), selected for their reliability, coverage, and consistency in providing historical financial data for sukuk markets. A total of 1,082 observations for each series is sufficient to ensure statistical reliability, allowing the models to capture volatility patterns more accurately and minimizing risks associated with small samples. The uniformity of observations across all funds and indices, with each covering the same time frame, also guarantees consistency in comparative analysis. This ensures that differences in model performance are genuinely due to model capabilities rather than inconsistencies in data availability.

A brief description of each sukuk fund and index included in the sample is provided in Table 5.1 to clarify their investment focus and coverage.

Table 5-1 Description of Sukuk dataset

Sakk Name	Definition
Emirates Global Sukuk Fund USD B Share Class Accumulation	Global sukuk investment fund focusing on capital appreciation through accumulation.
Amdynamic Sukuk - Class A	Malaysian sukuk fund offering stable Shariah-compliant fixed-income returns.
Franklin Global Sukuk Fund A(acc) USD	Global sukuk fund managed by Franklin Templeton, designed for capital growth.
Franklin Global Sukuk Fund I(acc) USD	Institutional accumulation share class providing global sukuk exposure.
Franklin Global Sukuk Fund A(mdis) USD	Distribution share class investing in diversified global sukuk portfolios.
Franklin Global Sukuk Fund N(acc) EUR	Euro-denominated accumulation share class focused on global sukuk investments.
Franklin Global Sukuk Fund A(acc) EUR	Euro accumulation class targeting Shariah-compliant sukuk returns.
AZ Fund 1 - AZ Islamic - Global Sukuk A-AZ Fund EUR Inc	European income share class investing in global sukuk under Islamic principles.
AZ Fund 1 - AZ Islamic - Global Sukuk B-AZ Fund EUR Inc	Alternative income share class focusing on diversified global sukuk.
AZ Fund 1 - AZ Islamic - Global Sukuk A-AZ Fund EUR Acc	Accumulation share class investing broadly in global sukuk.
AZ Fund 1 - AZ Islamic - Global Sukuk B-AZ Fund EUR Acc	Alternative accumulation share class under AZ Islamic sukuk strategy.
Principal Islamic Lifetime Enhanced Sukuk Fund	Sukuk fund enhancing exposure to diversified global Shariah-compliant bonds.
Principal Islamic Lifetime Sukuk	Core sukuk fund aiming for long-term stable returns under Islamic finance principles.
AZ Multi Asset - AZ Islamic - MAMG Global Sukuk A USD Inc	USD income fund investing in global sukuk, part of AZ Islamic multi-asset solutions.
Franklin Global Sukuk Fund X(qdis) USD	Distribution share class providing quarterly payouts from global sukuk investments.
AZ Multi Asset - AZ Islamic - MAMG Global Sukuk Master EUR Inc	Euro master fund structure investing in diversified global sukuk portfolios.
PB Aiman Sukuk Fund	Malaysian sukuk investment fund offering diversified Shariah-compliant returns.
Franklin Global Sukuk Fund W(qdis) USD	Wealth-focused sukuk fund offering periodic USD distributions.

Emirates NBD SICAV - Emirates Global Sukuk Fund A USD Inc	Income-generating sukuk fund managed under Emirates NBD's platform.
Emirates NBD SICAV - Emirates Global Sukuk Fund C USD Inc	Alternative share class for sukuk income strategies under Emirates NBD.
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala USD Acc	Sukuk accumulation fund managed by Rasmala with a global investment scope.
Franklin Global Sukuk Fund W(qdis) EUR-H1	Euro-hedged sukuk distribution fund minimizing currency exposure.
KAF Sukuk Fund	Malaysian sukuk fund investing in domestic Shariah-compliant fixed income.
DJ. (ex-Reinvestment)	Dow Jones Sukuk Total Return Index excluding coupon reinvestments, tracking global sukuk performance.
Emirates Global Sukuk Fund USD Institutional Share Class Accumulation	Institutional accumulation share class investing in global sukuk markets.
AZ Multi Asset - AZ Islamic - MAMG Global Sukuk Master EUR Inc .1	Subclass variant of AZ Islamic sukuk master fund offering global exposure.
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala USD Inc	Income share class of Rasmala’s global sukuk fund offering regular distributions.
S&P GCC Sukuk Index	Index tracking sukuk issued by Gulf Cooperation Council (GCC) countries.
S&P Global Sukuk Index	Global index measuring the performance of high-yield USD-denominated sukuk.
S&P MENA Sukuk Index	Index covering sukuk issued by Middle Eastern and North African entities.
Sukuk & Bonds Index	Tadawul index reflecting Saudi Arabia’s domestic sukuk and bond market performance.

Source: Author, based on Investing.com and S&P Global.

Chapter 6 : Empirical Results and Analysis

This chapter presents the empirical findings of the study in two structured parts. The first part investigates the volatility dynamics and evaluates the predictability of sukuk market volatility through detailed statistical and visual analyses. The second part assesses and compares the forecasting accuracy of selected statistical and machine learning models across different forecasting horizons. Together, these analyses provide a comprehensive understanding of sukuk volatility characteristics and the effectiveness of various forecasting methodologies.

1 Analysis of Sukuk Market Volatility and Predictability:

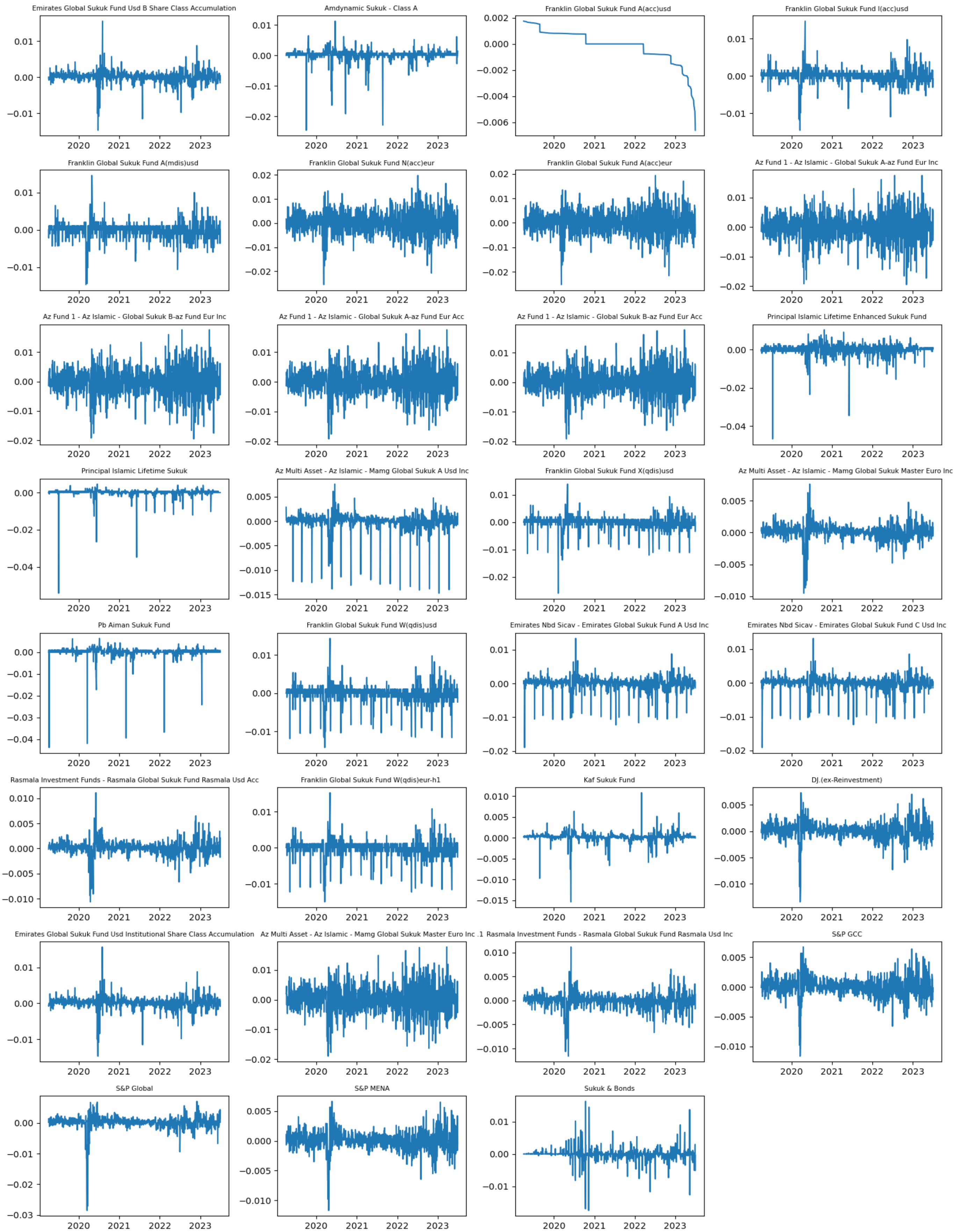
This section investigates the volatility dynamics and predictability of the sukuk market through comprehensive statistical and visual analyses. It includes an examination of log returns, descriptive statistics, return distributions, autocorrelation patterns, stationarity tests, volatility clustering, and conditional variance analysis. The insights gained provide the necessary foundation to evaluate the forecasting potential and suitability of different modeling approaches in subsequent sections.

1.1 Visual Analysis of Log Returns:

The results section begins with a preliminary analysis of the sukuk log returns series. **Figure 6.1** provides a detailed perspective on the sukuk markets dataset by plotting the returns of selected sukuk funds and indices over the sample period.

This visualization supports an initial examination of key features such as volatility clustering, mean shifts, and structural breaks, which are important for guiding later statistical testing and model selection.

Figure 6.1 Log Return Series of the Sukuk Market



As shown in **Figure 6.1** the plots illustrate daily log returns for various Sukuk instruments from around 2019 to early 2023. They reveal distinct volatility patterns and market dynamics over time. the returns exhibit stationarity, fluctuating closely around zero, suggesting that daily returns are typically stable and predictable for most Sukuk instruments. Except the Franklin Global Sukuk Fund AccUsd, which deviates from this pattern.

Notably, several Sukuk instruments experienced pronounced volatility clustering, particularly around early 2020, coinciding with the COVID-19 pandemic beginning. During this period, unexpected changes and spikes in returns were observed, reflecting a significant market uncertainty and external economic shocks influencing the Islamic bond market.

Despite general stability, certain Sukuk instruments show periods of amplified volatility characterized by sharp fluctuations. This varying volatility behaviour across instruments highlights heterogeneity in risk profiles among Sukuk securities. For example, certain Sukuk exhibit mild volatility with returns closely centred around zero, suggesting stable market conditions. Conversely, several instruments demonstrate higher volatility with frequent and pronounced spikes, highlighting greater sensitivity to market events and economic conditions.

1.2 Descriptive statistics:

The following table summarizes descriptive statistics for 31 sukuk instruments each comprising 1086 observations of daily log returns, providing an overview of their essential statistical measures. Metrics presented include mean return, standard deviation, minimum, median, maximum values, skewness, kurtosis, and results from the Jarque-Bera test for normality. These statistics provide a thorough initial assessment of the return characteristics and risk profiles across different sukuk funds.

Table 6-1 Descriptive Statistics

Sakk name	mean	std	Min	median	max	skew	kurtosis	Jarque_Bera_p_value
Emirates Global Sukuk Fund Usd B Share Class Accumulation	5.00E-05	0.001785	-0.01469	0.000199	0.015518	-1.33912	17.40307	0.00E+00
Amdynamic Sukuk Class A	0.000101	0.001952	-0.02453	0	0.011212	-5.58863	60.39673	0.00E+00
Franklin Global Sukuk Fund A(acc)usd	-0.00012	0.001266	-0.00661	0	0.001744	-1.42574	3.089304	5.54E-173
Franklin Global Sukuk Fund I(acc)usd	8.20E-05	0.001911	-0.01451	0	0.014634	-0.76057	12.41268	0.00E+00
Franklin Global Sukuk Fund A(mdis)usd	-0.0001	0.002095	-0.01466	0	0.014566	-0.80318	9.558686	0.00E+00
Franklin Global Sukuk Fund N(acc)eur	4.90E-05	0.004607	-0.0255	0	0.019757	-0.19836	2.036929	1.92E-42
Franklin Global Sukuk Fund A(acc)eur	8.50E-05	0.004642	-0.02528	0	0.019418	-0.19083	2.009773	2.92E-41
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	6.00E-05	0.004331	-0.01923	0.000159	0.017572	-0.21363	1.87313	1.80E-36
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	6.00E-05	0.004326	-0.01923	0.000158	0.017721	-0.21982	1.878648	8.87E-37
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	-0.0001	0.004532	-0.01947	0.0002	0.01754	-0.36874	1.941508	1.52E-42
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	-0.00011	0.00453	-0.01948	0.000198	0.017548	-0.36439	1.960299	3.93E-43
Principal Islamic Lifetime Enhanced Sukuk Fund	-2.00E-05	0.003044	-0.0468	0	0.010363	-5.59839	70.71244	0.00E+00
Principal Islamic Lifetime Sukuk	-1.10E-05	0.002503	-0.05419	0	0.004587	-13.6422	250.036	0.00E+00
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	-0.00013	0.002047	-0.01472	0.000198	0.00757	-4.17508	24.22175	0.00E+00
Franklin Global Sukuk Fund X(qdis)usd	-7.10E-05	0.002425	-0.02603	0	0.013986	-2.43404	19.22616	0.00E+00
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	0.000124	0.001257	-0.00954	0.000176	0.007626	-1.67844	13.31883	0.00E+00
Pb Aiman Sukuk Fund	8.00E-06	0.002894	-0.04383	0	0.006338	-11.1022	149.5292	0.00E+00
Franklin Global Sukuk Fund W(qdis)usd	-7.70E-05	0.002304	-0.01415	0	0.014278	-1.54234	9.841563	0.00E+00
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	-9.50E-05	0.002234	-0.01898	0.000209	0.013354	-2.46709	13.95694	0.00E+00
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	-0.00015	0.002235	-0.01914	0.000129	0.013188	-2.50134	14.06032	0.00E+00
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	0.000112	0.001458	-0.01064	0.000195	0.011141	-1.16341	13.57424	0.00E+00
Franklin Global Sukuk Fund W(qdis)eur-h1	-0.00015	0.002334	-0.01504	0	0.015232	-1.54617	10.47742	0.00E+00
Kaf Sukuk Fund	0.00017	0.001214	-0.01538	0.000256	0.01086	-3.20363	45.5952	0.00E+00
DJ.(ex-Reinvestment)	8.80E-05	0.00158	-0.0134	0.000148	0.007293	-1.1751	10.49321	0.00E+00
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	8.00E-05	0.001786	-0.01467	0.000191	0.015639	-1.33158	17.57961	0.00E+00
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	0.000162	0.00445	-0.01897	0.000285	0.017691	-0.19344	1.683246	1.35E-29
Rasmala Investment Funds - Rasmala Global Sukuk Fund Basmala Usd Inc	-2.70E-05	0.001564	-0.0116	0.000138	0.011148	-1.2239	11.65895	0.00E+00
S&P GCC	8.10E-05	0.001601	-0.01163	0.00014	0.00672	-0.85783	6.758738	0.00E+00
S&P Global	0.000168	0.002105	-0.02853	0.000316	0.007064	-5.38627	63.13195	0.00E+00
S&P MENA	0.000105	0.001591	-0.01165	0.000144	0.006674	-0.87843	7.021898	0.00E+00
Sukuk & Bonds	-4.70E-05	0.001786	-0.01749	0	0.016297	-0.5881	36.60556	0.00E+00

As depicted in **Table 6.1**, the mean returns highlight strong performance for instruments such as Amdynamic Sukuk - Class A (0.000101) and Kaf Sukuk Fund (0.00017). Conversely, instruments such as Franklin Global Sukuk Fund A(acc)usd (mean: -0.00012) and Emirates Nbd Sicav - Emirates Global Sukuk Fund C USD Inc (mean: -0.00015) exhibit negative average returns.

Volatility, measured by standard deviation, varies significantly, for instance the Franklin Global Sukuk Fund N(acc)eur shows high volatility (0.004607), indicating frequent substantial fluctuations and suggesting increased investment risk. In contrast, Franklin Global Sukuk Fund A(acc)usd demonstrates notably lower volatility (0.001266), reflecting more stable performance but potentially lower returns.

Examining the range from minimum to maximum returns reveals significant insights. For example, Franklin Global Sukuk Fund N(acc)eur presents the highest positive return (max: 0.019757), indicating significant potential for positive returns despite its high volatility. Conversely, Principal Islamic Lifetime Sukuk exhibits the broadest range, with notably negative minimum returns (min: -0.05419) and relatively modest maximum returns, highlighting its considerable potential for large losses relative to gains. In addition, Median returns are generally near zero across most instruments, suggesting limited frequency of substantial returns.

Further examination of distribution characteristics through skewness and kurtosis metrics highlights the asymmetrical risk of negative versus positive returns, as well as the probability of outlier impacts. For instance, a significant negative skew in certain instruments like Principal Islamic Lifetime Sukuk (-13.6422), indicates a higher likelihood of significant losses. Meanwhile, high kurtosis values, such as those observed in Principal Islamic Lifetime Enhanced Sukuk Fund (kurtosis: 70.71244), indicate an elevated risk of extreme returns, implying heavy-tailed distributions.

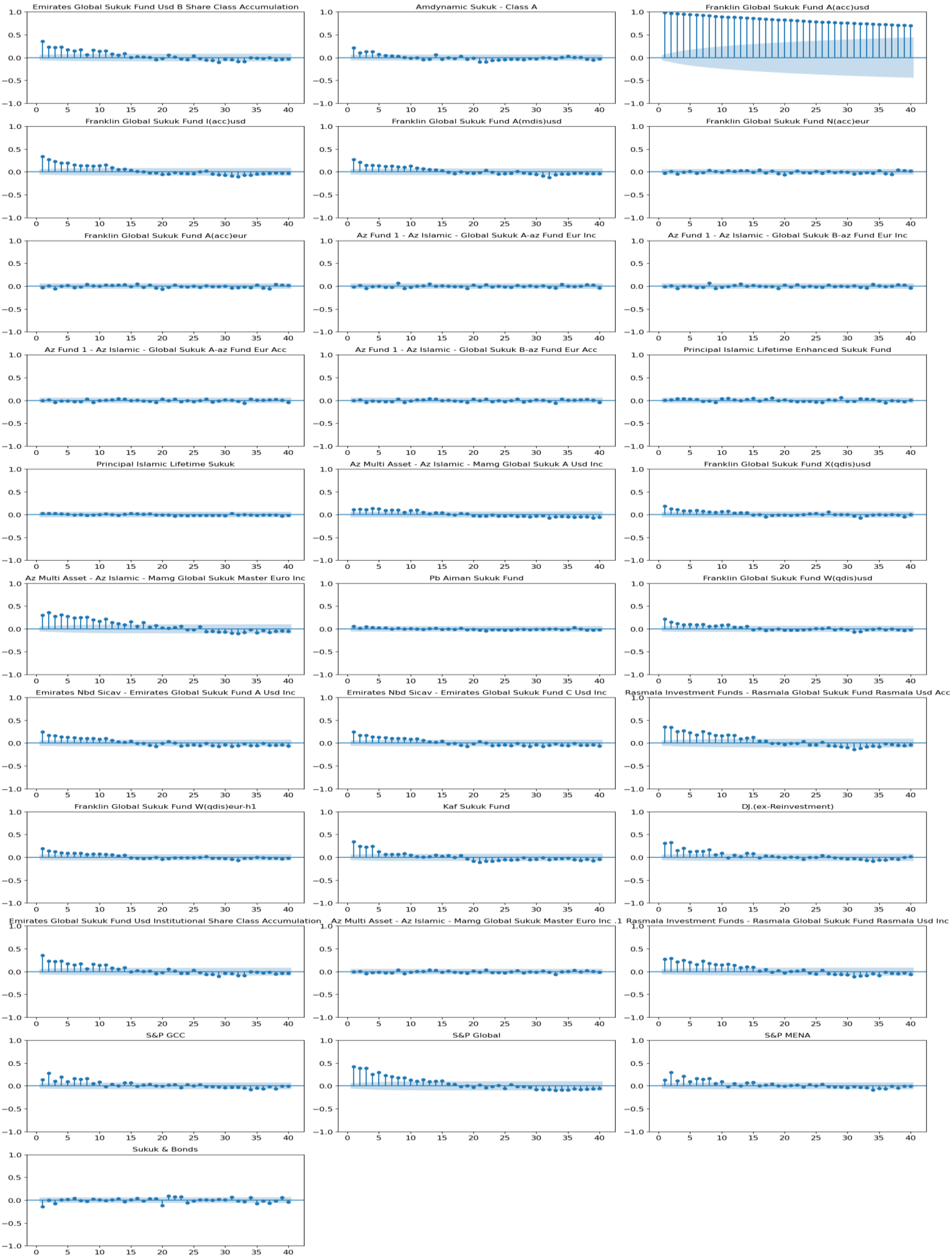
The Jarque-Bera test p-values (near zero for all instruments) firmly reject normality, indicating significant deviation from a Gaussian distribution. Consequently, traditional Gaussian-based models may inadequately capture the true complexity of sukuk returns.

Overall, the descriptive statistics reveal substantial heterogeneity in risk and return profiles across sukuk instruments. While mean returns are close to zero for most funds, significant differences in volatility, skewness, and kurtosis indicate varying levels of risk exposure. Some instruments display mild volatility and relatively symmetric return distributions, suggesting lower risk, whereas others exhibit high volatility, strong negative skewness, and heavy tails, reflecting greater vulnerability to extreme events.

1.3 Autocorrelation analysis:

After completing the descriptive statistics, the next step is to examine the autocorrelation structure of the sukuk dataset. This is done through the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots. This step helps examine dependencies between actual and previous values and reveals how current values are influenced by past observations. Understanding these relationships is essential before selecting and fitting an appropriate forecasting model.

Figure 6.2 Autocorrelation Function (ACF) plots



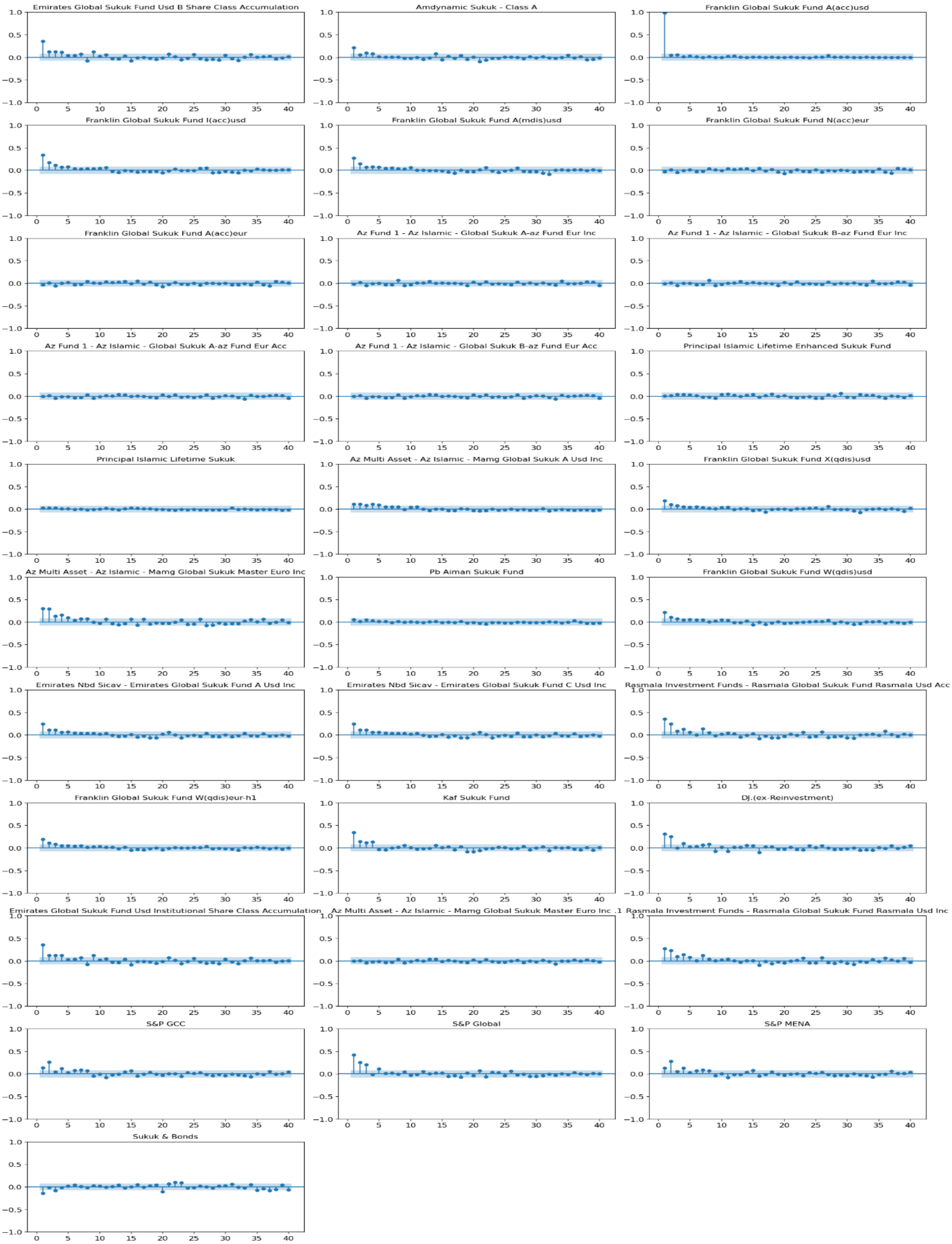
As displayed in **Figure 6.2**, the autocorrelation analysis of the Sukuk fund and index returns reveals three distinct patterns across the dataset. The first group of series exhibits short-term autocorrelation, where autocorrelation values are significant at lag 1 but decline quickly afterward. This indicates short memory and suggests the series are stationary. as seen in the Franklin Global Sukuk Fund A(acc)eur, S&P Global, and Az Fund 1 – Az Islamic – Global Sukuk A-eur Fund Eur Acc.

The second group shows persistent autocorrelation that decays slowly over multiple lags. This behaviour suggests possible long memory or structural persistence in the data. Examples include DJ (ex-Reinvestment), and S&P GCC.

The third group contains series with flat ACFs, showing no significant autocorrelation. These include nearly all the Az fund variants, except for two, as well as several other funds. The flat autocorrelation indicates a lack of serial dependence in the mean, making them resemble white noise processes.

Conversely, one series stands apart from all others which is the Franklin Global Sukuk Fund A(acc)usd, since its ACF remains high across all lags with minimal decay. This pattern strongly indicates non-stationarity and the presence of a unit root

Figure 6.3 Partial Autocorrelation Function (PACF) plots



As shown in **Figure 6.3**, the PACF plots for the sukuk dataset reveal several distinct partial autocorrelation structures across the different funds and indices.

Firstly, most of the sukuk funds show a clear, significant spike at lag 1, followed by values that fall within the confidence intervals, indicating short-term memory in the data. Examples include the Franklin Global Sukuk Fund (acc/usd), Az Fund 1 – Az Islamic Global Sukuk A-Eur Fund Eur Inc, and Principal Islamic Lifetime Sukuk.

In contrast, some funds show more complex dynamics. For example, the Emirates Global Sukuk Fund USD Institutional Share Class Accumulation, Rasmala Investment Funds – Rasmala Global Sukuk Fund Rasmala USD Inc, and S&P GCC have significant spikes at multiple lags, such as lags 1, 2, and 3, or even further. This indicates persistent autocorrelations.

Lastly Some funds, including KAF Sukuk Fund, S&P Global, and Sukuk & Bonds, show PACF values entirely within the confidence interval across all lags. This pattern indicates a white noise process, suggesting no significant autocorrelation. In such cases, historical returns offer no predictive power for future values.

1.4 Stationarity Test:

In order to check the stationarity of our sukuk dataset, which is crucial for time series analysis. we use the augmented dicky fuller test (ADF), considering three metrics the ADF statistic, p-value, and critical values at the 1%, 5%, and 10% confidence levels.

Table 6-2 ADF Unit Root Test

Sakk name	ADF Statistic	p-value	Critical Value (1%)	Critical Value (5%)	Critical Value (10%)
Emirates Global Sukuk Fund Usd B Share Class A	-6.636212	5.554388e-09	-3.436470	-2.864242	-2.568209
Amdynamic Sukuk - Class A	-12.598172	1.759951e-23	-3.436408	-2.864215	-2.568194
Franklin Global Sukuk Fund A(acc)usd	5.859764	1.000000e+00	-3.436408	-2.864215	-2.568194
Franklin Global Sukuk Fund I(acc)usd	-9.745310	8.286167e-17	-3.436414	-2.864217	-2.568196
Franklin Global Sukuk Fund A(mdis)usd	-7.073286	4.874421e-10	-3.436442	-2.864230	-2.568202
Franklin Global Sukuk Fund N(acc)eur	-33.972952	0.000000e+00	-3.436391	-2.864207	-2.568190
Franklin Global Sukuk Fund A(acc)eur	-34.109542	0.000000e+00	-3.436391	-2.864207	-2.568190
Az Fund 1 - Az Islamic - Global Sukuk A-az Fun...	-33.365150	0.000000e+00	-3.436391	-2.864207	-2.568190
Az Fund 1 - Az Islamic - Global Sukuk B-az Fun...	-33.331944	0.000000e+00	-3.436391	-2.864207	-2.568190
Az Fund 1 - Az Islamic - Global Sukuk A-az Fun...	-33.043505	0.000000e+00	-3.436391	-2.864207	-2.568190
Az Fund 1 - Az Islamic - Global Sukuk B-az Fun...	-32.978939	0.000000e+00	-3.436391	-2.864207	-2.568190
Principal Islamic Lifetime Enhanced Sukuk Fund	-32.680079	0.000000e+00	-3.436391	-2.864207	-2.568190
Principal Islamic Lifetime Sukuk	-31.927678	0.000000e+00	-3.436391	-2.864207	-2.568190
Az Multi Asset - Az Islamic - Mamg Global Suku...	-8.012304	2.184862e-12	-3.436431	-2.864225	-2.568200
Franklin Global Sukuk Fund X(qdis)usd	-10.424897	1.667562e-18	-3.436419	-2.864220	-2.568197
Az Multi Asset - Az Islamic - Mamg Global Suku...	-5.441599	2.768370e-06	-3.436482	-2.864247	-2.568212
Pb Aiman Sukuk Fund	-17.547876	4.157029e-30	-3.436403	-2.864212	-2.568193
Franklin Global Sukuk Fund W(qdis)usd	-9.326263	9.592418e-16	-3.436425	-2.864222	-2.568198
Emirates Nbd Sicav - Emirates Global Sukuk Fun...	-9.675213	1.246048e-16	-3.436419	-2.864220	-2.568197
Emirates Nbd Sicav - Emirates Global Sukuk Fun...	-9.679131	1.217933e-16	-3.436419	-2.864220	-2.568197
Rasmala Investment Funds - Rasmala Global Suku...	-6.582300	7.461660e-09	-3.436493	-2.864253	-2.568214
Franklin Global Sukuk Fund W(qdis)eur-h1	-9.343692	8.659868e-16	-3.436425	-2.864222	-2.568198
Kaf Sukuk Fund	-10.474350	1.260317e-18	-3.436408	-2.864215	-2.568194
DJ.(ex-Reinvestment)	-6.778700	2.531746e-09	-3.436476	-2.864245	-2.568210
Emirates Global Sukuk Fund Usd Institutional S...	-6.671823	4.567484e-09	-3.436470	-2.864242	-2.568209
Az Multi Asset - Az Islamic - Mamg Global Suku...	-32.994759	0.000000e+00	-3.436391	-2.864207	-2.568190
Rasmala Investment Funds - Rasmala Global Suku...	-7.337205	1.088644e-10	-3.436425	-2.864222	-2.568198
S&P GCC	-6.611982	6.343321e-09	-3.436476	-2.864245	-2.568210
S&P Global	-8.386600	2.428963e-13	-3.436414	-2.864217	-2.568196
S&P MENA	-6.579620	7.571757e-09	-3.436476	-2.864245	-2.568210
Sukuk & Bonds	-5.098120	1.414151e-05	-3.436517	-2.864263	-2.568220

As displayed in **Table 6.2** we see that all sukuk listed show evidence of stationarity, with their ADF statistics significantly more negative than the critical values at the 1% confidence level, and very low p-values, indicating a rejection of the null hypothesis of a unit root. This signifies that these sukuk time series have stable statistical properties, making them suitable for further analysis. The only exception is Franklin Global Sukuk Fund A(acc)usd, with an ADF statistic of 5.859764 and a p-value of 1, which typically suggests the presence of a unit root and indicating that the series is non-stationary.

1.5 Auto Regressive Conditional Heteroscedasticity (ARCH) Effect:

To determine if our dataset exhibits volatility clustering, we apply the ARCH effect test using the Lagrange Multiplier (LM) statistic. A significant LM p-value indicates the presence of volatility clustering.

Table 6-3 ARCH Effect Test

Sakk name	LM Statistic	LM p-value	LM Significance
Emirates Global Sukuk Fund Usd B Share Class A...	180.967287	0.000000e+00	0.05
Amdynamic Sukuk - Class A	12.580632	9.442018e-01	0.05
Franklin Global Sukuk Fund A(acc)usd	1082.420275	0.000000e+00	0.05
Franklin Global Sukuk Fund I(acc)usd	190.786246	0.000000e+00	0.05
Franklin Global Sukuk Fund A(mdis)usd	144.810510	0.000000e+00	0.05
Franklin Global Sukuk Fund N(acc)eur	21.079330	5.158401e-01	0.05
Franklin Global Sukuk Fund A(acc)eur	25.941141	2.542177e-01	0.05
Az Fund 1 - Az Islamic - Global Sukuk A-az Fun...	61.005026	1.584957e-05	0.05
Az Fund 1 - Az Islamic - Global Sukuk B-az Fun...	94.184536	6.584688e-11	0.05
Az Fund 1 - Az Islamic - Global Sukuk A-az Fun...	115.050524	1.365574e-14	0.05
Az Fund 1 - Az Islamic - Global Sukuk B-az Fun...	115.366157	1.199041e-14	0.05
Principal Islamic Lifetime Enhanced Sukuk Fund	0.321551	1.000000e+00	0.05
Principal Islamic Lifetime Sukuk	0.230003	1.000000e+00	0.05
Az Multi Asset - Az Islamic - Mamg Global Suku...	3.959089	9.999924e-01	0.05
Franklin Global Sukuk Fund X(qdis)usd	28.965390	1.458516e-01	0.05
Az Multi Asset - Az Islamic - Mamg Global Suku...	367.997823	0.000000e+00	0.05
Pb Aiman Sukuk Fund	40.379531	9.758884e-03	0.05
Franklin Global Sukuk Fund W(qdis)usd	71.174926	4.315756e-07	0.05
Emirates Nbd Sicav - Emirates Global Sukuk Fun...	139.194637	0.000000e+00	0.05
Emirates Nbd Sicav - Emirates Global Sukuk Fun...	137.153568	0.000000e+00	0.05
Rasmala Investment Funds - Rasmala Global Suku...	306.842576	0.000000e+00	0.05
Franklin Global Sukuk Fund W(qdis)eur-h1	86.643916	1.268184e-09	0.05
Kaf Sukuk Fund	141.137997	0.000000e+00	0.05
DJ.(ex-Reinvestment)	385.625079	0.000000e+00	0.05
Emirates Global Sukuk Fund Usd Institutional S...	179.299740	0.000000e+00	0.05
Az Multi Asset - Az Islamic - Mamg Global Suku...	16.500464	7.902958e-01	0.05
Rasmala Investment Funds - Rasmala Global Suku...	279.113126	0.000000e+00	0.05
S&P GCC	350.947359	0.000000e+00	0.05
S&P Global	397.069416	0.000000e+00	0.05
S&P MENA	354.593088	0.000000e+00	0.05
Sukuk & Bonds	189.381945	0.000000e+00	0.05

From **Table 6.3** we observe That the ARCH effect test results demonstrate a mix of significant and insignificant ARCH effects across different sukuk. Notably, 23 instances show significant ARCH effects, whereas 8 are insignificant.

Among those with significant ARCH effects, sukuk such as Emirates Global Sukuk Fund Usd B Share Class A, Franklin Global Sukuk Fund A(acc)usd, and Franklin Global Sukuk Fund I(acc)usd have extremely low LM p-values (≈ 0), confirming strong time-dependent volatility. Their high LM statistics, such as 1082.42 for Franklin Global Sukuk Fund A(acc)usd, suggest that past volatility plays a crucial role in predicting future fluctuations. These results indicate that volatility clustering is present, making models like GARCH suitable for capturing the dynamic behavior of these returns.

On the other hand, 8 sukuk fail to show significant ARCH effects, as their p-values are greater than or equal to 0.05, suggesting that their volatility remains stable over time. Examples include Amdynamic Sukuk - Class A (p-value = 0.944), Franklin Global Sukuk Fund N(acc)eur (p-value = 0.516), and Principal Islamic Lifetime Sukuk (p-value = 1.000). Their low LM statistics, such as 0.230 for Principal Islamic Lifetime Sukuk, further confirm the absence of volatility clustering for these sukuk.

1.6 Sukuk Markets Volatility analysis:

To examine volatility in our sukuk dataset, we apply the GARCH (1,1) model, a widely used method for modeling financial time series volatility. This model captures time-varying volatility by incorporating past returns and past volatility, allowing us to track fluctuations in sukuk markets. Since Assessing volatility of sukuk markets is essential for selecting suitable forecasting model in subsequent stages.

Table 6-4 Volatility Estimation of Sukuk Using the GARCH(1,1) Model

Sakk name	Alpha	Beta	Mu	Omega
Emirates Global Sukuk Fund Usd B Share Class A...	0.169787	0.829029	0.000203	4.52E-08
Amdynamic Sukuk - Class A	0.0635	0.90225	0.00051	9.07E-09
Franklin Global Sukuk Fund A(acc)usd	0.052257	0.898746	1.88E-06	1.51E-10
Franklin Global Sukuk Fund I(acc)usd	0.311383	0.687617	0.000144	1.54E-07
Franklin Global Sukuk Fund A(mdis)usd	0.243071	0.718124	-0.00013	3.04E-07
Franklin Global Sukuk Fund N(acc)eur	0.075287	0.910657	0.000167	3.38E-07
Franklin Global Sukuk Fund A(acc)eur	0.07822	0.905496	0.000206	3.94E-07
Az Fund 1 - Az Islamic - Global Sukuk A-az Fun...	0.039586	0.95254	-2.88E-05	1.76E-07
Az Fund 1 - Az Islamic - Global Sukuk B-az Fun...	0.04012	0.951556	-8.73E-06	1.80E-07
Az Fund 1 - Az Islamic - Global Sukuk A-az Fun...	0.060003	0.93035	0.000195	1.95E-07
Az Fund 1 - Az Islamic - Global Sukuk B-az Fun...	0.057847	0.93285	0.000199	1.90E-07
Principal Islamic Lifetime Enhanced Sukuk Fund	0.047792	0.943943	-1.04E-05	3.70E-08
Principal Islamic Lifetime Sukuk	0.052868	0.918211	-4.52E-05	1.18E-08
Az Multi Asset - Az Islamic - Mamg Global Suku...	0.00584	0.885823	-0.00012	4.50E-07
Franklin Global Sukuk Fund X(qdis)usd	0.006633	0.991717	-0.00011	1.49E-08
Az Multi Asset - Az Islamic - Mamg Global Suku...	0.167612	0.829863	0.000212	1.18E-08
Pb Aiman Sukuk Fund	0.051024	0.932062	-4.57E-05	3.13E-08
Franklin Global Sukuk Fund W(qdis)usd	0.015717	0.983096	-0.00016	1.30E-08
Emirates Nbd Sicav - Emirates Global Sukuk Fun...	0.007352	0.983948	-0.00011	3.83E-08
Emirates Nbd Sicav - Emirates Global Sukuk Fun...	0.009378	0.985503	-0.00017	2.43E-08
Rasmala Investment Funds - Rasmala Global Suku...	0.101639	0.895822	0.000198	1.41E-08
Franklin Global Sukuk Fund W(qdis)eur-h1	0.017812	0.96851	-0.00025	7.24E-08
Kaf Sukuk Fund	0.688163	0.309431	0.000355	3.46E-07
DJ.(ex-Reinvestment)	0.123334	0.870876	0.00018	2.25E-08
Emirates Global Sukuk Fund Usd Institutional S...	0.171325	0.827671	0.000236	4.49E-08
Az Multi Asset - Az Islamic - Mamg Global Suku...	0.055007	0.933499	0.000293	2.37E-07
Rasmala Investment Funds - Rasmala Global Suku...	0.062945	0.92484	4.48E-05	2.24E-08
S&P GCC	0.088112	0.905541	0.000153	2.00E-08
S&P Global	0.16904	0.829952	0.000349	3.95E-08
S&P MENA	0.091506	0.902103	0.00018	1.85E-08
Sukuk & Bonds	0.037306	0.959419	7.07E-05	1.00E-08

As observed from the **Table 6.4** the variability in these parameters among sukuk highlights differing volatility levels (low, moderate, high):

- **Low Volatility Sukuk:** Identified by their minimal Omega values, indicating low inherent volatility. Their Alpha and Beta sum being near or slightly below 1 suggests reduced market sensitivity. For instance, the Amdynamic Sukuk, with an Omega of 9.07E-09 and an Alpha and Beta sum of 0.965749, exhibits low inherent volatility and moderate market adaptability. The Principal Islamic Lifetime Sukuk, with an Omega of 1.18E-08 and an Alpha and Beta sum of 0.971079, also shows low volatility, indicating stability and moderate market responsiveness.
- **Moderate Volatility Sukuk:** These are characterized by moderate Omega values and Alpha plus Beta sums around 1, reflecting a balance between inherent volatility and market responsiveness. For example, the Emirates Global Sukuk Fund USD B Share Class Accumulation, with an Omega of 4.52E-08 and an Alpha and Beta sum of 0.998816, exhibits moderate volatility and significant market interaction. Similarly, S.P. Global, with an Omega of 3.98E-08 and an Alpha and Beta sum of 0.998911, maintains a profile of moderate volatility and market engagement.

- **High Volatility Sukuk:** These are marked by high Omega values or exceptionally high Alpha and Beta sums, indicating significant baseline volatility and market sensitivity. For instance, the Franklin Global Sukuk Fund I acc usd, with a high Omega of 1.54E-07 and an Alpha plus Beta of 0.999, shows considerable inherent volatility and market reactivity. The Kaf Sukuk Fund, despite a moderate Omega of 3.46E-07, has a notably high Alpha of 0.688163, suggesting a potential for increased volatility and strong market responsiveness.

1.7 Sukuk markets Dynamic Correlations:

to estimate the time varying correlations among our sukuk datasets. we use the DCC GARCH model.

Table 6-5 Sukuk Markets Dynamic Correlations Using DCC Grach Model

	Estimate	Std. Error	T-Statistic	P-Value
[Joint]dcca1	0.007127	0.001687	4.225283	2.39E-05
[Joint]dccb1	0.943053	0.021969	42.92555	0

Regarding the dynamic correlations among sukuk markets as presented in Table 5, the DCC-GARCH model estimates dcca1 at 0.007127 and at 0.943053, both of which are statistically significant. The low dcca1 value suggests that the short-term response of correlations to market shocks is minimal. Conversely, the high dccb1 value indicates that historical correlations have a significant influence on the current sukuk market dynamics.

Subsequently, the high statistical significance of both coefficients suggests strong evidence that sukuk correlations are predominantly influenced by past values rather than immediate market changes. Additionally, the sum of these coefficients being less than one (0.95018) ensures the stationarity of the correlation process, further supporting the reliability of the model in capturing the long-term stability of sukuk market relationships.

This section presents the empirical data analysis conducted on Sukuk market returns. The analysis aims to rigorously evaluate forecasting models based on real-world data, ensuring practical relevance of the findings. The dataset includes historical price and returns series of various Sukuk funds, carefully selected to represent diverse market dynamics, volatility levels, and investment profiles.

2 Forecasting Performance Evaluation of Time Series Models:

This section evaluates the forecasting accuracy of the selected models: GARCH(1,1), ARIMA, HAR, Prophet, ANN, and LSTM across short (one week), medium (one month), and long (200 days) horizons. Performance is assessed using error metrics such as MAE, MSE, and RMSE, along with visual comparisons of predicted and actual returns. The results reveal each model's strengths and limitations in capturing sukuk market volatility.

2.1 ARIMA MODEL FORECASTING PERFORMANCE:

This section evaluates the performance of the ARIMA model over various time horizons. By dividing the analysis into short-term, medium-term, and long-term periods, we aim to provide a thorough understanding of the model's effectiveness and reliability in predicting market volatility. to achieve the best ARIMA fit, we selected the suitable orders for each instrument based on Akaike Information Criterion (AIC) metric. Table 5 shows the appropriate ARIMA order for each sakk.

Table 6-6 ARIMA Orders

Sakk Name	ARIMA Orders
Sukuk & Bonds	**(1, 1, 2)**
Emirates Global Sukuk Fund Usd B Share Class Accumulation	**(1, 1, 1)**
Amdynamic Sukuk - Class A	**(4, 0, 0)**
Franklin Global Sukuk Fund A(acc)usd	**(2, 2, 4)**
Franklin Global Sukuk Fund I(acc)usd	**(1, 0, 2)**
Franklin Global Sukuk Fund A(mdis)usd	**(1, 0, 2)**
Franklin Global Sukuk Fund N(acc)eur	**(0, 0, 0)**
Franklin Global Sukuk Fund A(acc)eur	**(0, 0, 0)**
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	**(0, 0, 0)**
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	**(0, 0, 0)**
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	**(0, 0, 0)**
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	**(0, 0, 0)**
Principal Islamic Lifetime Enhanced Sukuk Fund	**(0, 0, 0)**
Principal Islamic Lifetime Sukuk	**(0, 0, 0)**
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	**(2, 0, 1)**
Franklin Global Sukuk Fund X(qdis)usd	**(5, 0, 0)**
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	**(2, 0, 2)**
Pb Aiman Sukuk Fund	**(1, 0, 0)**
Franklin Global Sukuk Fund W(qdis)usd	**(5, 0, 0)**
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	**(2, 0, 2)**
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	**(1, 0, 2)**
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	**(3, 0, 2)**
Franklin Global Sukuk Fund W(qdis)eur-h1	**(5, 0, 0)**
Kaf Sukuk Fund	**(1, 0, 1)**
DJ.(ex-Reinvestment)	**(2, 1, 3)**
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	**(0, 1, 2)**
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	**(0, 0, 0)**
S&P GCC	**(2, 1, 3)**
S&P Global	**(3, 0, 0)**
S&P MENA	**(2, 1, 3)**

2.1.1 ARIMA Model Performance in the Short Term (One Week):

In the short term, the performance of the ARIMA model is assessed over a one-week period. This section includes the visualization of the model's predictions compared to the actual values, along with forecasting assessment through performance metrics.

Table 6-7 Short Term ARIMA Model Performance Error Metrics

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	1.051425e-06	0.001025	0.000977
Amdynamic Sukuk - Class A	1.571458e-06	0.001254	0.000914
Franklin Global Sukuk Fund A(acc)usd	6.391531e-06	0.002528	0.002516
Franklin Global Sukuk Fund I(acc)usd	2.411392e-06	0.001553	0.001278
Franklin Global Sukuk Fund A(mdis)usd	2.764203e-06	0.001663	0.001152
Franklin Global Sukuk Fund N(acc)eur	2.387087e-05	0.004886	0.004415
Franklin Global Sukuk Fund A(acc)eur	2.279755e-05	0.004775	0.004269
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	1.446806e-05	0.003804	0.003126
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	1.410836e-05	0.003756	0.003101
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	1.390690e-05	0.003729	0.003114
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	1.427036e-05	0.003778	0.003133
Principal Islamic Lifetime Enhanced Sukuk Fund	7.806886e-07	0.000884	0.000771
Principal Islamic Lifetime Sukuk	2.462184e-10	0.000016	0.000016
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	5.355866e-07	0.000732	0.000532
Franklin Global Sukuk Fund X(qdis)usd	2.612398e-06	0.001616	0.001231
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	4.079217e-07	0.000639	0.000509
Pb Aiman Sukuk Fund	2.019492e-07	0.000449	0.000231
Franklin Global Sukuk Fund W(qdis)usd	2.312546e-06	0.001521	0.001339
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	6.854323e-07	0.000828	0.000744
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	6.298992e-07	0.000794	0.000701
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	2.504382e-06	0.001583	0.001355
Franklin Global Sukuk Fund W(qdis)eur-h1	1.525132e-06	0.001235	0.000963
Kaf Sukuk Fund	1.163280e-08	0.000108	0.000088
DJ. (ex-Reinvestment)	2.666810e-06	0.001633	0.001400
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	4.027857e-07	0.000635	0.000543
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	1.347728e-05	0.003671	0.002947
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	2.673162e-06	0.001635	0.001406
S&P GCC	3.612599e-06	0.001901	0.001664
S&P Global	4.111519e-06	0.002028	0.001627
S&P MENA	3.310611e-06	0.001820	0.001553
Sukuk & Bonds	7.732292e-06	0.002781	0.002093

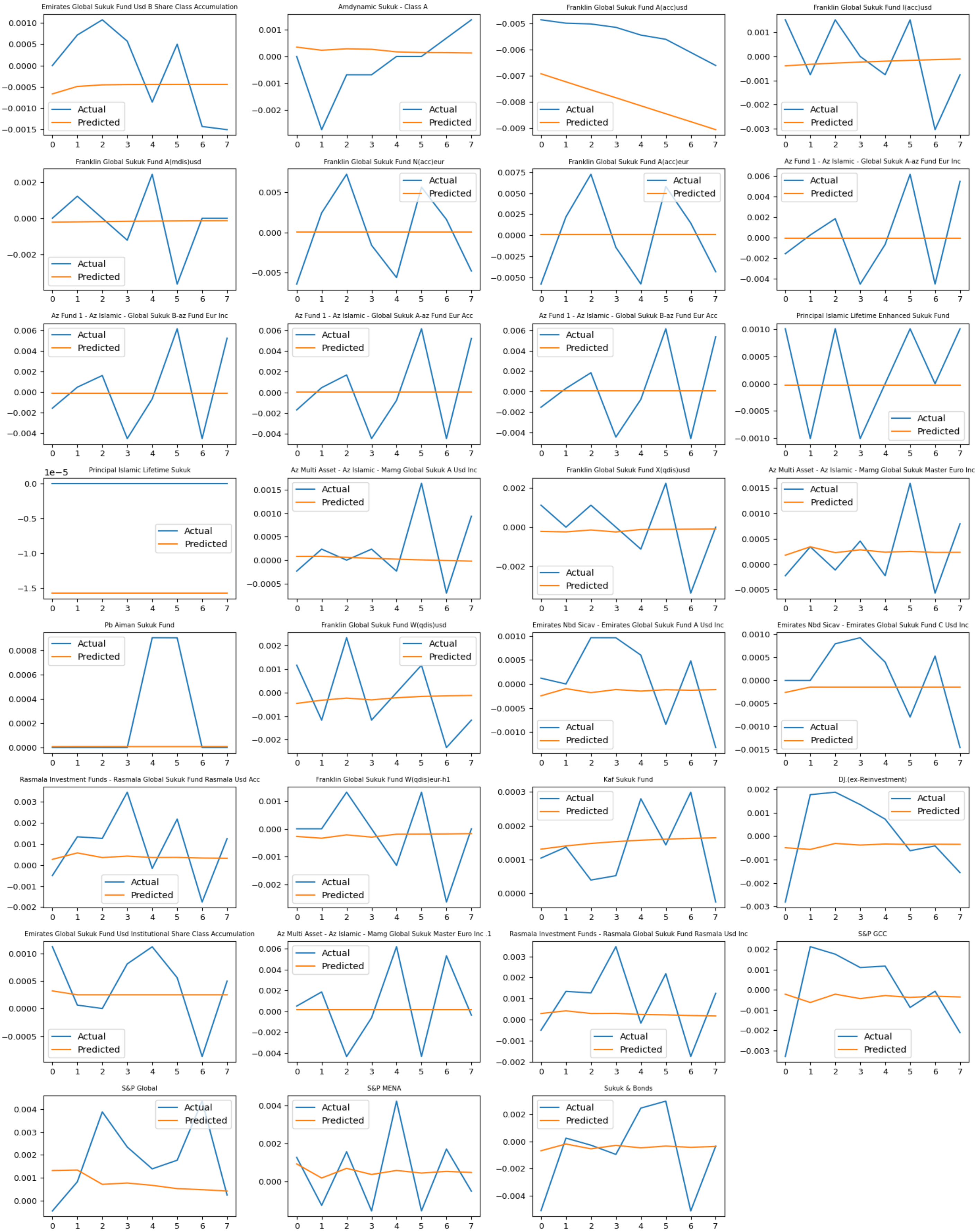
For comprehensive evaluation about ARIMA model performance in the short run, **Table 6.7** provide quantitative measurements. As shown the performance metrics table reveals a diverse range of outcomes, characterized by three main indicators: Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE). These indicators provide a comprehensive view of the model's predictive capabilities across different funds, highlighting areas of strength and potential improvement.

Firstly, the metrics demonstrate a distinct variance in model accuracy among the funds. Exceptionally low MSE, RMSE, and MAE values, such as those seen in the "Principal Islamic Lifetime Sukuk" and "Pb Aiman Sukuk Fund," suggest that the ARIMA model performs exceedingly well in these instances. This high degree of accuracy indicates that the model's parameters are well-suited to the characteristics and market dynamics of these specific funds, allowing it to capture and predict their movements with great precision.

On the other end of the spectrum, funds like "Franklin Global Sukuk Fund N(acc)eur" and "S&P Global" display significantly higher values for all four metrics. These elevated figures suggest considerable prediction errors, which could be attributed to several factors. For instance, the model may not be capturing all relevant market dynamics due to its inherent linear nature, or it may be inadequately tuned to address the specific volatilities and trends of these funds. The high MSE and RMSE values are particularly concerning as they emphasize the presence of large errors, potentially exacerbated by volatile market movements that the model fails to anticipate.

The consistent relationship between MSE, RMSE, and MAE across most funds indicates that the model's error distribution is stable across different scenarios. However, the variable magnitude of these metrics across the funds suggests that the underlying data complexity and market behaviour significantly influence model performance. This insight is critical for identifying funds where the model is less effective, signalling a need for reconsideration of the model's applicability.

Figure 6.4 One Week Predicted with ARIMA Model Vs One-Week Actual Values.



As depicted in the **Figure 6.4**, the ARIMA model exhibits a varied level of accuracy and highlights several key issues and considerations in its application across the short run. Initially, the model demonstrates a decent ability to follow the general trends of the funds' returns. Notably, in some funds like the S&P MENA, the predicted values align closely with the actual values, suggesting a good fit and effective parameter settings for these specific market conditions. However, this level of accuracy is not consistent across all funds.

A significant challenge for the ARIMA model in the short run evident from the plots is its handling of volatile markets. In funds with sharp fluctuations, such as the Emirates Global Sukuk Fund, the model struggles to accurately predict peaks and troughs, which results in larger discrepancies between the predicted and actual values. This could be attributed to the intrinsic limitations of ARIMA models, which are primarily designed for data with trends and seasonality but may not perform as well with high volatility unless appropriately modified.

Another issue observed is the systematic bias in some of the predictions, where the model either consistently overpredicts or underpredicts across certain funds. For example, plots like those for "the Franklin Global Sukuk Fund AccUsd" show a pattern where the predictions lag behind the actual values. This lag indicates that the model's parameters might not be capturing the latest trends efficiently, possibly due to a lag parameter that is set too high or insufficient differencing, which could be adjusted to improve responsiveness.

Overall, while the ARIMA model offers a baseline capability for predicting short term returns in certain less volatile Sukuk funds, the mixed results across different funds suggest that it might require fine-tuning or even replacement for more complex funds. The model's linear nature limits its ability to handle non-linear dynamics often observed in financial markets, suggesting a potential benefit in exploring alternative forecasting methods.

2.1.2 ARIMA Model Performance in The Medium Term (One Month):

For the medium term, we extended the evaluation period to one month. This section discusses the model's predictive capability over this time frame.

Table 6-8 Medium Term ARIMA Model Performance Error Metrics

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	1.187810e-06	0.001090	0.000914
Amdynamic Sukuk - Class A	1.611192e-06	0.001269	0.000590
Franklin Global Sukuk Fund A(acc)usd	5.412024e-05	0.007357	0.007074
Franklin Global Sukuk Fund I(acc)usd	1.808629e-06	0.001345	0.000982
Franklin Global Sukuk Fund A(mdis)usd	2.258397e-06	0.001503	0.001072
Franklin Global Sukuk Fund N(acc)eur	1.868164e-05	0.004322	0.003568
Franklin Global Sukuk Fund A(acc)eur	1.844843e-05	0.004295	0.003535
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	1.254121e-05	0.003541	0.002856
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	1.212907e-05	0.003483	0.002815
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	1.222510e-05	0.003496	0.002823
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	1.234829e-05	0.003514	0.002832
Principal Islamic Lifetime Enhanced Sukuk Fund	4.072941e-07	0.000638	0.000412
Principal Islamic Lifetime Sukuk	1.025665e-07	0.000320	0.000142
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	5.829034e-07	0.000763	0.000611
Franklin Global Sukuk Fund X(qdis)usd	1.861229e-06	0.001364	0.000981
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	5.689023e-07	0.000754	0.000608
Pb Aiman Sukuk Fund	2.897870e-07	0.000538	0.000268
Franklin Global Sukuk Fund W(qdis)usd	1.741802e-06	0.001320	0.001041
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	1.148130e-06	0.001072	0.000882
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	1.162189e-06	0.001078	0.000882
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	1.271803e-06	0.001128	0.000877
Franklin Global Sukuk Fund W(qdis)eur-h1	1.737431e-06	0.001318	0.001014
Kaf Sukuk Fund	1.882566e-08	0.000137	0.000120
DJ.(ex-Reinvestment)	2.123975e-06	0.001457	0.001176
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	1.385535e-06	0.001177	0.000905
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	1.155590e-05	0.003399	0.002680
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	2.053548e-06	0.001433	0.001029
S&P GCC	2.720933e-06	0.001650	0.001289
S&P Global	3.893438e-06	0.001973	0.001303
S&P MENA	4.036659e-06	0.002009	0.001693
Sukuk & Bonds	2.694695e-06	0.001642	0.001075

To evaluate the medium-run performance of the ARIMA model, quantitative measurements are provided in **Table 6.8**. As shown, the ARIMA model displays varying levels of accuracy across different Sukuk funds and indices. Among the funds, the Kaf Sukuk Fund stands out with the lowest error metrics, indicating the highest accuracy. The ARIMA model predicts this fund with exceptional precision, as reflected by its MSE of 1.882566e-08, RMSE of 0.000137, and MAE of 0.000120. Similarly, the Principal Islamic Lifetime Sukuk also shows very high accuracy, with low error metrics (MSE: 1.025665e-07, RMSE: 0.000320, MAE: 0.000142).

Several other funds show moderate accuracy. For instance, the Emirates Global Sukuk Fund USD B Share Class Accumulation has moderate error values (MSE: 1.187810e-06, RMSE: 0.001090, MAE: 0.000914). The Franklin Global Sukuk Fund A (MDIS) USD also falls into this category, with an MSE of 2.258397e-06, RMSE of 0.001503, and MAE of 0.001072, suggesting that the model captures the general trend but misses some finer details.

The model's accuracy drops significantly for certain funds, indicating poor performance. The Franklin Global Sukuk Fund A (ACC) USD records notably higher error metrics (MSE: 5.412024e-05, RMSE: 0.007357, MAE: 0.007074), suggesting that the ARIMA model struggles to predict this fund accurately. The high values across all metrics indicate substantial discrepancies between the predicted and actual values. The S&P MENA Index also exhibits higher errors (MSE: 4.036659e-06, RMSE: 0.002009, MAE: 0.001693), reflecting the model's lower accuracy for this index.

Figure 6.5 One Month Predicted with ARIMA Model Vs One-Month Actual Values.

As illustrated in **Figure 6.5**, The plots show that, in most cases, the model fails to produce accurate medium-term forecasts. Given that in majority of the funds and indices, the predicted series appears flat or disconnected from the actual data. Examples include "the Kaf Sukuk Fund", "Principal Islamic Lifetime Sukuk", "Emirates Global Sukuk Fund USD B Share Class Accumulation", "S&P MENA", and "Sukuk & Bonds". In these cases, the ARIMA model does not respond to volatility or shifts in direction, leading to forecasts that miss both the magnitude and the timing of movements. This indicates that the model is not effectively capturing the underlying dynamics of these return series.

This pattern of underperformance is consistent across most of the remaining funds. Predicted values frequently fail to replicate the amplitude or frequency of actual returns, resulting in substantial deviation. The model tends to revert to static or near-zero forecasts, especially for series with non-linear patterns or frequent fluctuations.

A partial exception is the "Franklin Global Sukuk Fund A(acc) USD". Here, the ARIMA model successfully identifies the downward trend present in the actual series. However, despite capturing the direction, the predicted values remain systematically below the actual returns throughout the forecast horizon. This consistent gap highlights the model’s inability to match the true scale of the trend, even when the direction is correctly estimated.

Overall, the ARIMA model demonstrates poor performance in medium-term forecasting across most of the Sukuk sample. Its limited responsiveness and inability to track volatility suggest that it lacks the flexibility needed for accurate return prediction in this context.

2.1.3 ARIMA Model Performance in The Long Term (200 Days):

The long-term performance of the GARCH (1,1) model was assessed over a 200-day period. This part of the analysis focuses on the model's stability and consistency in volatility prediction over an extended period.

Table 6-9 Long Term ARIMA Model Performance Error Metrics.

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	3.71E-06	0.001927	0.001418
Amdynamic Sukuk - Class A	1.02E-06	0.001008	0.000631
Franklin Global Sukuk Fund A(acc)usd	1.50E-03	0.038725	0.035056
Franklin Global Sukuk Fund I(acc)usd	5.57E-06	0.00236	0.001787
Franklin Global Sukuk Fund A(mdis)usd	6.33E-06	0.002515	0.001884
Franklin Global Sukuk Fund N(acc)eur	2.94E-05	0.005419	0.004201
Franklin Global Sukuk Fund A(acc)eur	2.99E-05	0.005467	0.004212
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	3.24E-05	0.005694	0.004314
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	3.25E-05	0.0057	0.004309
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	2.85E-05	0.005334	0.004138
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	2.85E-05	0.005335	0.004137
Principal Islamic Lifetime Enhanced Sukuk Fund	1.79E-06	0.001337	0.000678
Principal Islamic Lifetime Sukuk	2.05E-06	0.001433	0.000649
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	4.64E-06	0.002154	0.001151
Franklin Global Sukuk Fund X(qdis)usd	7.62E-06	0.00276	0.001993
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	1.60E-06	0.001264	0.000942
Pb Aiman Sukuk Fund	3.69E-06	0.001922	0.000653
Franklin Global Sukuk Fund W(qdis)usd	7.64E-06	0.002763	0.001988
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	4.91E-06	0.002215	0.001488
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	4.92E-06	0.002219	0.001491
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	2.80E-06	0.001673	0.00122
Franklin Global Sukuk Fund W(qdis)eur-h1	7.61E-06	0.002759	0.001912
Kaf Sukuk Fund	8.00E-07	0.000895	0.000503
DJ.(ex-Reinvestment)	3.96E-06	0.001991	0.001529
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	3.53E-06	0.001879	0.001361
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	2.86E-05	0.005348	0.004145
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	3.36E-06	0.001834	0.00132
S&P GCC	4.36E-06	0.002089	0.001594
S&P Global	3.19E-06	0.001786	0.001296
S&P MENA	4.38E-06	0.002093	0.001663
Sukuk & Bonds	4.34E-06	0.002082	0.001107

As depicted in Table 6.9, the ARIMA model shows limited effectiveness in long-term forecasting across the sukuk dataset. Only a few funds display strong performance with low error metrics. "Principal Islamic Lifetime Sukuk" has an RMSE of 0.001433 and MAE of 0.000649 (not near-zero errors), followed by "Kaf Sukuk Fund" (RMSE: 0.000895, MAE: 0.000503), "Pb Aiman Sukuk Fund" (RMSE: 0.001922, MAE: 0.000653), and "MAMG Global Sukuk Master Euro Inc" (RMSE: 0.001264, MAE: 0.000942).

Moderate accuracy appears in funds like "Principal Islamic Lifetime Enhanced Sukuk" (RMSE: 0.001337), "Amdynamic Sukuk – Class A" (RMSE: 0.001008), and "Emirates Global Sukuk Fund USD B" (RMSE: 0.001927), where RMSE ranges from approximately 0.0010 to 0.0019. In these cases, the model tracks the direction but misses finer movements.

Most funds, however, fall into poor performance. "Franklin Global Sukuk Fund A(acc)usd" (RMSE: 0.038725, MAE: 0.035056) shows extremely high errors. Other Franklin funds, "Rasmala Funds" (e.g., RMSE: 0.001673–0.001834), and S&P indices (RMSE: 0.001786–0.002093) record RMSE values between 0.0015 and 0.0026, showing the model's inability to capture volatility or structural changes. The weakest results are in euro-denominated funds such as "Franklin Global Sukuk Fund N(acc)eur" (RMSE: 0.005419, MAE: 0.004201) and "Az Fund 1" classes (RMSE: 0.005334–0.005700, MAE over 0.004), indicating persistent forecast errors.

Overall, these findings confirm that ARIMA lacks robustness for long-term sukuk forecasting and performs best only in stable, low-noise conditions.

Figure 6.6 200 Predicted Days with Arima Model Vs 200 Actual Values.

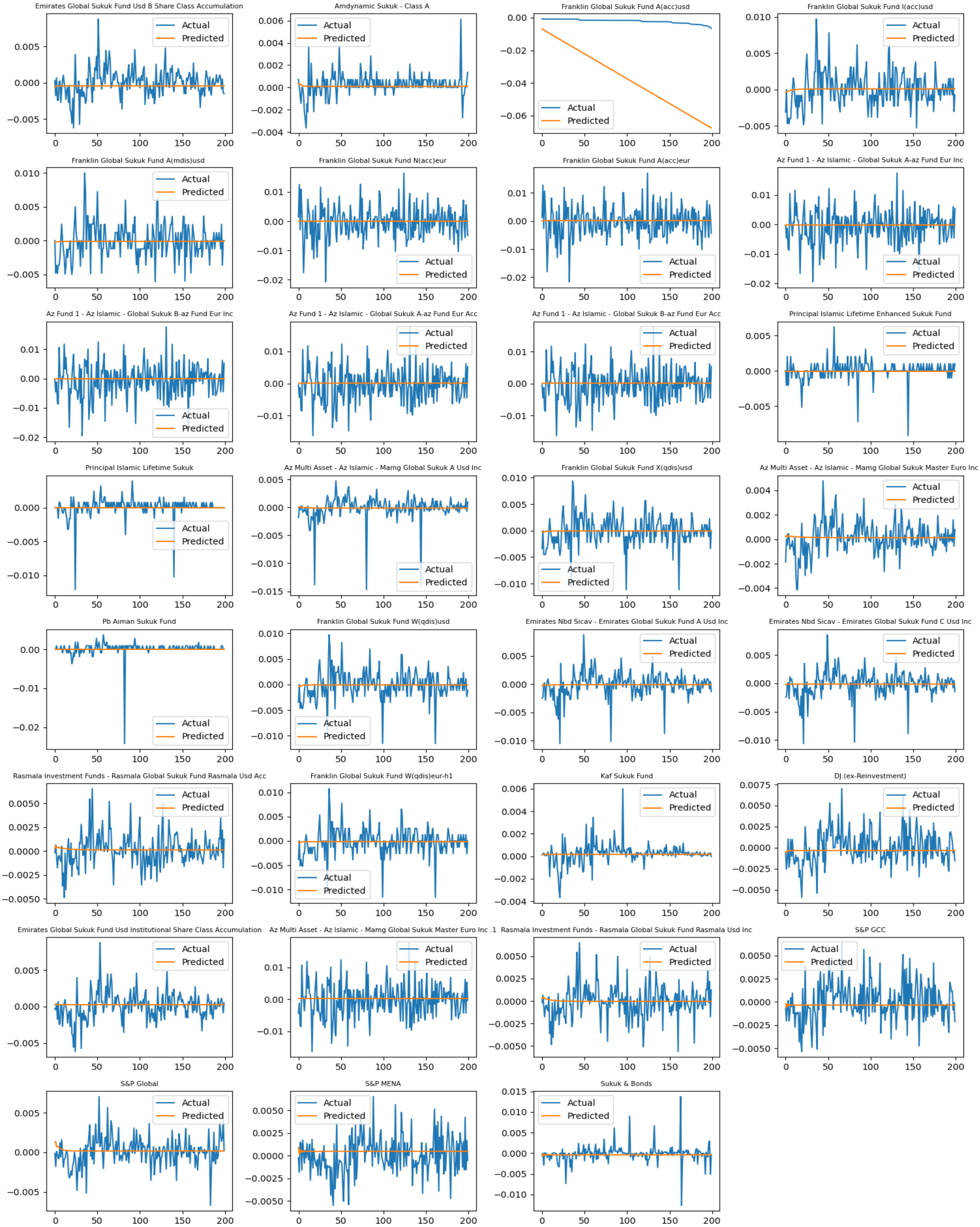


Figure 6.6 illustrates the performance of an ARIMA model in predicting the returns of various Sukuk funds over 200 days.

In most plots, the predicted lines are flat or smoothed, failing to capture the volatility or direction of the actual returns. This is clear in funds like Kaf Sukuk Fund, Rasmala, Principal Islamic Lifetime Enhanced Sukuk, and Az Multi Asset – MAMG Global Sukuk, where the model underreacts to market movements.

The Franklin Global Sukuk Fund A(acc) USD shows the worst fit. The prediction follows a linear decline while actual returns are volatile, indicating a complete mismatch. Similar issues appear in Franklin W(qdis) and W(dis) EUR-h1, where the model fails to respond to return fluctuations.

Even in more stable cases like PB Aiman Sukuk Fund or Principal Islamic Lifetime Sukuk, the model captures general direction but misses the scale of variation. For the S&P indices and Sukuk & Bonds, the ARIMA forecasts stay flat, ignoring visible shifts in the actual data.

These plots confirm that ARIMA lacks the flexibility to handle long-term forecasting for most sukuk funds. It fails to track trends, underestimates volatility, and struggles with non-linear patterns.

2.1.4 Comparative Summary of ARIMA Model Performance Across Forecast Horizons:

Comparing the ARIMA model's performance across different time horizons reveals declining performance as the forecasting horizon extends from short to long term.

In the short run (7 days), the model performs relatively well for low-volatility funds. Error metrics are lowest in this horizon, especially for funds like "Principal Islamic Lifetime Sukuk" (RMSE: 0.000016), "Kaf Sukuk Fund", and "Pb Aiman Sukuk Fund". Visual plots confirm a reasonable match between actual and predicted returns for these cases. However, accuracy declines in funds with higher volatility or noise, where predictions appear flat or lagging.

In the medium run (30 days), predictive accuracy deteriorates. RMSE and MAE values increase for most funds. While "Kaf Sukuk Fund" and "Principal Islamic Lifetime Sukuk" still show low errors, many others such as "Franklin Global Sukuk Fund A(acc) USD" (RMSE: 0.007357) reveal a growing gap between predictions and actual movements. Visual inspection confirms this decline. Most predicted lines fail to follow the dynamic shifts in the actual data, indicating reduced responsiveness to volatility and trend changes.

In the long run (200 days), the ARIMA model consistently underperforms. The model generates smoothed or nearly flat forecasts, failing to capture return variability. RMSE values peak for several funds, including "Franklin Global Sukuk Fund N(acc)eur" and "Az Fund 1 classes", with errors over 0.004. Visual plots reinforce this result, given that predictions often diverge from actual returns or completely miss trend direction, especially in volatile series.

Across all three horizons, only a few funds with stable and low-variance behaviour maintain consistent prediction quality. In contrast, most funds especially those affected by volatility, nonlinearity, or structural shifts show a rapid decline in forecast accuracy beyond the short term.

These results confirm that ARIMA is only suitable for short-term forecasting under low volatility conditions. Its limitations become apparent in medium and long horizons, where more adaptive or hybrid models are required for reliable performance.

2.2 GARCH (1.1) FORECASTING PERFORMANCE:

This section examines the performance of the GARCH (1,1) model over different time horizons. We analyze its effectiveness in predicting volatility over short-term, medium-term, and long-term periods. The results are categorized based on these different time frames to provide a comprehensive evaluation.

2.2.1 GARCH (1.1) Model Performance in The Short Term (one week):

In the short term, the GARCH (1,1) model's performance is evaluated over a one-week period. This section covers the associated performance metrics and the visualization of the model's predictions versus the actual values.

Table 6-10 Short Term GARCH (1.1) Model Performance Error Metrics

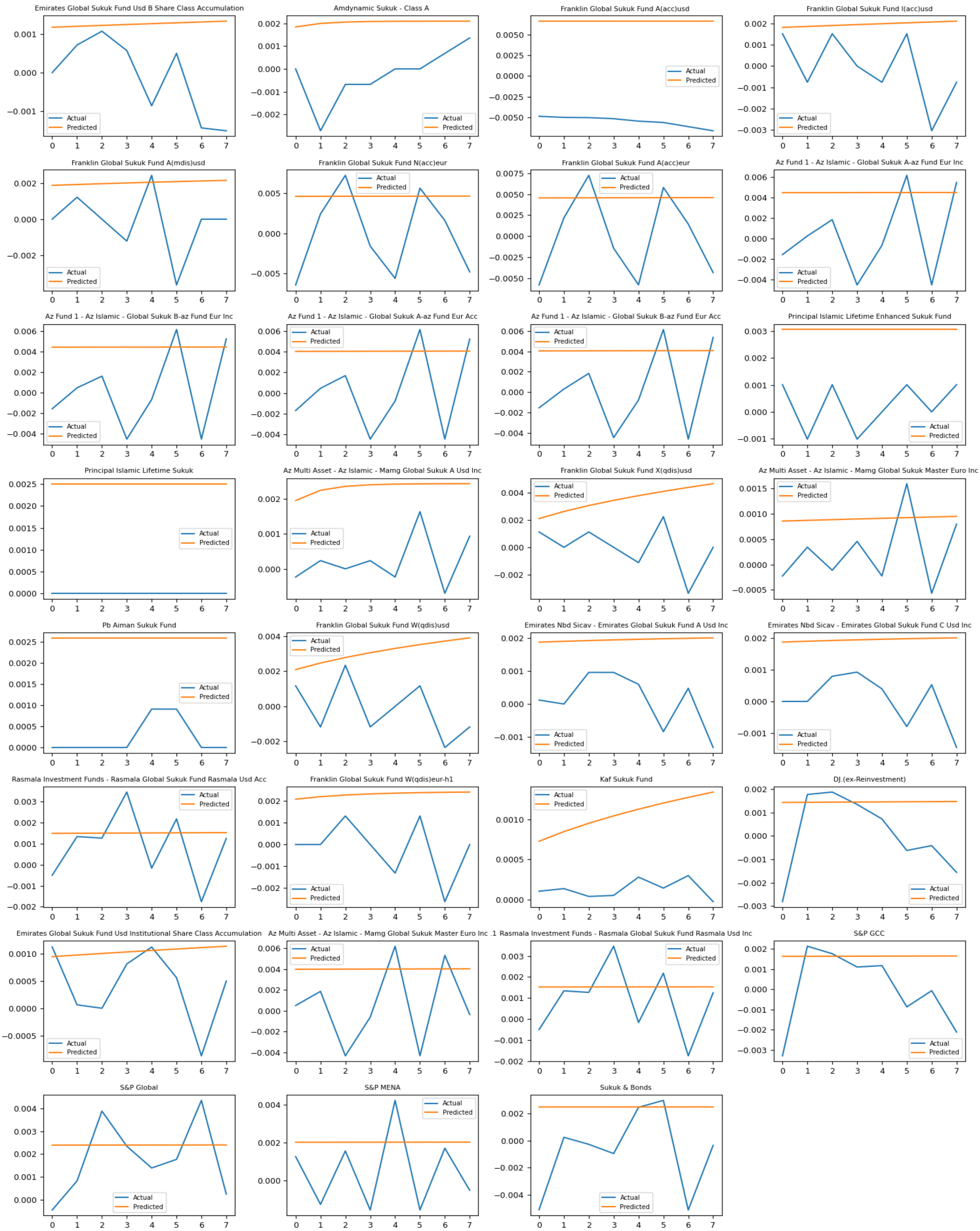
Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	2.846195e-06	0.001687	0.001371
Amdynamic Sukuk - Class A	6.538869e-06	0.002557	0.002305
Franklin Global Sukuk Fund A(acc)usd	1.462646e-04	0.012094	0.012080
Franklin Global Sukuk Fund I(acc)usd	6.679146e-06	0.002584	0.002068
Franklin Global Sukuk Fund A(mdis)usd	7.527137e-06	0.002744	0.002266
Franklin Global Sukuk Fund N(acc)eur	4.703025e-05	0.006858	0.005720
Franklin Global Sukuk Fund A(acc)eur	4.464766e-05	0.006682	0.005653
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	3.174334e-05	0.005634	0.004842
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	3.139712e-05	0.005603	0.004802
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	2.813304e-05	0.005304	0.004595
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	2.848775e-05	0.005337	0.004624
Principal Islamic Lifetime Enhanced Sukuk Fund	8.583569e-06	0.002930	0.002807
Principal Islamic Lifetime Sukuk	6.308844e-06	0.002512	0.002512
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	4.869511e-06	0.002207	0.002104
Franklin Global Sukuk Fund X(qdis)usd	1.669070e-05	0.004085	0.003528
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	8.310904e-07	0.000912	0.000813
Pb Aiman Sukuk Fund	5.736180e-06	0.002395	0.002363
Franklin Global Sukuk Fund W(qdis)usd	1.384533e-05	0.003721	0.003248
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	3.971515e-06	0.001993	0.001829
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	4.250628e-06	0.002062	0.001907
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	2.744316e-06	0.001657	0.001280
Franklin Global Sukuk Fund W(qdis)eur-h1	7.704254e-06	0.002776	0.002477
Kaf Sukuk Fund	9.228619e-07	0.000961	0.000937
DJ. (ex-Reinvestment)	4.504611e-06	0.002122	0.001607
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	8.144099e-07	0.000902	0.000689
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	2.546126e-05	0.005046	0.004358
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	2.762131e-06	0.001662	0.001289
S&P GCC	6.063001e-06	0.002462	0.001819
S&P Global	2.819788e-06	0.001679	0.001460
S&P MENA	6.108845e-06	0.002472	0.002096
Sukuk & Bonds	1.856520e-05	0.004309	0.003377

As depicted in **Table 6.10** the GARCH(1.1) model performance for predicting short term returns varies significantly across different funds. First, it performs well for several funds with stable volatility patterns. For example, "Emirates Global Sukuk Institutional" (RMSE = 0.000902, MAE = 0.000689), "Az Multi Asset - Mamg Global Sukuk Master Euro Inc" (RMSE = 0.000912, MAE = 0.000813), and "Kaf Sukuk Fund" (RMSE = 0.000961, MAE = 0.000937) show very low error values. These results suggest that GARCH(1,1) effectively captures short-term volatility when the data exhibit consistent clustering and limited noise.

In contrast, the model struggles with several Franklin funds. Indeed, "The Franklin Global Sukuk Fund A(acc)usd" records the highest error (RMSE = 0.012094, MAE = 0.012080), and similar issues appear in N(acc)eur (RMSE = 0.006858, MAE = 0.005720) and A(acc)eur (RMSE = 0.006682, MAE = 0.005653). These high errors indicate that GARCH(1,1) cannot capture the complexity of their return patterns. The likely causes are structural breaks, nonlinearity, or regime changes in the data that the model cannot handle.

For sukuk indices, the model shows moderate performance. For example, "S&P GCC Index" (RMSE = 0.002462), "S&P Global Index" (RMSE = 0.001679), and "S&P MENA Index" (RMSE = 0.002472) have acceptable error levels. These indices, due to their aggregated nature, tend to be less volatile and smoother, which allows GARCH(1,1) to perform more reliably.

Figure 6.7 One Week Predicted with GARCH (1.1) Model Vs One-Week Actual Values.



As illustrated in **Figure 6.7**, the visualization shows that the GARCH(1,1) model fails to capture short-term return dynamics. In almost all cases, the predicted lines are flat or barely change, while actual returns fluctuate sharply. which indicates poor predictive accuracy.

Even for relatively stable funds like "KAF Sukuk Fund" or "Emirates Global Sukuk Fund," the model doesn’t follow the return movements. For volatile funds like "Franklin Global Sukuk Fund" and "Rasmala Global Sukuk Fund," the mismatch is more obvious.

The same issue appears in sukuk indices. The predicted values do not react to market changes. The model fails to reflect actual volatility, suggesting it is not suitable for this data.

This suggests that GARCH(1,1) model may not be suitable for short-term prediction for these particular Sukuk funds, due to its inability to model the observed return volatility.

2.2.2 GARCH (1.1) Model Performance in The Medium Term (one month):

For the medium term, we extended the evaluation period to one month. This section explores the model's predictive capabilities over this timeframe, by analysing the performance metrics before comparing predicted and real values plots.

Table 6-11 Medium Term GARCH (1.1) Model Performance Error Metrics

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	0.000004	0.002115	0.001858
Amdynamic Sukuk - Class A	0.000005	0.002182	0.002040
Franklin Global Sukuk Fund A(acc)usd	0.000122	0.011051	0.011024
Franklin Global Sukuk Fund I(acc)usd	0.000009	0.002937	0.002720
Franklin Global Sukuk Fund A(mdis)usd	0.000009	0.002972	0.002670
Franklin Global Sukuk Fund N(acc)eur	0.000045	0.006706	0.005442
Franklin Global Sukuk Fund A(acc)eur	0.000044	0.006651	0.005367
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	0.000033	0.005736	0.004852
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	0.000032	0.005696	0.004811
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	0.000029	0.005421	0.004564
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	0.000030	0.005444	0.004576
Principal Islamic Lifetime Enhanced Sukuk Fund	0.000009	0.002926	0.002864
Principal Islamic Lifetime Sukuk	0.000006	0.002403	0.002386
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	0.000006	0.002542	0.002435
Franklin Global Sukuk Fund X(qdis)usd	0.000045	0.006686	0.006352
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	0.000002	0.001233	0.001114
Pb Aiman Sukuk Fund	0.000006	0.002373	0.002325
Franklin Global Sukuk Fund W(qdis)usd	0.000027	0.005163	0.004931
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	0.000006	0.002544	0.002323
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	0.000007	0.002594	0.002377
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	0.000003	0.001732	0.001536
Franklin Global Sukuk Fund W(qdis)eur-h1	0.000009	0.003000	0.002719
Kaf Sukuk Fund	0.000002	0.001572	0.001494
DJ. (ex-Reinvestment)	0.000005	0.002153	0.001790
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	0.000003	0.001821	0.001550
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	0.000029	0.005389	0.004572
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	0.000004	0.002035	0.001677
S&P GCC	0.000006	0.002393	0.001974
S&P Global	0.000008	0.002853	0.002360
S&P MENA	0.000007	0.002593	0.002005
Sukuk & Bonds	0.000011	0.003255	0.002841

Table 6.11 shows the performance metrics of the GARCH (1,1) model in predicting medium-term (30 days) returns for Sukuk funds, its accuracy varies significantly across sukuk funds and indices. A few funds demonstrate very low error values, indicating strong forecasting accuracy. For instance, "Az Multi Asset - MAMG Global Sukuk Master Euro Inc," "KAF Sukuk Fund," and both accumulation and income classes of the "Rasmala Global Sukuk Fund" show RMSE values well below 0.002. These results suggest that the model performs reliably when return series are relatively smooth and exhibit consistent volatility clustering.

A larger group of funds falls into the moderate performance range, with RMSE values between 0.002 and 0.003. Examples include "Principal Islamic Lifetime Enhanced Sukuk Fund," "Emirates Global Sukuk Fund Institutional Share Class," and various classes of "Az Multi Asset" and "Emirates NBD" funds. In these cases, the model captures general volatility behaviour but fails to fully reflect more subtle changes in return dynamics. This implies that while the GARCH(1,1) model is somewhat effective, its limitations become more visible as market conditions vary.

In contrast, several funds—particularly those from Franklin—exhibit poor forecasting results. "Franklin Global Sukuk Fund A(acc) USD" shows the highest error (RMSE = 0.011051, MAE = 0.011024), followed by other Franklin share classes such as N(acc) EUR and A(acc) EUR, where RMSE values exceed 0.006. These high errors reveal that the GARCH(1,1) model struggles to adapt to the volatility structure of these funds, likely due to nonlinear patterns, structural breaks, or irregular return behaviour.

The model's performance on sukuk indices is more stable but not highly accurate. "S&P GCC," "S&P MENA," "S&P Global," and the "Sukuk & Bonds" index present RMSE values ranging from 0.0024 to 0.0032. These moderate errors suggest that while GARCH(1,1) can handle aggregated index-level data better than some individual funds, it still lacks the flexibility to capture medium-term variations in full detail.

Figure 6.8 One Month Predicted with GARCH (1.1) Model Vs One-Month Actual Values.

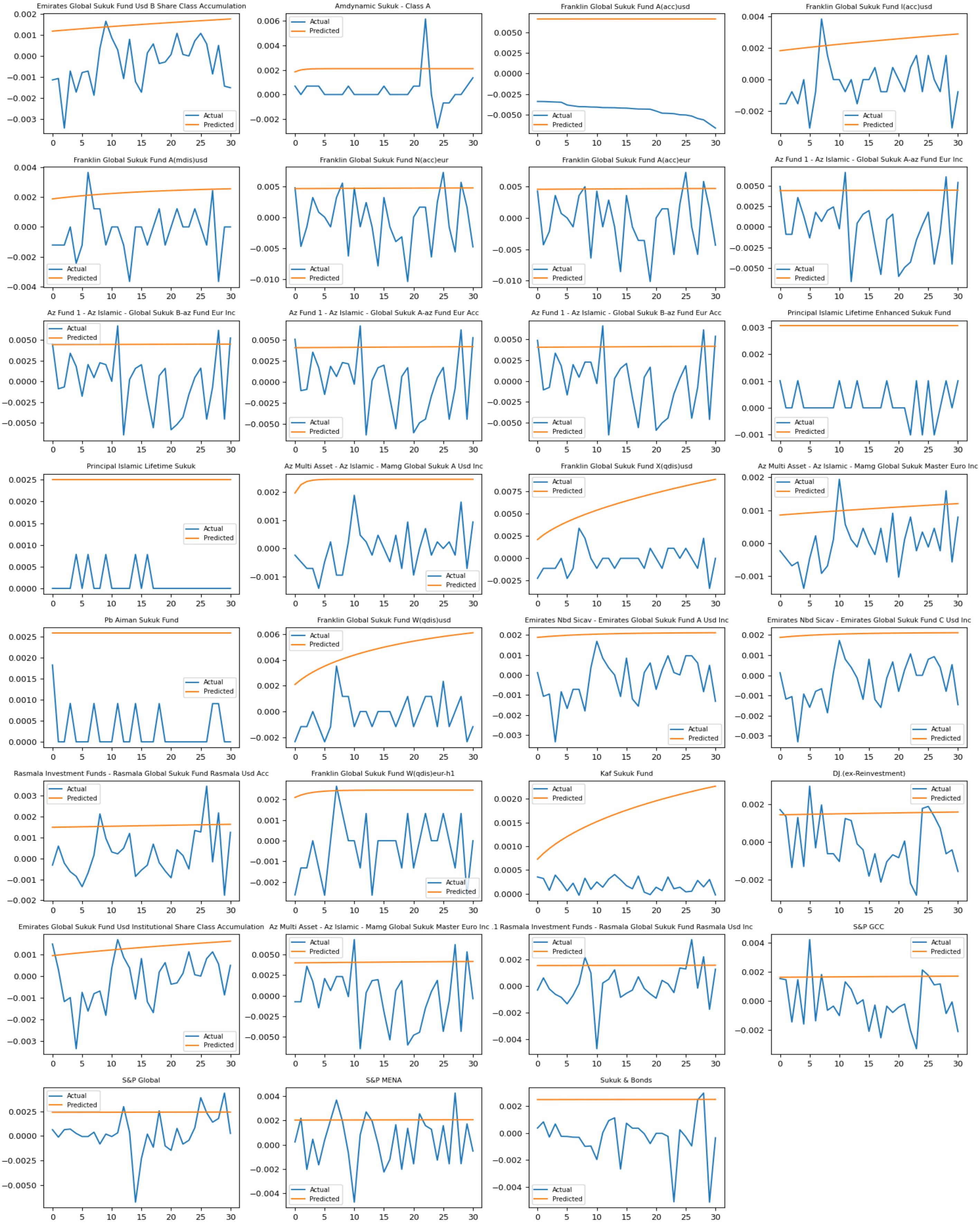


Figure 6.8 illustrates the medium-term performance of the GARCH (1,1) model for Sukuk funds, revealing significant limitations in accurately predicting returns over a 30-day period. The model consistently fails to capture the observed volatility and fluctuations in actual returns across various funds. Instead, it predicts nearly constant returns that do not align with the dynamic nature of the actual data. For instance, funds like "Franklin Global Sukuk Fund A(acc) USD", "N(acc) EUR", and "W(qdis) USD" exhibit particularly poor fits, as the predicted returns show no reaction to spikes or jumps in actual returns. Even in funds where return volatility is lower, such as "Az Multi Asset - MAMG Global Sukuk Master Euro Inc" the model fails to match the observed variations, which points to a structural mismatch between the model and the data.

This indicates that the GARCH (1,1) model may not be suitable for medium-term return prediction for these Sukuk funds, suggesting a need for more sophisticated models or additional explanatory variables to better capture the underlying return dynamics.

2.2.3 GARCH (1.1) Model Performance in the long Term (200 days):

The GARCH (1,1) model's long-term performance was tested over a 200-day period. In this part of the analysis, we focus on assessing the model's stability and reliability in predicting volatility over an extended duration.

Table 6-12 Long Term GARCH (1.1) Model Performance Error Metrics.

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	0.000010	0.003181	0.002803
Amdynamic Sukuk - Class A	0.000004	0.002082	0.001911
Franklin Global Sukuk Fund A(acc)usd	0.000078	0.008850	0.008765
Franklin Global Sukuk Fund I(acc)usd	0.000024	0.004920	0.004432
Franklin Global Sukuk Fund A(mdis)usd	0.000014	0.003681	0.003097
Franklin Global Sukuk Fund N(acc)eur	0.000056	0.007503	0.006224
Franklin Global Sukuk Fund A(acc)eur	0.000056	0.007506	0.006217
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	0.000059	0.007701	0.006206
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	0.000059	0.007695	0.006205
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	0.000051	0.007124	0.005850
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	0.000051	0.007125	0.005845
Principal Islamic Lifetime Enhanced Sukuk Fund	0.000011	0.003287	0.003036
Principal Islamic Lifetime Sukuk	0.000008	0.002808	0.002439
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	0.000011	0.003350	0.002628
Franklin Global Sukuk Fund X(qdis)usd	0.000258	0.016069	0.015042
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	0.000004	0.002027	0.001761
Pb Aiman Sukuk Fund	0.000010	0.003112	0.002467
Franklin Global Sukuk Fund W(qdis)usd	0.000056	0.007473	0.006998
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	0.000010	0.003127	0.002494
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	0.000010	0.003166	0.002531
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	0.000006	0.002439	0.002090
Franklin Global Sukuk Fund W(qdis)eur-h1	0.000014	0.003804	0.003050
Kaf Sukuk Fund	0.000008	0.002870	0.002736
DJ. (ex-Reinvestment)	0.000007	0.002599	0.002183
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	0.000009	0.003065	0.002682
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	0.000050	0.007084	0.005809
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	0.000006	0.002457	0.002019
S&P GCC	0.000007	0.002724	0.002254
S&P Global	0.000009	0.002927	0.002538
S&P MENA	0.000008	0.002883	0.002409
Sukuk & Bonds	0.000011	0.003371	0.002931

As represented in **Table 6.12**, The long-term forecasting results using the GARCH(1,1) model show persistent performance limitations, with a few exceptions. A small group of funds achieved relatively low RMSE and MAE values, such as "Az Multi Asset - MAMG Global Sukuk Master Euro Inc" (RMSE = 0.002027, MAE = 0.001761), "Rasmala Global Sukuk Fund" in both accumulation and income classes (RMSE = 0.00245, MAE = 0.0020), and "Principal Islamic Lifetime Sukuk" (RMSE = 0.002808, MAE = 0.002439). These results suggest that in cases where long-term volatility is more stable, the model can still offer basic trend approximations.

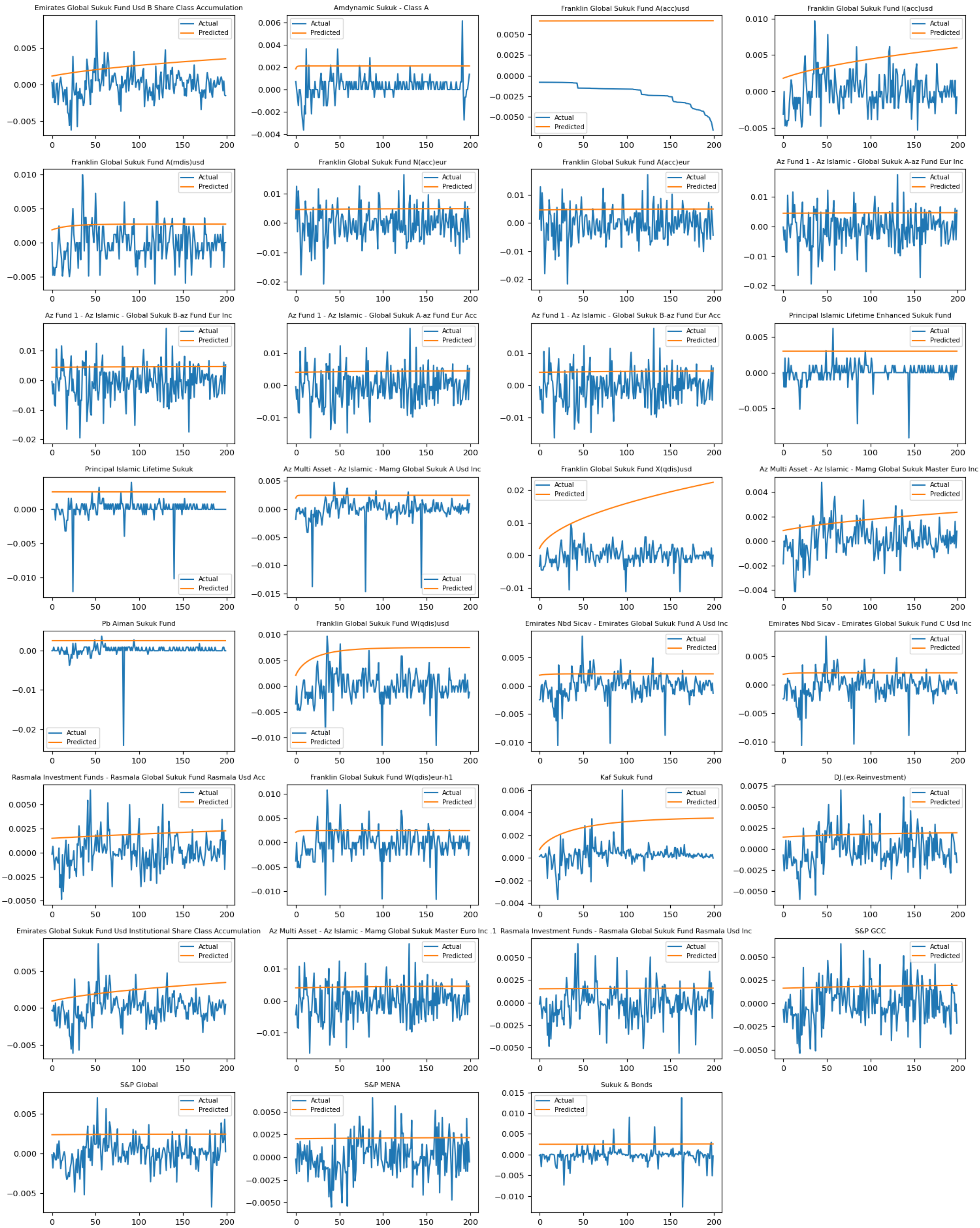
However, many funds show a sharp drop in predictive quality. "Franklin Global Sukuk Fund X(qdis) USD" stands out with the worst performance (RMSE = 0.016069, MAE = 0.015042), far exceeding other funds. Several other Franklin and Az Fund share classes also record high errors, with RMSE values above 0.007 and MAE often exceeding 0.006. This indicates the model’s inability to capture long-term volatility dynamics in more complex or irregular series.

For other funds like "Emirates Global Sukuk", "Pb Aiman Sukuk, and Az Islamic Sukuk A/B EUR", the model delivers only moderate accuracy. Errors range from 0.003 to 0.007, showing a mismatch between predictions and actual long-term return behaviours. These results suggest that the model may track some average volatility levels but fails to adapt to shifts or nonlinear movements.

On the index level, the GARCH(1,1) model maintains a similar moderate trend. "S&P GCC", "S&P MENA", and "Sukuk & Bonds" indices show RMSE values between 0.0027 and 0.0034, with low MAE values. Although errors are not extreme, the model still lacks the flexibility needed for accurate long-term tracking, even in smoother aggregated series.

Overall, the long-term results confirm the structural weakness of GARCH(1,1) for extended horizons. While it performs adequately in a few stable series, it consistently underperforms for volatile or complex return patterns.

Figure 6.9 200 Predicted Days with GARCH (1.1) Model Vs 200 Actual Values.



As shown in **Figure 6.9**, The long-term performance of the GARCH (1,1) model for Sukuk funds highlights significant limitations in accurately predicting returns over a 200-day period. Across the various funds, the model consistently fails to capture the observed volatility and fluctuations in actual returns, resulting in relatively stable predicted returns that do not align with the dynamic nature of the actual data.

The performance is particularly poor in several Franklin funds, such as "Franklin Global Sukuk Fund A(acc) USD" and "Franklin Global Sukuk Fund X(qdis) USD". These plots show extreme divergence, with predicted lines declining or drifting in one direction while actual returns remain highly volatile. In "Pb Aiman Sukuk Fund", the model entirely fails to react to a sharp drop, showing no responsiveness.

Even in cases with lower volatility, like "KAF Sukuk Fund" or "Rasmala Global Sukuk Fund", the predicted values do not align with observed patterns. Although volatility is lower, the model still produces overly smoothed predictions that ignore shifts and corrections present in actual returns.

This suggests that the GARCH (1,1) model may not be suitable for long-term prediction of returns for these particular Sukuk funds, and there is a need for more sophisticated models or additional explanatory variables to better capture the underlying return dynamics.

2.2.4 Comparative Summary of GARCH (1,1) Model Performance Across Forecast Horizons:

In general, the performance of the GARCH(1,1) model varies significantly across different forecast horizons. For many funds, prediction errors increase as the horizon extends, indicating that the model's accuracy weakens over time. This trend is especially visible in funds such as "Franklin Global Sukuk Fund A(acc) USD", where high errors persist across all periods, highlighting a consistent mismatch between model output and actual return behavior.

In contrast, some funds such as "Amdynamic Sukuk Class A" and "Az Multi Asset MAMG Global Sukuk Master Euro Inc" maintain low errors across short, medium, and long terms, suggesting that the model performs well when the return series is relatively stable and predictable. A few other funds, including "Principal Islamic Lifetime Sukuk" and "S&P GCC", show moderate but consistent performance, with only slight increases in error over longer horizons.

Overall, the GARCH(1,1) model offers limited short-term predictive power under stable conditions but performs poorly for most funds as the forecast window extends. The results confirm that its rigid assumptions and limited flexibility make it unsuitable for capturing the complex dynamics of sukuk returns over time. More advanced or adaptive models are needed to improve forecasting accuracy, especially for medium and long-term horizons.

2.3 HAR Model forecasting Performance:

The third traditional model we have chosen is Heterogeneous autoregressive, in this section, we assess the HAR model's performance across multiple time horizons. By examining its ability to predict volatility in the short-term, medium-term, and long-term, we provide a detailed evaluation. The findings are grouped according to these distinct periods for a thorough analysis.

2.3.1 HAR Model Performance in The Short Term (one week):

For the short-term analysis, we assess the HAR model's performance over a one-week period.

Table 6-13 Short Term HAR Model Performance Error Metrics.

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	8.888392e-07	0.000943	0.000788
Amdynamic Sukuk - Class A	1.472967e-06	0.001214	0.000882
Franklin Global Sukuk Fund A(acc)usd	6.200183e-08	0.000249	0.000182
Franklin Global Sukuk Fund I(acc)usd	2.795084e-06	0.001672	0.001358
Franklin Global Sukuk Fund A(mdis)usd	3.962574e-06	0.001991	0.001372
Franklin Global Sukuk Fund N(acc)eur	2.374584e-05	0.004873	0.004397
Franklin Global Sukuk Fund A(acc)eur	2.266756e-05	0.004761	0.004253
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	1.413391e-05	0.003760	0.003092
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	1.379762e-05	0.003715	0.003066
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	1.375113e-05	0.003708	0.003092
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	1.413922e-05	0.003760	0.003112
Principal Islamic Lifetime Enhanced Sukuk Fund	7.724623e-07	0.000879	0.000774
Principal Islamic Lifetime Sukuk	5.425897e-10	0.000023	0.000021
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	6.658459e-07	0.000816	0.000597
Franklin Global Sukuk Fund X(qdis)usd	3.024966e-06	0.001739	0.001304
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	6.540364e-07	0.000809	0.000640
Pb Aiman Sukuk Fund	1.755870e-07	0.000419	0.000239
Franklin Global Sukuk Fund W(qdis)usd	2.587681e-06	0.001609	0.001405
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	6.900025e-07	0.000831	0.000681
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	6.834361e-07	0.000827	0.000661
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	3.616486e-06	0.001902	0.001734
Franklin Global Sukuk Fund W(qdis)eur-h1	1.852488e-06	0.001361	0.000999
Kaf Sukuk Fund	1.478230e-08	0.000122	0.000105
DJ. (ex-Reinvestment)	2.149671e-06	0.001466	0.001248
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	5.309306e-07	0.000729	0.000617
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	1.337016e-05	0.003657	0.002943
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	3.548091e-06	0.001884	0.001695
S&P GCC	3.265786e-06	0.001807	0.001543
S&P Global	4.574041e-06	0.002139	0.001765
S&P MENA	4.792109e-06	0.002189	0.001973
Sukuk & Bonds	8.568949e-06	0.002927	0.002298

As shown in **Table 6.13**, the HAR model demonstrates strong predictive accuracy for short-term sukuk returns, significantly outperforming the earlier GARCH(1,1) results. Many funds, including "Franklin Global Sukuk Fund A(acc) USD" (RMSE = 0.000249, MAE = 0.000182), "KAF Sukuk Fund" (RMSE = 0.000122, MAE = 0.000105), and "Principal Islamic Lifetime Sukuk" (RMSE = 0.000023, MAE = 0.000021), achieve remarkably low error values, highlighting the model's effectiveness in capturing short-term volatility dynamics.

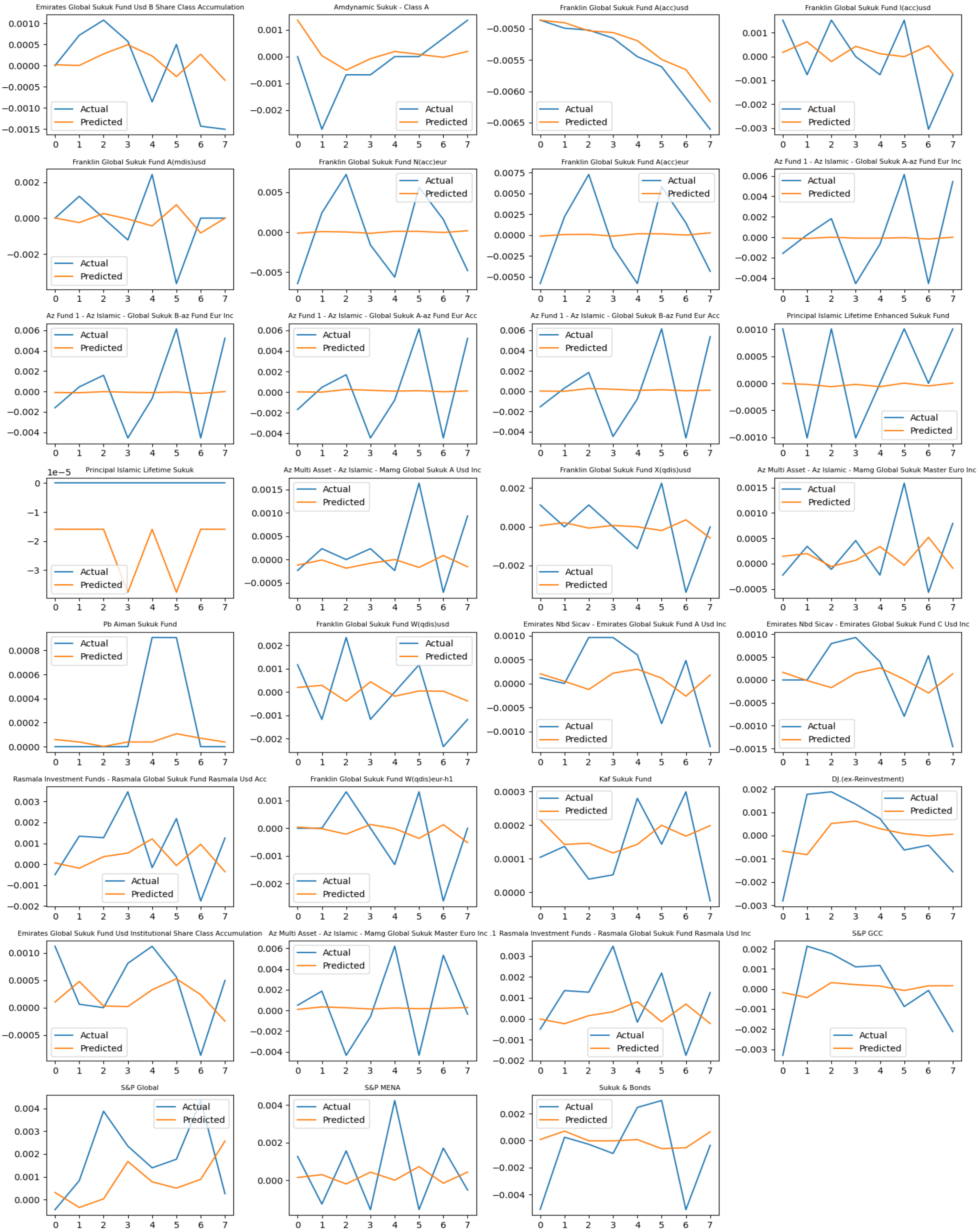
Additionally, funds such as "Az Multi Asset - MAMG Global Sukuk Master Euro Inc" and "Emirates Global Sukuk Institutional" also exhibit strong performance, with RMSE values consistently below 0.001, reinforcing the HAR model's capability to accurately forecast stable and moderately volatile returns.

However, certain funds, notably "Franklin Global Sukuk Fund N(acc) EUR" and "A(acc) EUR", exhibit comparatively higher errors (RMSE ≈ 0.0048), indicating that the HAR model faces challenges in capturing highly volatile or irregular return patterns within these specific series.

At the index level, the HAR model shows moderate to good accuracy, with "S&P GCC", "S&P Global", and "S&P MENA" indices achieving RMSE values around 0.002. Although slightly higher than individual funds, these error levels still represent satisfactory short-term predictive performance.

Overall, the HAR model clearly provides substantial improvements over GARCH(1,1) in short-term forecasting, effectively capturing the volatility structure in most sukuk funds, though challenges remain in handling extremely volatile or irregular series.

Figure 6.10 Short Term HAR Model Performance Error Metrics



As shown in **Figure 6.10**, the visual plots confirm the HAR model's strong short-term forecasting capability for sukuk returns. In most cases, the predicted lines closely track the actual returns, demonstrating high predictive accuracy and responsiveness. Funds such as "Franklin Global Sukuk Fund A(acc) USD", "Principal Islamic Lifetime Sukuk", "KAF Sukuk Fund", and "Emirates Global Sukuk Institutional" show remarkably close matches between predicted and actual values, capturing even sharp fluctuations effectively.

Moderate fits are observed for funds like "S&P GCC", "S&P MENA", and "Az Multi Asset - MAMG Global Sukuk Master Euro Inc", where predictions generally follow the actual returns but occasionally differ in magnitude or timing. Despite minor mismatches, these forecasts remain reasonably accurate.

However, a few funds display weaker predictions, notably "Franklin Global Sukuk Fund N(acc) EUR", "A(acc) EUR", and most of "AZ Funds" variant , where the predicted lines deviate significantly in certain periods, reflecting challenges in capturing particularly volatile or irregular short-term movements.

Overall, the visual analysis strongly supports the HAR model's superior short-term predictive performance relative to the previously examined GARCH(1,1) model. The HAR model's flexibility enables it to adjust effectively to the short-term volatility dynamics present in most sukuk funds, making it a more reliable forecasting tool for short-term horizons.

2.3.2 HAR Model Performance in The Medium Term (one month):

For the medium term, we extended the evaluation period to one month. This section explores the model's predictive capabilities over this timeframe, by analysing the performance metrics, before comparing predicted versus real values plots.

Table 6-14 Medium Term HAR Model Performance Error Metrics.

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	1.182310e-06	0.001087	0.000867
Amdynamic Sukuk - Class A	1.624542e-06	0.001275	0.000613
Franklin Global Sukuk Fund A(acc)usd	2.577621e-08	0.000161	0.000092
Franklin Global Sukuk Fund I(acc)usd	2.058518e-06	0.001435	0.001057
Franklin Global Sukuk Fund A(mdis)usd	2.716524e-06	0.001648	0.001178
Franklin Global Sukuk Fund N(acc)eur	1.858668e-05	0.004311	0.003565
Franklin Global Sukuk Fund A(acc)eur	1.832507e-05	0.004281	0.003532
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	1.265131e-05	0.003557	0.002879
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	1.225661e-05	0.003501	0.002844
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	1.248722e-05	0.003534	0.002846
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	1.262612e-05	0.003553	0.002859
Principal Islamic Lifetime Enhanced Sukuk Fund	4.053461e-07	0.000637	0.000415
Principal Islamic Lifetime Sukuk	1.043226e-07	0.000323	0.000150
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	5.723443e-07	0.000757	0.000599
Franklin Global Sukuk Fund X(qdis)usd	2.002322e-06	0.001415	0.001028
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	5.809777e-07	0.000762	0.000609
Pb Aiman Sukuk Fund	2.811812e-07	0.000530	0.000296
Franklin Global Sukuk Fund W(qdis)usd	1.896111e-06	0.001377	0.001078
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	1.103782e-06	0.001051	0.000851
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	1.098925e-06	0.001048	0.000844
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	1.330916e-06	0.001154	0.000878
Franklin Global Sukuk Fund W(qdis)eur-h1	1.903648e-06	0.001380	0.001074
Kaf Sukuk Fund	2.153399e-08	0.000147	0.000126
DJ. (ex-Reinvestment)	2.066529e-06	0.001438	0.001215
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	1.108103e-06	0.001053	0.000839
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	1.174210e-05	0.003427	0.002699
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	2.111735e-06	0.001453	0.001061
S&P GCC	2.791568e-06	0.001671	0.001340
S&P Global	3.603449e-06	0.001898	0.001267
S&P MENA	4.252553e-06	0.002062	0.001801
Sukuk & Bonds	2.959252e-06	0.001720	0.001164

As depicted in **Table 6.14**, the HAR model continues to demonstrate robust predictive accuracy for sukuk returns at the medium-term horizon. Funds such as "Franklin Global Sukuk Fund A(acc) USD" (RMSE = 0.000161, MAE = 0.000092), "KAF Sukuk Fund" (RMSE = 0.000147, MAE = 0.000126), and "Principal Islamic Lifetime Sukuk" (RMSE = 0.000323, MAE = 0.000150) achieve exceptionally low errors, highlighting the model's sustained accuracy beyond short-term horizons.

Funds including "Az Multi Asset - MAMG Global Sukuk Master Euro Inc", "Principal Islamic Lifetime Enhanced Sukuk Fund", and "Emirates Global Sukuk Institutional" also show good medium-term performance, maintaining RMSE values well below 0.001. This indicates the HAR model's strong ability to capture moderate volatility and maintain accuracy as the forecasting horizon lengthens.

However, certain "Franklin funds" particularly "Franklin Global Sukuk Fund N(acc) EUR" and "A(acc) EUR" continue to pose forecasting challenges, reflected by higher errors (RMSE ≈ 0.0043). These errors, though reduced from the short-term scenario, remain higher than most other funds, suggesting persistent volatility or irregular patterns that challenge even robust models.

At the index level, the HAR model demonstrates moderate to strong predictive power, with indices like "S&P GCC", "S&P Global", and "Sukuk & Bonds" showing RMSE values consistently around or below 0.002. Although slightly less precise compared to individual low-volatility funds, these results still indicate a reliable medium-term forecasting capability.

Overall, the HAR model maintains excellent predictive performance into the medium term, clearly outperforming simpler models such as GARCH(1,1), but its performance is highly fund-specific, necessitating adjustments to enhance its applicability across a broader range of Sukuk funds.

Figure 6.11 One Month Predicted With HAR Model Vs One-Month Actual Values.

As displayed in **Figure 6.10**, the medium-term visualization confirms the HAR model's consistent predictive strength across most sukuk funds. For many funds, the predicted returns closely follow the actual series, reflecting the model's effective capturing of volatility patterns. Notably, "Franklin Global Sukuk Fund A(acc) USD", "KAF Sukuk Fund", and "Principal Islamic Lifetime Sukuk" display excellent visual alignment, with predictions accurately tracking even significant fluctuations.

Funds like "Emirates Global Sukuk Institutional" and "Az Multi Asset - MAMG Global Sukuk Master Euro Inc" also show strong predictive accuracy, with predictions closely aligned to actual returns but occasionally underestimating sharper volatility spikes.

However, the HAR model still faces difficulties with more complex or erratic series, particularly "Franklin Global Sukuk Fund N(acc) EUR", "A(acc) EUR", and most of "Az Funds" variant, where the predicted returns remain nearly flat or show minimal variation, completely missing the sharp volatility spikes or sudden directional movements observed in the actual returns. This indicates the model's limitations when faced with highly volatile or structurally complex series.

Indices such as S&P GCC, S&P MENA, and Sukuk & Bonds demonstrate generally good alignment, indicating the model's reliability in forecasting aggregated market movements, although some mismatches persist during volatile periods.

Overall, visual evidence strongly supports the HAR model's robust medium-term forecasting capabilities, highlighting its substantial improvement over simpler models like GARCH(1,1), despite minor limitations in handling highly volatile or irregular data.

2.3.3 HAR Model Performance in the long Term (200 days):

In this section, we analyze the performance of the HAR model over a 200-day period to assess its long-term predictive capabilities. The following subsections provide a detailed examination of the model's performance through visualizations and performance metrics.

Table 6-15 Long Term HAR Model Performance Error Metrics.

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	3.096144e-06	0.001760	0.001294
Amdynamic Sukuk - Class A	9.568686e-07	0.000978	0.000620
Franklin Global Sukuk Fund A(acc)usd	9.400885e-09	0.000097	0.000033
Franklin Global Sukuk Fund I(acc)usd	4.978864e-06	0.002231	0.001725
Franklin Global Sukuk Fund A(mdis)usd	5.812837e-06	0.002411	0.001809
Franklin Global Sukuk Fund N(acc)eur	2.978179e-05	0.005457	0.004232
Franklin Global Sukuk Fund A(acc)eur	3.031798e-05	0.005506	0.004245
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	3.267990e-05	0.005717	0.004321
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	3.275474e-05	0.005723	0.004320
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	2.872444e-05	0.005360	0.004157
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	2.873644e-05	0.005361	0.004158
Principal Islamic Lifetime Enhanced Sukuk Fund	1.780257e-06	0.001334	0.000692
Principal Islamic Lifetime Sukuk	2.047159e-06	0.001431	0.000666
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	4.547207e-06	0.002132	0.001148
Franklin Global Sukuk Fund X(qdis)usd	7.203385e-06	0.002684	0.001954
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	1.403195e-06	0.001185	0.000892
Pb Aiman Sukuk Fund	3.726188e-06	0.001930	0.000689
Franklin Global Sukuk Fund W(qdis)usd	7.179401e-06	0.002679	0.001952
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	4.674048e-06	0.002162	0.001460
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	4.710047e-06	0.002170	0.001464
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	2.668484e-06	0.001634	0.001202
Franklin Global Sukuk Fund W(qdis)eur-h1	7.208448e-06	0.002685	0.001892
Kaf Sukuk Fund	7.912776e-07	0.000890	0.000494
DJ. (ex-Reinvestment)	3.593141e-06	0.001896	0.001472
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	3.066056e-06	0.001751	0.001278
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	2.888187e-05	0.005374	0.004165
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	3.335884e-06	0.001826	0.001327
S&P GCC	4.266915e-06	0.002066	0.001601
S&P Global	3.012862e-06	0.001736	0.001265
S&P MENA	4.291949e-06	0.002072	0.001616
Sukuk & Bonds	4.248287e-06	0.002061	0.001084

As presented in **Table 6.15**, in the long term the HAR model continues to deliver strong predictive accuracy for many sukuk funds, though errors generally increase compared to short- and medium-term horizons. Remarkably accurate predictions persist for funds such as "Franklin Global Sukuk Fund A(acc) USD" (RMSE = 0.000097, MAE = 0.000033), indicating exceptional performance even over extended forecasting periods. Similarly, "KAF Sukuk Fund" and "Amdynamic Sukuk - Class A" maintain low errors (RMSE below 0.001), demonstrating the model's effectiveness in capturing stable long-term volatility structures.

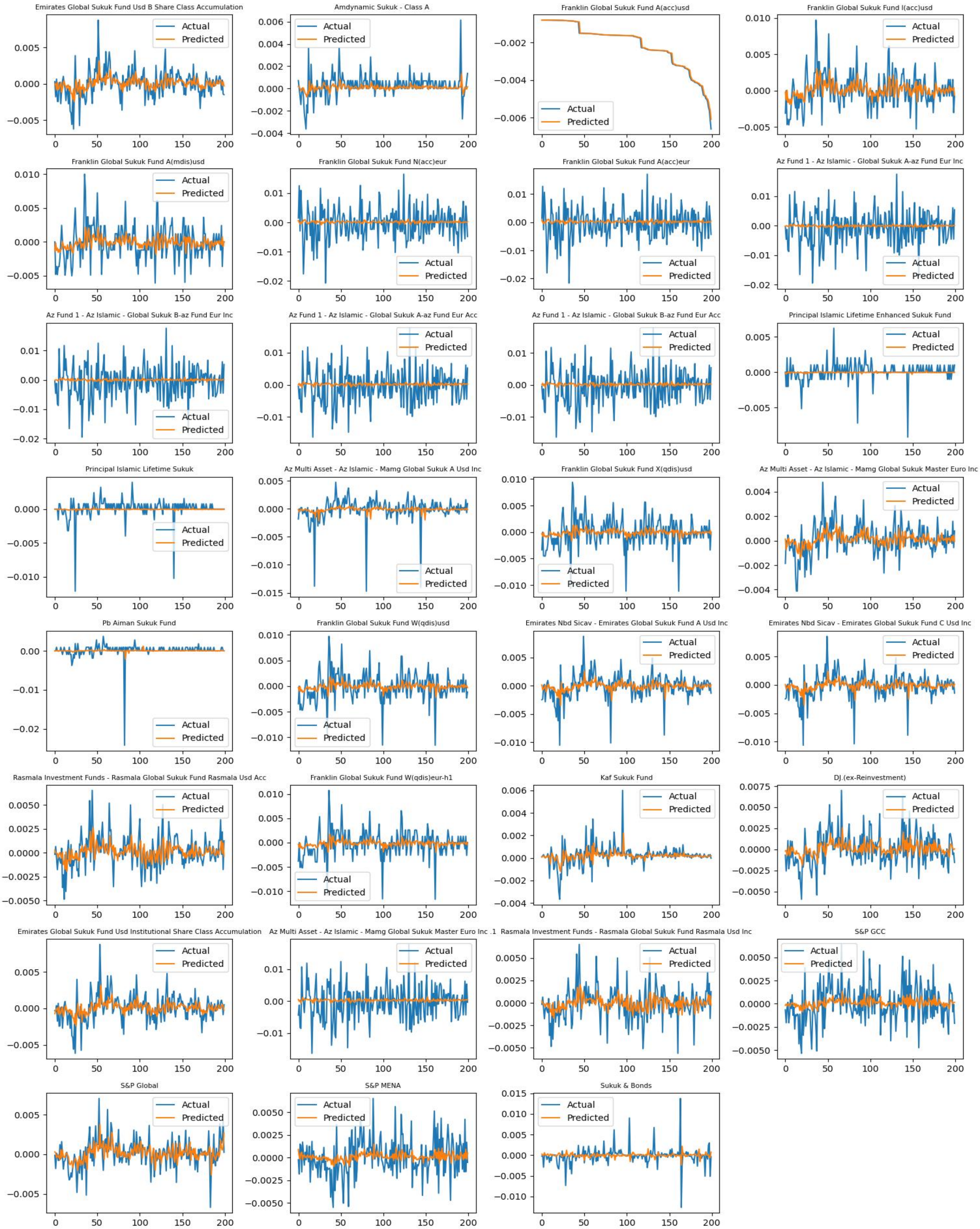
Moderate accuracy appears in funds such as "Principal Islamic Lifetime Enhanced Sukuk Fund" and "Pb Aiman Sukuk Fund", where RMSE values range between 0.0013 and 0.002. While slightly increased from shorter horizons, these remain acceptable and indicate consistent long-term predictive capabilities.

However, forecasting accuracy diminishes notably for more complex or volatile funds. Specifically, "Franklin Global Sukuk Fund N(acc) EUR, A(acc) EUR", and various classes of "Az Islamic Global Sukuk Funds" record relatively higher errors, with RMSE values surpassing 0.005. These higher errors reflect persistent challenges in accurately forecasting returns for funds characterized by more complex volatility patterns or irregular return behaviours.

At the index level, including "S&P GCC", "S&P Global", and "Sukuk & Bonds", the model achieves moderately strong performance (RMSE around 0.002), reinforcing its capability to reliably forecast long-term volatility for aggregated market indices.

Overall, the HAR model demonstrates substantial predictive effectiveness in long-term sukuk forecasting, significantly outperforming simpler models like GARCH(1,1). Nonetheless, its limitations in predicting highly complex or volatile series suggest opportunities for further improvement or the use of complementary forecasting methods.

Figure 6.12 . 200 Predicted Days with HAR Model Vs 200 Actual Values.



The visual plots in **Figure 6.12**, confirm the strong predictive performance of the HAR model over the long term for many sukuk funds. Notably, funds like "Franklin Global Sukuk Fund A(acc) USD", "KAF Sukuk Fund", "Principal Islamic Lifetime Sukuk", and "Emirates Global Sukuk Institutional" show excellent visual alignment, with predicted returns closely tracking actual volatility patterns across an extended horizon.

Moderate but consistent predictive accuracy is visible in funds such as "Pb Aiman Sukuk Fund", "Principal Islamic Lifetime Enhanced Sukuk Fund", and indices including "S&P GCC" and "S&P MENA", where predictions follow the general trend effectively but occasionally underestimate sharper volatility spikes.

Though, significant challenges remain evident in certain highly volatile series, particularly "Franklin Global Sukuk Fund N(acc) EUR, A(acc) EUR", and several "Az Islamic Global Sukuk Funds". In these cases, the predicted lines remain relatively flat or show limited variation, clearly unable to capture frequent volatility jumps and sharp directional changes present in the actual returns.

Overall, the visual evidence strongly supports the HAR model's effectiveness for long-term sukuk forecasting. Its flexibility allows it to track volatility accurately in most series, marking significant improvement over simpler models. However, the remaining difficulties with extremely volatile series highlight potential areas for model refinement or the integration of more advanced predictive approaches.

2.3.4 Comparative Summary of HAR Model Performance Across Forecast Horizons:

In general, the HAR model demonstrates high predictive accuracy in the short run for many Sukuk funds, indicated by low error metrics such as MSE, RMSE, , and MAE. In the short run, the model captures the return dynamics effectively, providing strong performance for most funds.

As the prediction horizon extends to the medium and long run, there is a noticeable increase in error metrics for several funds, indicating that the predictive accuracy of the model diminishes over longer periods. This trend is expected, as the complexity and variability of financial markets make long-term predictions more challenging.

Despite this, the HAR model still performs reasonably well in the medium run, maintaining relatively low error metrics for many funds. For example, funds like the Franklin Global Sukuk Fund A(acc)usd and Principal Islamic Lifetime Sukuk continue to exhibit low errors, showcasing the model's robustness for medium-term forecasts.

In the long run, while the HAR model shows higher errors compared to the short and medium runs, it remains a useful tool for predicting sukuk returns. The model's performance varies significantly across different funds, with some funds like Franklin Global Sukuk Fund A(acc)usd continuing to exhibit low error metrics, whereas others like Franklin Global Sukuk Fund N(acc)eur show higher errors.

2.4 PROPHET Model Forecasting Performance:

This section delves into the PROPHET model's performance over different time periods. We thoroughly analyze its volatility predictions across short-term, medium-term, and long-term horizons. The results are organized by these timeframes to offer an in-depth evaluation of the model's effectiveness.

2.4.1 PROPHET Model Performance in The Short Term (one week):

For the short-term evaluation, we assess the PROPHET model's performance over a one-week period. This section includes visual comparisons of the model's predictions against actual values, along with an analysis of the relevant performance metrics.

Table 6-16 Short Term PROPHET Model Performance Error Metrics.

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	1.628833e-06	0.001276	0.001130
Amdynamic Sukuk - Class A	1.912971e-06	0.001383	0.001045
Franklin Global Sukuk Fund A(acc)usd	1.754824e-06	0.001325	0.001192
Franklin Global Sukuk Fund I(acc)usd	2.270981e-06	0.001507	0.001272
Franklin Global Sukuk Fund A(mdis)usd	2.734952e-06	0.001654	0.001074
Franklin Global Sukuk Fund N(acc)eur	2.502805e-05	0.005003	0.004410
Franklin Global Sukuk Fund A(acc)eur	2.380158e-05	0.004879	0.004264
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	1.404845e-05	0.003748	0.003107
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	1.378647e-05	0.003713	0.003082
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	1.360015e-05	0.003688	0.003097
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	1.395457e-05	0.003736	0.003116
Principal Islamic Lifetime Enhanced Sukuk Fund	1.123051e-06	0.001060	0.000979
Principal Islamic Lifetime Sukuk	1.065022e-07	0.000326	0.000325
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	4.959998e-07	0.000704	0.000542
Franklin Global Sukuk Fund X(qdis)usd	2.655693e-06	0.001630	0.001211
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	4.397400e-07	0.000663	0.000546
Pb Aiman Sukuk Fund	9.633018e-07	0.000981	0.000902
Franklin Global Sukuk Fund W(qdis)usd	2.318454e-06	0.001523	0.001361
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	5.726982e-07	0.000757	0.000641
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	5.567347e-07	0.000746	0.000617
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	2.623460e-06	0.001620	0.001396
Franklin Global Sukuk Fund W(qdis)eur-h1	1.559070e-06	0.001249	0.000867
Kaf Sukuk Fund	5.696388e-07	0.000755	0.000742
DJ. (ex-Reinvestment)	2.522424e-06	0.001588	0.001406
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	1.438638e-06	0.001199	0.001100
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	1.319223e-05	0.003632	0.003029
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	2.907000e-06	0.001705	0.001454
S&P GCC	3.331622e-06	0.001825	0.001575
S&P Global	4.232696e-06	0.002057	0.001602
S&P MENA	3.733396e-06	0.001932	0.001718
Sukuk & Bonds	8.136854e-06	0.002853	0.002136

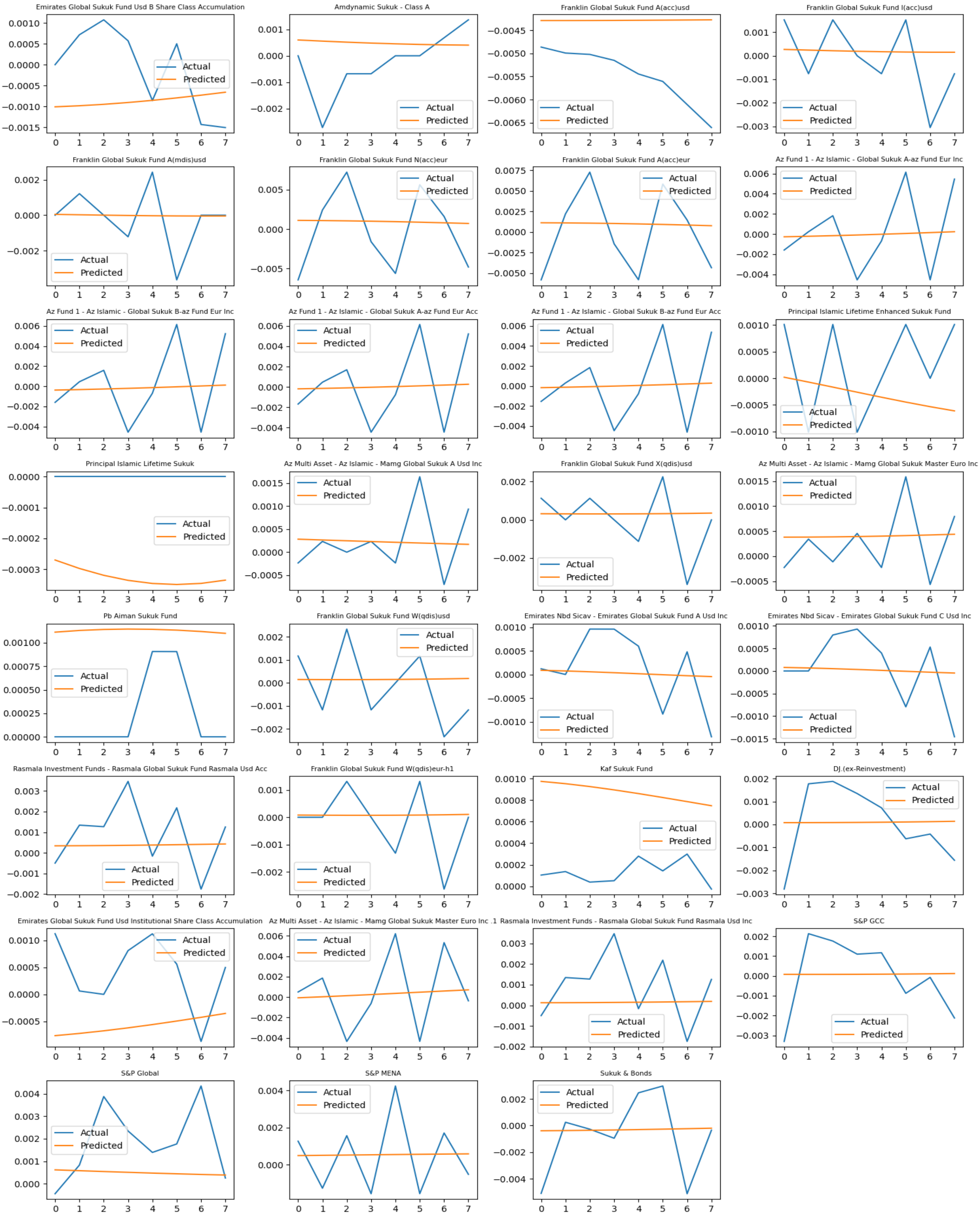
As illustrated in **Table 6.16**, The performance of the Prophet model in predicting the short-term returns of Sukuk funds reveals notable variations across different funds. several funds, including "Principal Islamic Lifetime Sukuk" (RMSE = 0.000326), "Az Multi Asset - MAMG Global Sukuk Master Euro Inc" (RMSE = 0.000663), and "KAF Sukuk Fund" (RMSE = 0.000755), achieve notably low prediction errors, highlighting PROPHET's suitability for capturing short-term volatility patterns in these stable series.

Moderate but acceptable accuracy is observed in funds like "Emirates Global Sukuk Institutional" (RMSE = 0.001199) and "Franklin Global Sukuk Fund" A(acc) USD (RMSE = 0.001325), where predictions remain reasonably accurate but slightly less precise compared to HAR results. These outcomes still reflect PROPHET's overall effectiveness in forecasting relatively stable series.

However, funds with higher volatility or complexity, such as "Franklin Global Sukuk Fund N(acc) EUR" (RMSE ≈ 0.0050) and "A(acc) EUR" (RMSE ≈ 0.0049), show substantially larger errors. This indicates that PROPHET struggles similarly to other models when facing highly volatile or irregular short-term patterns.

Overall, through the performance error metrics outputs, the PROPHET proves to be an effective short-term forecasting tool for most sukuk funds, though slightly less accurate compared to the HAR model. Its ease of application makes it valuable, yet its performance limitations in highly volatile series suggest potential areas for further model refinement or complementary modeling approaches.

Figure 6.13 One Week Predicted With PROPHET Model Vs One-Week Actual Values.



Contrary to the performance error metrics, the graphical plots in **Figure 6.13** show that the short-term PROPHET model demonstrates limited forecasting capability for most sukuk funds. In general, predicted lines remain nearly flat or show slight variation, failing to reflect actual volatility movements. Funds like "Emirates Global Sukuk Institutional", "Franklin Global Sukuk Fund A(acc) USD", and "KAF Sukuk Fund" fail to capture significant directional changes or volatility spikes in actual returns.

A particularly noticeable weakness is evident in funds with strong volatility, such as "Franklin Global Sukuk Fund N(acc) EUR, A(acc) EUR", and "Az Islamic Global Sukuk Funds", where the predicted series remains consistently flat, entirely failing to capture the real volatility dynamics.

Indices like "S&P GCC", "S&P MENA", and "Sukuk & Bonds" similarly show static predictions, disconnected from the actual short-term return fluctuations. This highlights PROPHET’s limitations in tracking short-term market movements accurately.

Overall, the visual evidence indicates the PROPHET model’s structural inadequacy for short-term sukuk forecasting, since it provides overly simplistic predictions, unable to effectively adapt to rapid changes or volatility clustering in sukuk returns.

2.4.2 PROPHET Model Performance in The Medium Term (one month):

The PROPHET model's medium-term performance is evaluated over a one-week period. This section includes analysis of the related performance error metrics, and comparison of the model's predictions with actual values graphically.

Table 6-17 Medium Term PROPHET Model Performance Error Metrics.

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	1.048806e-06	0.001024	0.000775
Amdynamic Sukuk - Class A	1.674286e-06	0.001294	0.000746
Franklin Global Sukuk Fund A(acc)usd	5.571924e-07	0.000746	0.000546
Franklin Global Sukuk Fund I(acc)usd	2.334671e-06	0.001528	0.001178
Franklin Global Sukuk Fund A(mdis)usd	2.487072e-06	0.001577	0.001169
Franklin Global Sukuk Fund N(acc)eur	1.833366e-05	0.004282	0.003577
Franklin Global Sukuk Fund A(acc)eur	1.816619e-05	0.004262	0.003551
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	1.405281e-05	0.003749	0.002911
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	1.353052e-05	0.003678	0.002866
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	1.363275e-05	0.003692	0.002908
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	1.378141e-05	0.003712	0.002930
Principal Islamic Lifetime Enhanced Sukuk Fund	8.172455e-07	0.000904	0.000790
Principal Islamic Lifetime Sukuk	2.892923e-07	0.000538	0.000452
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	6.134806e-07	0.000783	0.000622
Franklin Global Sukuk Fund X(qdis)usd	2.313979e-06	0.001521	0.001237
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	7.381828e-07	0.000859	0.000721
Pb Aiman Sukuk Fund	6.010297e-07	0.000775	0.000700
Franklin Global Sukuk Fund W(qdis)usd	2.056397e-06	0.001434	0.001204
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	1.254623e-06	0.001120	0.000864
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	1.287337e-06	0.001135	0.000875
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	1.274637e-06	0.001129	0.000923
Franklin Global Sukuk Fund W(qdis)eur-h1	1.968314e-06	0.001403	0.001089
Kaf Sukuk Fund	2.568547e-07	0.000507	0.000476
DJ. (ex-Reinvestment)	2.352795e-06	0.001534	0.001342
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	1.136532e-06	0.001066	0.000829
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	1.315675e-05	0.003627	0.002757
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	1.971005e-06	0.001404	0.001015
S&P GCC	2.881995e-06	0.001698	0.001415
S&P Global	3.775876e-06	0.001943	0.001237
S&P MENA	3.977378e-06	0.001994	0.001660
Sukuk & Bonds	2.845701e-06	0.001687	0.001036

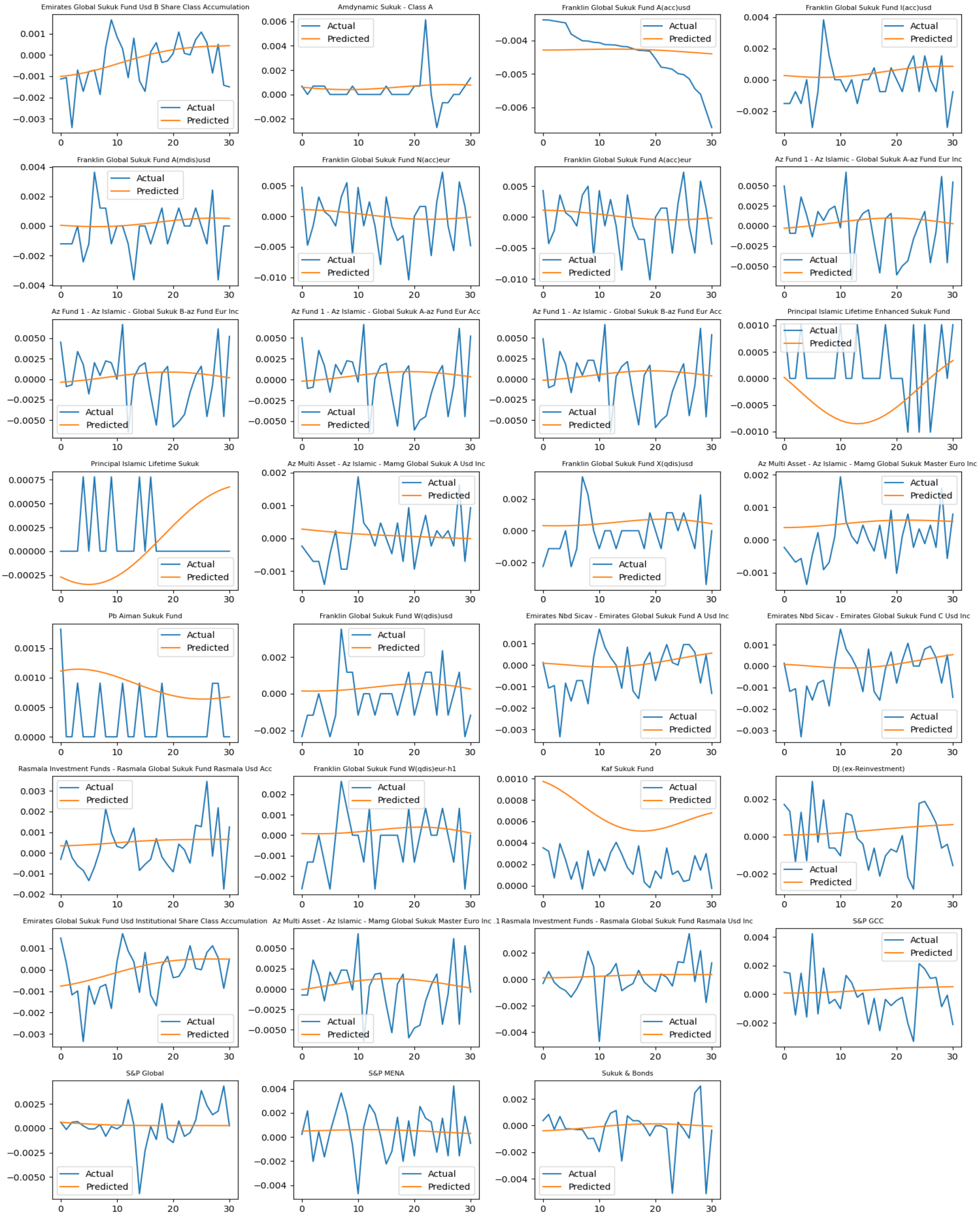
As illustrated in **Table 6.17**, the model demonstrates mixed outcomes across different funds and indices. Interestingly, certain funds exhibit notably low errors, pointing toward reliable forecasting in specific cases. For instance, "the Emirates Global Sukuk Fund B Share Class" and "KAF Sukuk Fund" achieve relatively precise predictions, with RMSE values at 0.001024 and 0.000507 respectively, suggesting the model may effectively handle stable or less complex volatility structures.

Yet, even in cases with moderate accuracy such as "Principal Islamic Lifetime Sukuk" (RMSE = 0.000538) and "Az Multi Asset - MAMG Global Sukuk Master Euro Inc" (RMSE = 0.000783). Additionally, higher error levels in funds like "Franklin Global Sukuk Fund N(acc) EUR" (RMSE ≈ 0.0043) and "A(acc) EUR" (RMSE ≈ 0.0042) highlight persistent limitations, clearly illustrating the difficulty PROPHET faces in capturing volatile or less predictable dynamics.

At the broader market level, PROPHET maintains reasonable performance with indices such as "S&P GCC" (RMSE = 0.001698), "S&P Global" (RMSE = 0.001943), and "Sukuk & Bonds" (RMSE = 0.001687).

Overall, while the Prophet model performs well for stable funds, its accuracy diminishes for more volatile ones, highlighting areas for potential improvement.

Figure 6.14 One Month Predicted With PROPHET Model Vs One-Month Actual Values.



As depicted in **Figure 6.14**, the graphical representation reveals a fundamental weakness in PROPHET's medium-term predictive capabilities for sukuk returns. Across nearly all series, PROPHET generates overly simplistic predictions characterized by flat or gently sloped lines, clearly disconnected from actual market dynamics. For instance, in series with significant volatility, such as "the Franklin Global Sukuk Fund A(acc) EUR, N(acc) EUR", and several "Az Islamic Global Sukuk Funds", predictions remain persistently flat, failing entirely to capture abrupt movements and volatility spikes.

Even in funds previously suggested by numerical metrics to have decent accuracy, like "Emirates Global Sukuk Institutional" and "KAF Sukuk Fund", the visual evidence contradicts that impression, showing predictions that appear rigid and unresponsive to real world fluctuations. The model occasionally captures broad trends but consistently misses essential short-term shifts, making it unsuitable for nuanced forecasting or decision-making that requires sensitivity to market changes.

In essence, the visual assessment clearly reveals PROPHET's significant limitations in medium-term sukuk market forecasting. Despite the potential appeal of its simplicity, PROPHET's rigid forecasting structure severely restricts its accuracy and effectiveness in capturing reliable market behaviour.

2.4.3 PRORHET Model Performance in the long Term (200 days):

In this section, we evaluate the HAR model's performance over a 200-day period to assess its long-term predictive capabilities. By concentrating on an extended timeframe, the following subsections offer a detailed analysis of the model's performance using visualizations and performance metrics.

Table 6-18 Long Term PROPHET Model Performance Error Metrics.

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	3.464182e-06	0.001861	0.001381
Amdynamic Sukuk - Class A	1.140843e-06	0.001068	0.000715
Franklin Global Sukuk Fund A(acc)usd	8.748238e-06	0.002958	0.002855
Franklin Global Sukuk Fund I(acc)usd	5.802174e-06	0.002409	0.001837
Franklin Global Sukuk Fund A(mdis)usd	6.517347e-06	0.002553	0.001936
Franklin Global Sukuk Fund N(acc)eur	2.958687e-05	0.005439	0.004219
Franklin Global Sukuk Fund A(acc)eur	3.011280e-05	0.005488	0.004234
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	3.247319e-05	0.005699	0.004286
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	3.245155e-05	0.005697	0.004279
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	2.838974e-05	0.005328	0.004102
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	2.842651e-05	0.005332	0.004105
Principal Islamic Lifetime Enhanced Sukuk Fund	1.825481e-06	0.001351	0.000791
Principal Islamic Lifetime Sukuk	2.127125e-06	0.001458	0.000753
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	4.688906e-06	0.002165	0.001147
Franklin Global Sukuk Fund X(qdis)usd	8.308016e-06	0.002882	0.002100
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	1.676898e-06	0.001295	0.000986
Pb Aiman Sukuk Fund	4.100580e-06	0.002025	0.000827
Franklin Global Sukuk Fund W(qdis)usd	8.335556e-06	0.002887	0.002101
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	4.906827e-06	0.002215	0.001474
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	4.967756e-06	0.002229	0.001488
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	2.747441e-06	0.001658	0.001233
Franklin Global Sukuk Fund W(qdis)eur-h1	8.313852e-06	0.002883	0.002039
Kaf Sukuk Fund	9.257308e-07	0.000962	0.000623
DJ. (ex-Reinvestment)	3.998541e-06	0.002000	0.001568
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	3.569647e-06	0.001889	0.001397
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	2.851962e-05	0.005340	0.004107
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	3.257851e-06	0.001805	0.001304
S&P GCC	4.386339e-06	0.002094	0.001613
S&P Global	3.137948e-06	0.001771	0.001305
S&P MENA	4.261315e-06	0.002064	0.001595
Sukuk & Bonds	4.381828e-06	0.002093	0.001067

From Table 6.18, the long-term forecasting metrics for the PROPHET model offer mixed insights into its effectiveness. Unlike shorter horizons, the model's limitations become more pronounced with increasing forecast length. Several funds, notably those from Franklin such as "A(acc) USD" and "X(qdis) USD", exhibit significantly higher errors, reaching RMSE values as high as 0.0029, clearly demonstrating the model's declining predictive reliability over extended periods.

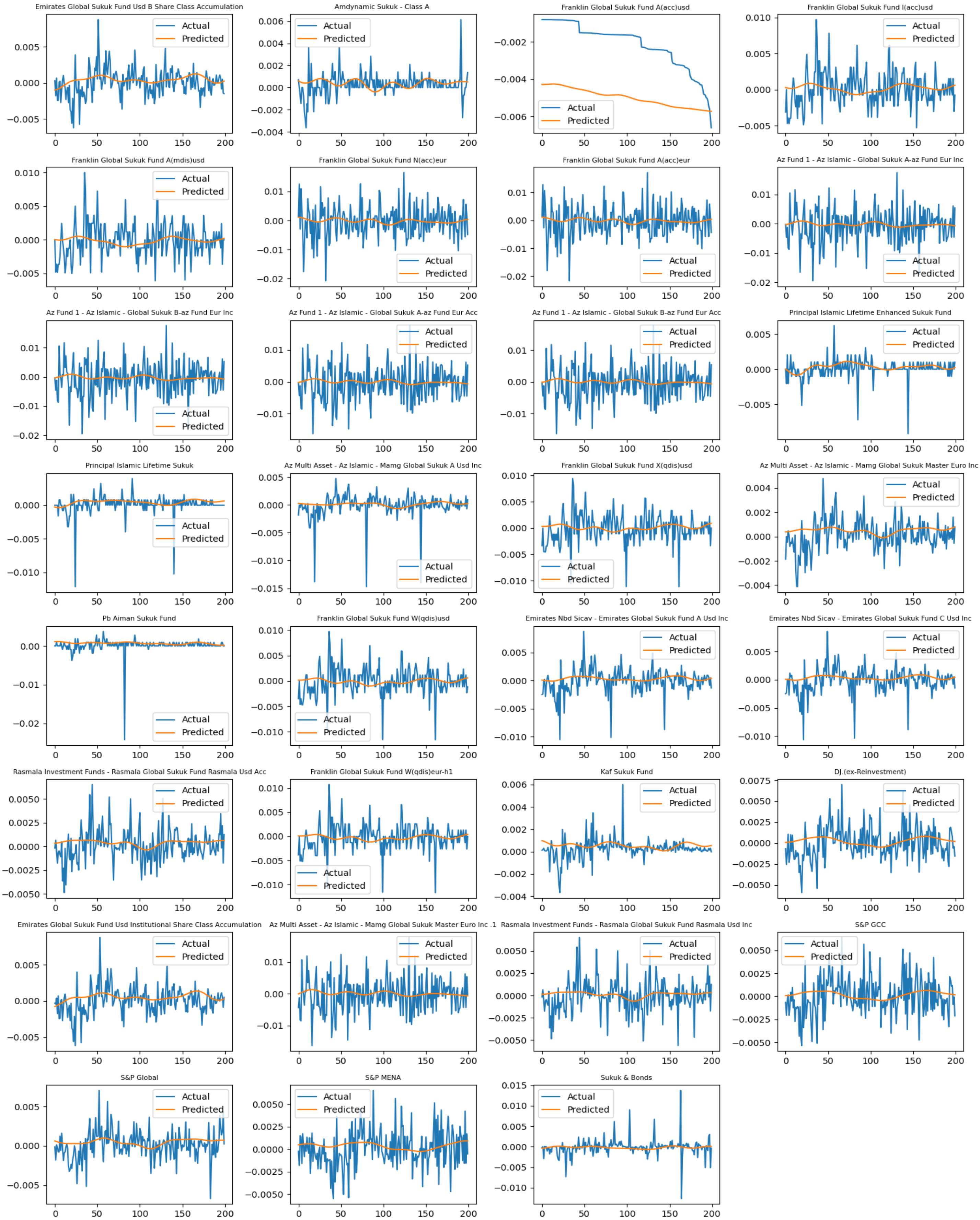
However, a few funds still manage to hold relatively modest errors, for instance the "Amdynamic Sukuk – Class A" and "KAF Sukuk Fund stand out positively", with RMSE at approximately 0.0011 and 0.00096, respectively, suggesting that PROPHET may still handle certain stable volatility patterns reasonably well, even in the long run.

Yet, performance for the more volatile and complex series, especially the "Franklin Global Sukuk Fund N(acc) EUR" and various "Az Islamic Global Sukuk Funds", depreciates noticeably, approaching RMSE values above 0.005. Such outcomes suggest that PROPHET's forecasting becomes significantly less reliable as fund complexity or volatility increases.

At the index level, long-term predictions for benchmarks such as "S&P GCC" (RMSE = 0.002094) and "S&P MENA" (RMSE = 0.002064) remain moderate, but clearly reveal limitations compared to shorter horizons, where predictions were tighter.

Ultimately, while PROPHET offers some practical simplicity and utility, its long-term forecasting accuracy sharply declines for volatile and complex sukuk returns, indicating that reliance on this model alone for long-term strategies might be problematic.

Figure 6.15 200 Predicted Days With PROPHET Model Vs 200 Actual Values.



As shown in **Figure 6.15**, The visual evidence clearly highlights significant weaknesses in the PROPHET model's long-term predictive ability for sukuk returns. Given that across almost all the series the PROPHET predictions appear as smooth, nearly flat lines that notably fail to track the volatility and directional shifts in actual returns. Even for series previously identified numerically as moderate performers, such as "KAF Sukuk Fund" and "Amdynamic Sukuk – Class A", the visual plots reveal predictions that do not really reflect the market's actual volatility dynamics.

Notable cases include the "Franklin Global Sukuk Fund A(acc) EUR", "N(acc) EUR", and the "Az Islamic Global Sukuk Funds", where the predicted returns are essentially static and fail entirely to match the sharp spikes and sudden directional changes present in the actual data. Such rigid forecasts clearly illustrate the model’s inability to adapt to real-world complexity and volatility in long-term horizons.

Index predictions for 'S&P GCC,' 'S&P MENA,' and 'Sukuk & Bonds' also reflect PROPHET’s typical flat output, capturing only a basic long-term trend with minimal added accuracy.

In summary, visual evaluation strongly underscores PROPHET’s limitations for long-term sukuk market forecasting.

2.4.4 Comparative summary of Prophet Model Performance Across Forecast Horizons:

As observed across the forecast results, the Prophet model's forecasting performance consistently declines as the prediction horizon extends. In the short term, numerical error metrics initially suggest acceptable accuracy for several funds; however, visual analysis reveals that predictions remain largely flat and fail to reflect actual volatility. Even funds that appear numerically accurate, such as "KAF Sukuk Fund" and "Emirates Global Sukuk Institutional", visually show poor responsiveness to short-term market movements.

As the horizon shifts to the medium term, Prophet's shortcomings become even clearer. Predictions continue to exhibit a rigid, simplified trend across almost all sukuk series, missing critical fluctuations and volatility shifts entirely. Funds with complex or volatile patterns, particularly the "Franklin and Az Islamic series", clearly illustrate Prophet's limitations as errors increase and the model struggles more significantly to capture actual dynamics.

In the long-term horizon, Prophet's predictive weaknesses intensify further. The flat, trend-like forecasts observed across all series become nearly meaningless, completely disconnected from the reality of volatile markets. Even previously moderate-performing funds show visible discrepancies between predicted and actual returns. Index-level forecasts, although appearing stable, lack the necessary sensitivity to reflect real market behaviour effectively.

Overall, the PROTHER model is basically limited by its rigidity and simplified trend assumptions, making it unsuitable for accurately capturing sukuk volatility, especially at medium and long-term horizons. Its declining performance across time highlights the necessity of more responsive, adaptive forecasting models to improve prediction quality in financial contexts.

2.5 ANN Model forecasting performance:

The second deep learning model we have selected is the artificial neural network (ANN). In this section, we evaluate the ANN model's performance across various time horizons. By examining its ability to predict volatility in the short-term, medium-term, and long-term, we offer a detailed assessment.

2.5.1 ANN Model Performance in The Short Term (one week):

For the short-term analysis, we evaluate the ANN model's performance over a one-week period. This section includes a comparison between the model's predictions and the actual values, along with a detailed analysis of the performance metrics.

Table 6-19 Short Term ANN Model Performance Error Metrics.

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	8.104224e-07	0.000900	0.000786
Amdynamic Sukuk - Class A	1.630744e-06	0.001277	0.001058
Franklin Global Sukuk Fund A(acc)usd	8.736836e-07	0.000935	0.000861
Franklin Global Sukuk Fund I(acc)usd	3.112961e-06	0.001764	0.001502
Franklin Global Sukuk Fund A(mdis)usd	4.188815e-06	0.002047	0.001497
Franklin Global Sukuk Fund N(acc)eur	2.651223e-05	0.005149	0.004730
Franklin Global Sukuk Fund A(acc)eur	2.568276e-05	0.005068	0.004582
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	1.624852e-05	0.004031	0.003198
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	1.547193e-05	0.003933	0.003145
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	1.499755e-05	0.003873	0.003131
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	1.537803e-05	0.003921	0.003084
Principal Islamic Lifetime Enhanced Sukuk Fund	2.087695e-06	0.001445	0.001297
Principal Islamic Lifetime Sukuk	1.771489e-06	0.001331	0.001331
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	1.042262e-06	0.001021	0.000771
Franklin Global Sukuk Fund X(qdis)usd	4.317010e-06	0.002078	0.001592
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	8.423765e-07	0.000918	0.000771
Pb Aiman Sukuk Fund	5.216282e-07	0.000722	0.000619
Franklin Global Sukuk Fund W(qdis)usd	2.859515e-06	0.001691	0.001425
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	6.981154e-07	0.000836	0.000695
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	8.003073e-07	0.000895	0.000776
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	3.905969e-06	0.001976	0.001802
Franklin Global Sukuk Fund W(qdis)eur-h1	2.138815e-06	0.001462	0.001150
Kaf Sukuk Fund	1.835369e-08	0.000135	0.000115
DJ. (ex-Reinvestment)	2.265501e-06	0.001505	0.001185
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	6.243834e-07	0.000790	0.000679
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	1.383909e-05	0.003720	0.002936
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	3.969051e-06	0.001992	0.001794
S&P GCC	3.408686e-06	0.001846	0.001525
S&P Global	4.483948e-06	0.002118	0.001695
S&P MENA	6.413852e-06	0.002533	0.002214
Sukuk & Bonds	9.873710e-06	0.003142	0.002622

As depicted in **Table 6.19**, The short-term forecasting metrics for the ANN model indicate promising accuracy across several sukuk funds, highlighting the model's potential strength. Notably, funds like "KAF Sukuk Fund" (RMSE = 0.000135), "Emirates Global Sukuk Institutional" (RMSE = 0.000790), and "Emirates Global Sukuk USD B Share Class" (RMSE = 0.000900) achieve exceptionally low prediction errors, suggesting ANN's effectiveness in capturing nuanced volatility patterns in stable market conditions.

However, this impressive accuracy isn't universal. Several funds, especially the "Franklin Global Sukuk Funds (N(acc) EUR" and "A(acc) EUR) " demonstrate significantly higher errors (RMSE ≈ 0.005), indicating persistent challenges for ANN in dealing with complex or highly volatile return series. Similar struggles are evident in certain "Az Islamic Global Sukuk Funds", with errors typically around RMSE ≈ 0.004.

In terms of indices, the ANN model offers mixed results, for instance the "S&P GCC" index shows reasonable short-term accuracy (RMSE = 0.001846), whereas other indices, particularly "Sukuk & Bonds" (RMSE = 0.003142), reveal weaker predictions, suggesting that aggregated indices may require more adaptive ANN configurations to fully capture short-term market dynamics.

In summary, the ANN model demonstrates considerable promise in short-term forecasting, especially for stable and predictable sukuk funds, but its accuracy quickly diminishes as fund volatility increases.

Figure 6.16 One Week Predicted With ANN Model Vs One-Week Actual Values.

As shown in **Figure 6.16**, The ANN model demonstrates good performance in the short term for predicting Sukuk fund returns, and it reinforces much of what the metrics suggest.

Several plots show close alignment between actual and predicted returns. Funds like "KAF Sukuk Fund", "Az Multi Asset - MAMG Global Sukuk Master Euro Inc", and "Emirates Global Sukuk Institutional" demonstrate reasonable tracking between the predicted and actual values, even if the model doesn't always capture the extremes. These cases indicate that ANN handles smooth, stable patterns well in short-term settings.

Nevertheless, for a large number of funds, especially "Franklin Global Sukuk Fund N(acc) EUR", "Franklin A(acc) USD", and "Rasmala Global Sukuk Funds", the ANN model visibly struggles, since the Predicted lines often follow a general trend but miss sharp turns and underestimate spikes, or lag behind actual changes. In some cases, the predictions flatten out or swing mildly while the real values fluctuate sharply.

Indices such as " S&P Global" shows perfect fit, while indices such as "S&P GCC", "S&P MENA", and "Sukuk & Bonds" show partial fit predictions, generally move in the same direction as the actual series but with noticeable gaps in volatility and timing.

In summary, the plots validate that ANN models can offer strong short-term predictions for low-volatility funds. But for more erratic or volatile returns, ANN struggles to respond with sufficient precision, highlighting the need for further optimization, deeper architectures, or ensemble strategies to enhance responsiveness and accuracy.

2.5.2 ANN Model Performance in The Medium Term (one month):

For the medium-term analysis, we extended the evaluation period to one month. This section examines the model's predictive capabilities over this timeframe through an analysis of the performance metrics, followed by comparison between predicted and actual values.

Table 6-20 Medium Term ANN Model Performance Error Metrics.

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	1.112141e-06	0.001055	0.000869
Amdynamic Sukuk - Class A	2.008067e-06	0.001417	0.000836
Franklin Global Sukuk Fund A(acc)usd	1.747232e-06	0.001322	0.001287
Franklin Global Sukuk Fund I(acc)usd	2.115387e-06	0.001454	0.001072
Franklin Global Sukuk Fund A(mdis)usd	2.829500e-06	0.001682	0.001228
Franklin Global Sukuk Fund N(acc)eur	2.097339e-05	0.004580	0.003857
Franklin Global Sukuk Fund A(acc)eur	2.060513e-05	0.004539	0.003827
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	1.227375e-05	0.003503	0.002823
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	1.195783e-05	0.003458	0.002818
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	1.177218e-05	0.003431	0.002787
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	1.226774e-05	0.003503	0.002822
Principal Islamic Lifetime Enhanced Sukuk Fund	8.197856e-07	0.000905	0.000715
Principal Islamic Lifetime Sukuk	4.609870e-07	0.000679	0.000564
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	6.861828e-07	0.000828	0.000645
Franklin Global Sukuk Fund X(qdis)usd	2.528056e-06	0.001590	0.001135
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	6.206407e-07	0.000788	0.000633
Pb Aiman Sukuk Fund	4.622798e-07	0.000680	0.000552
Franklin Global Sukuk Fund W(qdis)usd	1.844816e-06	0.001358	0.001028
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	1.170706e-06	0.001082	0.000921
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	1.098775e-06	0.001048	0.000882
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	1.436909e-06	0.001199	0.000890
Franklin Global Sukuk Fund W(qdis)eur-h1	1.912971e-06	0.001383	0.001101
Kaf Sukuk Fund	4.594115e-08	0.000214	0.000169
DJ. (ex-Reinvestment)	2.237998e-06	0.001496	0.001182
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	1.065791e-06	0.001032	0.000844
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	1.216807e-05	0.003488	0.002778
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	2.406249e-06	0.001551	0.001122
S&P GCC	3.278058e-06	0.001811	0.001423
S&P Global	4.277614e-06	0.002068	0.001482
S&P MENA	5.273048e-06	0.002296	0.002031
Sukuk & Bonds	3.483378e-06	0.001866	0.001239

As illustrated in **Table 6.20**, The Artificial Neural Network (ANN) model maintains strong predictive performance in the medium term, particularly for sukuk funds with stable return patterns. Several funds achieve low RMSE and MAE values, indicating that the model generalizes well across an extended horizon without significant loss of accuracy. This suggests the ANN model can effectively adapt to medium-term dynamics when the underlying data structure remains relatively consistent.

Top-performing funds in this category include the "KAF Sukuk Fund", which records exceptionally low prediction errors (RMSE = 0.000214, MAE = 0.000169), confirming high consistency over time. "The Principal Islamic Lifetime Sukuk" also shows strong results (RMSE = 0.000679, MAE = 0.000564), capturing both direction and magnitude with high accuracy. Other funds like "Pb Aiman Sukuk Fund" and "Az Multi Asset - MAMG Global Sukuk Master Euro Inc" report RMSE values below 0.001 and low MAEs, demonstrating solid model performance over the medium-term window.

A group of funds exhibits moderate performance, with acceptable but higher error rates. "Amdynamic Sukuk Class A", "Emirates Global Sukuk Institutional", and "Principal Islamic Lifetime Enhanced" with RMSE between 0.001 and 0.0014. These values remain reasonable for medium-term forecasting. Most Az Islamic Sukuk funds fall within this range as well, with RMSEs around 0.0034–0.0035 and MAEs below 0.0029, reflecting stable but less precise outcomes.

Weaker performance appears in funds with higher volatility or more complex return structures. "Franklin Global Sukuk Fund N(acc) EUR" and "A(acc) EUR" both record RMSE above 0.0045 and MAE exceeding 0.0038, indicating the ANN model struggles with high-frequency fluctuations and irregular trends. Similar degradation is observed in other Franklin variants such as "A(mdis) " and "X(qdis) ", where prediction errors increase significantly.

Index-level forecasts remain robust. S&P GCC, S&P Global, and Sukuk & Bonds all report RMSEs between 0.0018 and 0.0020, with MAEs under 0.0015. S&P MENA shows slightly higher error (RMSE = 0.002296) but still falls within acceptable limits for medium-term forecasting.

Overall, the ANN model demonstrates reliable predictive accuracy across most sukuk funds and indices in the medium term. Its strengths lie in handling smoother volatility patterns, while its limitations become more visible in cases of irregular or unstable returns. These findings support the use of ANN models for medium-term sukuk forecasting, particularly when return behavior is more consistent and volatility is moderate.

Figure 6.17 One Month Predicted With ANN Model Vs One-Month Actual Values.

As shown in **Figure 6.17**, The visual inspection of the medium-term ANN model predictions confirms the numerical findings and reveals deeper insights into the model's behaviour across different fund types.

In several cases, the model performs well, with strong alignment between predicted and actual values, for instance "the Emirates Global Sukuk Institutional", "KAF Sukuk Fund", and "Az Multi Asset - MAMG Global Sukuk Master Euro Inc" exhibit close correspondence, where the predicted lines track market fluctuations with minimal lag. This consistency supports the earlier observation that ANN generalizes well when the underlying data patterns remain stable.

In other series, the model shows partial fit with noticeable gaps. Funds such as "Az Islamic Sukuk Funds", and "Rasmala Global Sukuk Funds" reflect this behavior. While the ANN captures overall direction and key turning points, it underestimates volatility magnitude and sometimes lags in response. These results suggest that the model identifies the structural trend but struggles with amplitude and timing when returns shift more frequently.

Prediction failures are most visible in highly volatile series. "Franklin Global Sukuk Fund A(acc) USD" and "N(acc) EUR" are prime examples, where the ANN predictions either diverge sharply from actual values or collapse into nearly flat lines while real returns exhibit significant changes. Similar issues are seen in "Pb Aiman Sukuk Fund" and "Principal Islamic Lifetime Sukuk", where the predicted output lacks responsiveness or fails to reflect market direction. These cases show that the ANN model is limited in capturing erratic or structurally complex behaviors without further adjustments.

At the index level, forecasts for S&P Global, S&P MENA, and Sukuk & Bonds maintain a reasonable level of tracking. The predicted lines follow the actual return movements closely enough to be useful for identifying overall trends. While not perfect, this level of performance supports the model's applicability to broader market indices, where noise is typically lower than in individual fund returns.

Overall, the ANN model demonstrates strong medium-term forecasting capability when applied to funds with moderate or low volatility. It accurately captures trend direction and general movements but faces challenges with sudden shifts and noisy data.

2.5.3 ANN Model Performance in the long Term (200 days):

In this section, we evaluate the ANN model's performance over a 200-day period to assess its long-term predictive capabilities. By focusing on this extended timeframe, the following subsections provide a detailed analysis of the model's performance, incorporating visualizations and performance metrics.

Table 6-21 Long Term ANN Model Performance Error Metrics.

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	3.137392e-06	0.001771	0.001339
Amdynamic Sukuk - Class A	1.176620e-06	0.001085	0.000752
Franklin Global Sukuk Fund A(acc)usd	6.790477e-07	0.000824	0.000725
Franklin Global Sukuk Fund I(acc)usd	5.081474e-06	0.002254	0.001740
Franklin Global Sukuk Fund A(mdis)usd	5.802066e-06	0.002409	0.001826
Franklin Global Sukuk Fund N(acc)eur	3.090576e-05	0.005559	0.004321
Franklin Global Sukuk Fund A(acc)eur	3.295868e-05	0.005741	0.004455
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	3.317709e-05	0.005760	0.004375
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	3.522933e-05	0.005935	0.004525
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	3.049436e-05	0.005522	0.004277
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	3.065282e-05	0.005536	0.004293
Principal Islamic Lifetime Enhanced Sukuk Fund	2.595722e-06	0.001611	0.001113
Principal Islamic Lifetime Sukuk	3.027144e-06	0.001740	0.001008
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	5.285677e-06	0.002299	0.001237
Franklin Global Sukuk Fund X(qdis)usd	7.814437e-06	0.002795	0.002080
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	1.295123e-06	0.001138	0.000887
Pb Aiman Sukuk Fund	5.587974e-06	0.002364	0.001197
Franklin Global Sukuk Fund W(qdis)usd	7.378904e-06	0.002716	0.001998
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	5.072983e-06	0.002252	0.001521
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	5.118320e-06	0.002262	0.001556
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	2.617299e-06	0.001618	0.001201
Franklin Global Sukuk Fund W(qdis)eur-h1	7.582982e-06	0.002754	0.001988
Kaf Sukuk Fund	9.609168e-07	0.000980	0.000588
DJ. (ex-Reinvestment)	3.733331e-06	0.001932	0.001474
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	3.029047e-06	0.001740	0.001293
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	3.085336e-05	0.005555	0.004301
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	3.467337e-06	0.001862	0.001349
S&P GCC	4.684345e-06	0.002164	0.001668
S&P Global	3.660380e-06	0.001913	0.001491
S&P MENA	4.926005e-06	0.002219	0.001719
Sukuk & Bonds	4.983364e-06	0.002232	0.001178

From **Table 6.21**, the long-term performance of the ANN model shows a relatively stable and consistent behaviour across most sukuk funds, with clear variation depending on volatility and structural characteristics. Several funds demonstrate low RMSE and MAE values, suggesting strong long-horizon forecasting accuracy, particularly when the data exhibits less noise or smoother volatility cycles. For example, "Franklin Global Sukuk Fund A(acc) USD" achieves one of the lowest RMSE values at 0.000824, with a MAE of 0.000725, indicating that the ANN model effectively captures long-term trends in this fund with minimal deviation.

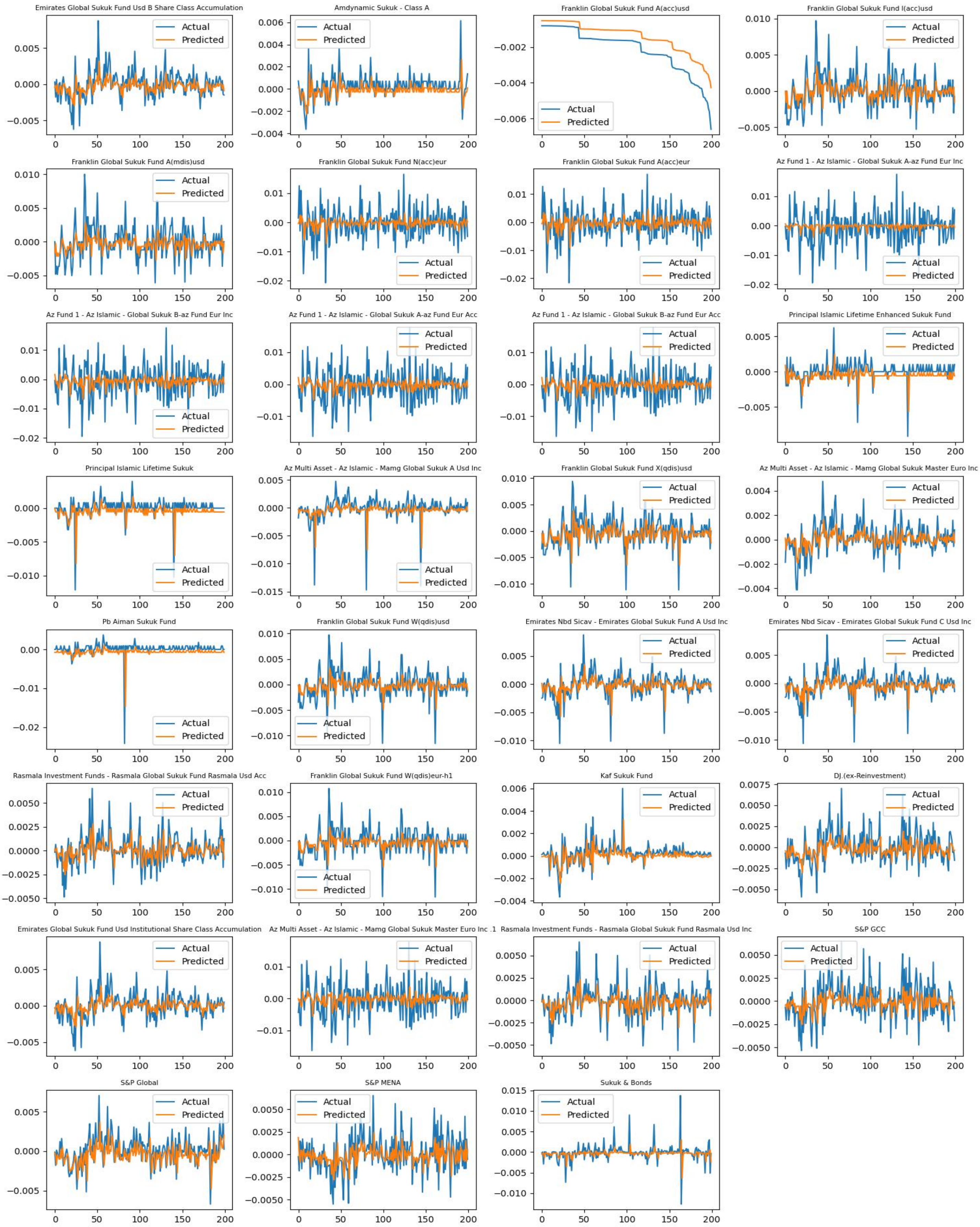
Similarly, the "KAF Sukuk Fund" stands out with RMSE at 0.000980 and MAE at just 0.000588. This confirms the model's robustness in low volatility environments. Other funds such as "Amdynamic Sukuk – Class A" and "Az Multi Asset – MAMG Global Sukuk Master Euro Inc" also perform well over the long term, maintaining RMSE values near or below 0.0011, reflecting the ANN model's ability to generalize patterns over time without severe degradation.

In contrast, several funds with higher volatility, especially in the Franklin and Az Islamic Global Sukuk series show significantly larger errors. "Franklin Global Sukuk Fund N(acc) EUR" and "A(acc) EUR" record RMSE values above 0.0055 and MAEs exceeding 0.0043, indicating that the model struggles with more complex or irregular return patterns. The same trend is visible in most of Az Fund class, where RMSE values range from 0.0055 to 0.0059. This suggests that ANN, while capable in smoother scenarios, becomes less reliable when long-term volatility is persistent or structural shifts occur.

Index-level forecasts are reasonably consistent, with "S&P GCC", "S&P Global", and "Sukuk & Bonds" all showing RMSE values between 0.0021 and 0.0022. The ANN model handles aggregated series moderately well, though its accuracy is lower than for individual low-volatility funds. These results imply the model captures general direction and longer-term trends but may underperform in the presence of frequent or sudden shifts.

Overall, the ANN model provides solid long-term forecasting performance, especially for structured, less volatile sukuk funds. As it captures persistent patterns and smooth transitions effectively. However, the model's sensitivity to irregular behaviour limits its predictive strength in more volatile funds and complex series. This reinforces the need for either model tuning or hybrid techniques when applied to long-term sukuk volatility forecasting.

Figure 6.18 200 predicted days with ANN model vs 200 actual values.



From **Figure 6.18**, the plots illustrate the long-term forecasting performance of the ANN model, and they confirm several patterns noted in the error metrics, while also exposing nuances that raw numbers alone do not capture.

In many cases, the ANN model tracks the actual return series quite well. Funds like "Emirates Global Sukuk Fund Institutional", "Az Multi Asset - MAMG Global Sukuk Master Euro Inc", and "KAF Sukuk Fund" show predictions that closely follow the direction and amplitude of actual returns. As well the predicted lines don’t simply move with the trend, they visibly reflect the day-to-day noise, showing that the model can retain some sensitivity over long-term horizons in stable series.

For a wider group of funds, such as "Az Islamic Sukuk Funds" and "S&P" indices, the model maintains good alignment in trend direction and fluctuation rhythm, but the predicted volatility is slightly diminished. As the predicted series follows the actual one reasonably well but often smooths out extreme values, leading to underestimation of volatility spikes.

The model visibly struggles with a few complex series, particularly "Franklin Global Sukuk Fund A(acc) USD", "Franklin N(acc) EUR", and "Pb Aiman Sukuk Fund". These predictions either flatten out or deviate progressively from actual values, especially in the presence of structural changes or persistent directional movements.

In summary, the plots support the conclusion that ANN is a viable long-term forecasting tool for many sukuk funds, particularly those with stable volatility and trend behaviour, but its performance is irregular across high-volatility and structurally shifting funds.

2.5.4 Comparative Summary of ANN Model Performance Across Forecast Horizons:

In total, The ANN model demonstrates a clear performance gradient across forecast horizons, with its accuracy generally decreasing as the prediction window extends. In the short-term run, the model achieves its best performance. Most funds record low RMSE and MAE values, and the predicted lines closely follow actual return movements. The model responds well to daily fluctuations, especially in smoother funds like "KAF Sukuk Fund", "Az Multi Asset – MAMG Global Sukuk", and "Emirates Global Sukuk Institutional", where both numerical metrics and plots show tight fit and strong directional accuracy.

Moving to the medium term, ANN performance remains relatively stable for many funds. Errors increase slightly, but predictions still track general patterns well. Visual inspection confirms that the model maintains its ability to follow market rhythm, although with some smoothing of peaks and occasional lag in volatility shifts. This is particularly evident in funds like "Principal Islamic Lifetime Sukuk" and "Pb Aiman Sukuk Fund". However, in volatile or complex series, such as the "Franklin Global Sukuk Fund N(acc) EUR", the model starts to lose sharpness. The accuracy of turning points weakens, and prediction intervals flatten slightly, though not enough to entirely invalidate the forecasts.

In the long-term run, the ANN model shows the most visible decline in forecasting accuracy. While it still performs well for funds with persistent trends and controlled variance such as "KAF Sukuk Fund" and "Az Multi Asset – MAMG Global Sukuk Master Euro Inc". The predictions become increasingly disconnected from actual returns in funds with structural breaks or high volatility, this is evident in "Franklin" and "Az Islamic" series, where predicted lines fail to adapt to downward trends or sharp fluctuations. In some cases, long-term forecasts appear smoothed or delayed, indicating the model's reduced sensitivity to long-range non-linear dynamics.

Overall, the ANN model demonstrates strong predictive performance in the short run, providing relatively low error metrics for several Sukuk funds. This performance is moderately sustained in the medium term, with slight increases in prediction errors reflecting the model's adaptability to monthly trends. In the long term, while the model's errors increase, it still provides reasonable predictive capabilities for several funds, though some exhibit higher volatility and prediction deviations. These metrics collectively underscore the ANN model's effectiveness in forecasting Sukuk fund returns, with varying accuracy across different timeframes.

2.6 LSTM Model forecasting performance:

This section provides an in-depth assessment of the LSTM model's performance over different time horizons. We evaluate its effectiveness in predicting volatility for short-term, medium-term, and long-term periods. The analysis is structured to offer a comprehensive evaluation of the model's performance across these varying time frames.

2.6.1 LSTM Model Performance in The Short Term (one week):

In the short term, we evaluate the LSTM model's performance over a one-week period quantitively and visually.

Table 6-22 Short Term LSTM Model Performance Error Metrics.

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	8.037694e-07	0.000897	0.000803
Amdynamic Sukuk - Class A	1.900108e-06	0.001378	0.001013
Franklin Global Sukuk Fund A(acc)usd	2.590121e-06	0.001609	0.001580
Franklin Global Sukuk Fund I(acc)usd	2.833225e-06	0.001683	0.001374
Franklin Global Sukuk Fund A(mdis)usd	3.775859e-06	0.001943	0.001402
Franklin Global Sukuk Fund N(acc)eur	2.565136e-05	0.005065	0.004640
Franklin Global Sukuk Fund A(acc)eur	2.439407e-05	0.004939	0.004471
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	1.558965e-05	0.003948	0.003176
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	1.519775e-05	0.003898	0.003151
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	1.516972e-05	0.003895	0.003167
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	1.567153e-05	0.003959	0.003193
Principal Islamic Lifetime Enhanced Sukuk Fund	1.623640e-06	0.001274	0.001173
Principal Islamic Lifetime Sukuk	1.706331e-10	0.000013	0.000013
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	8.214910e-07	0.000906	0.000691
Franklin Global Sukuk Fund X(qdis)usd	3.703978e-06	0.001925	0.001482
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	7.294607e-07	0.000854	0.000729
Pb Aiman Sukuk Fund	1.723949e-07	0.000415	0.000269
Franklin Global Sukuk Fund W(qdis)usd	2.725433e-06	0.001651	0.001434
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	7.066099e-07	0.000841	0.000730
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	6.791375e-07	0.000824	0.000700
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	3.620817e-06	0.001903	0.001734
Franklin Global Sukuk Fund W(qdis)eur-h1	1.955700e-06	0.001398	0.001083
Kaf Sukuk Fund	1.863891e-08	0.000137	0.000104
DJ. (ex-Reinvestment)	2.106821e-06	0.001451	0.001163
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	5.983561e-07	0.000774	0.000668
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	1.477672e-05	0.003844	0.003094
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	3.678047e-06	0.001918	0.001736
S&P GCC	3.155409e-06	0.001776	0.001486
S&P Global	4.089096e-06	0.002022	0.001561
S&P MENA	5.906901e-06	0.002430	0.002145
Sukuk & Bonds	9.401758e-06	0.003066	0.002442

From **Table 6.22**, the short-term performance of the LSTM model shows that it delivers strong predictive accuracy across many sukuk funds, particularly in environments where volatility is moderate, and structure is consistent.

Several funds exhibit excellent performance with low RMSE and MAE values, for instance "KAF Sukuk Fund" again stands out, achieving an RMSE of just 0.000137 and MAE of 0.000104 among the lowest in the table. Other top performers include "Principal Islamic Lifetime Sukuk" (RMSE = 0.000013, MAE = 0.000013) and "Pb Aiman Sukuk Fund" (RMSE = 0.000415), indicating that LSTM is highly effective in handling short-term fluctuations for funds with consistent volatility profiles.

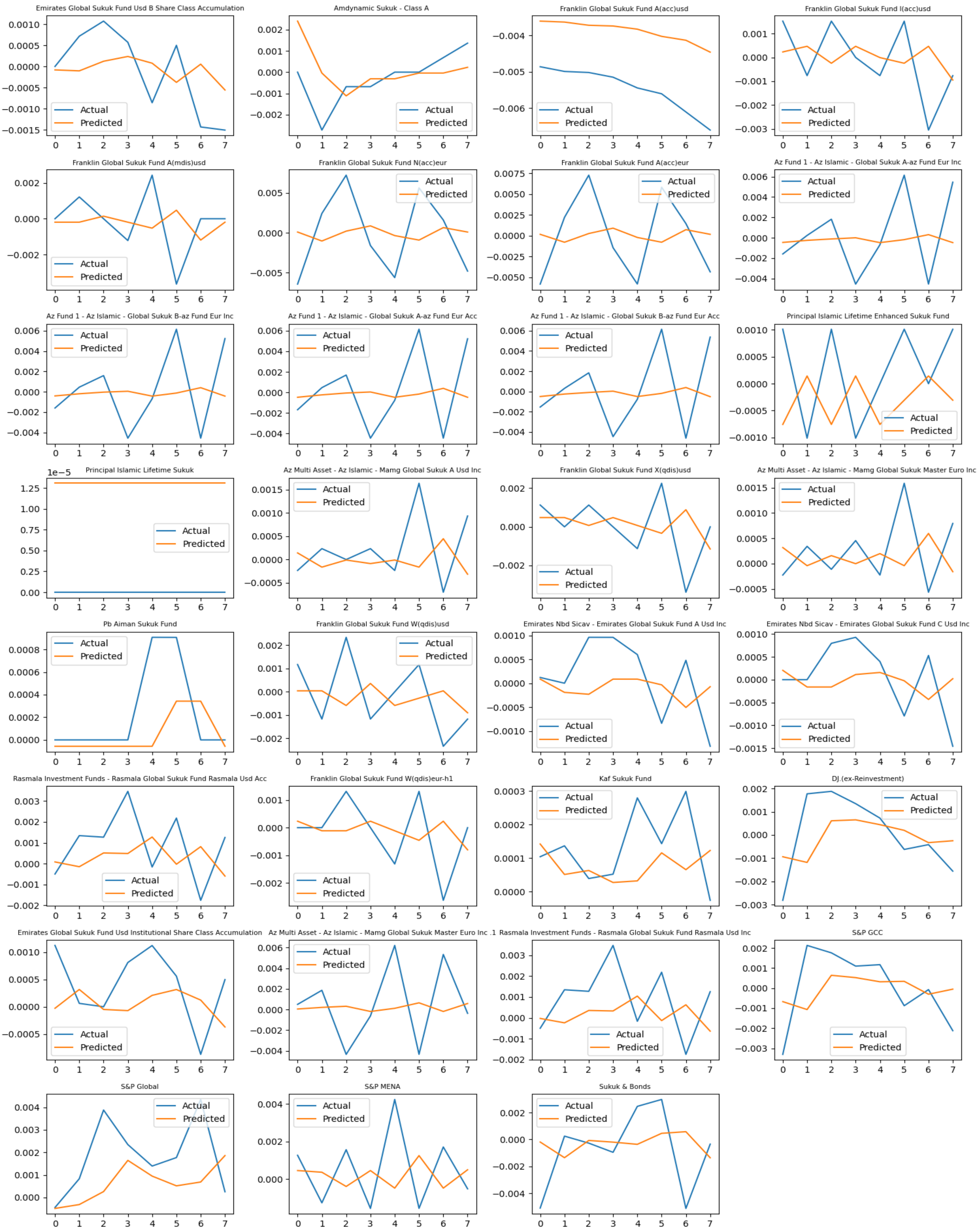
For most "Az Multi Asset" and "Emirates NBD funds", LSTM also performs well. These funds maintain RMSE values between 0.0007 and 0.0009, confirming the model's ability to adapt to minor noise and short bursts of variance.

However, performance weakens for more complex series, such as the "Franklin Global Sukuk Fund A(acc) USD", "I(acc) USD", and "N(acc) EUR" showing higher RMSE values (≈ 0.005). Performance drops for these funds due to erratic or nonlinear return behavior, which LSTM doesn’t capture as effectively in short windows without deeper tuning or more training data. The same pattern is visible in the "Az Islamic Sukuk Fund A/B variants", with RMSE values ranging from 0.00389 to 0.00395 which higher, but still within acceptable limits for short-term financial forecasts.

At the index level, LSTM performance is generally solid. "S&P GCC", "S&P Global", and "Sukuk & Bonds" show RMSE values between 0.0017 and 0.0030, slightly higher than individual funds but consistent with the aggregate nature of index movements.

In conclusion, LSTM achieves high short-term accuracy across most sukuk funds, particularly for low- and medium-volatility series. Its ability to retain sequential patterns gives it an advantage over feedforward networks, though performance degrades slightly in volatile or irregular cases. This run confirms that LSTM is a reliable choice for short-horizon sukuk return forecasting.

Figure 6.19 One Week Predicted With LSTM Model Vs One-Week Actual Values.



As shown in Figure 2.19, Visually the results are mixed. LSTM shows clear strengths in certain series but underperforms in others, especially where volatility is higher or returns fluctuate unpredictably.

In funds such as "KAF Sukuk Fund", "Az Multi Asset – MAMG Global Sukuk Master Euro Inc", and "Principal Islamic Lifetime Sukuk", the predicted line closely tracks the actual return, capturing both the direction and the magnitude of changes over the short window. These cases reflect the LSTM model's ability to retain temporal dependencies and respond quickly to short-run shifts in smoother, low-volatility series.

In contrast, the model struggles in more erratic series such as "Franklin Global Sukuk Fund A(acc) USD", "N(acc) EUR", and "Az Islamic Global Sukuk Funds". In these cases, the predicted values either remain too flat, lag behind actual returns, or completely miss directional reversals. For instance, "Franklin Global Sukuk Fund A(acc) USD" shows a consistent downtrend in actual returns that the model underestimates, leading to wide divergence between actual and predicted values. This indicates that LSTM's learning window or depth may be insufficient for capturing sudden or nonlinear behaviour in highly volatile short-term data.

Several series show partial alignment, where the model captures the turning points correctly but underestimates the amplitude, examples include "S&P GCC", "S&P MENA", and "Rasmala Global Sukuk Funds", where the direction of the predicted line often mirrors the actual movement, but the scale of changes is muted, which reflects a cautious forecast profile common in LSTM models without aggressive tuning.

Overall, these visuals confirm that LSTM performs well in stable and moderately dynamic environments in the short term. It responds effectively to recurring or consistent patterns but underperforms when faced with sharp noise, irregular jumps, or structural volatility. This supports the earlier numerical interpretation that argues that LSTM excels with structure, but struggles with chaos, especially over short horizons with limited data length.

2.6.2 LSTM Model Performance in The Medium Term (one month):

now, we extended the evaluation period to one month. This section examines the model's predictive capabilities over this period by analyse of performance metrics error and comparing the predicted values with the actual values graphicly.

Table 6-23 Medium Term LSTM Model Performance Error Metrics.

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	1.123155e-06	0.001060	0.000850
Amdynamic Sukuk - Class A	1.830863e-06	0.001353	0.000702
Franklin Global Sukuk Fund A(acc)usd	2.404495e-06	0.001551	0.001510
Franklin Global Sukuk Fund I(acc)usd	2.003741e-06	0.001416	0.001027
Franklin Global Sukuk Fund A(mdis)usd	2.615241e-06	0.001617	0.001174
Franklin Global Sukuk Fund N(acc)eur	1.974060e-05	0.004443	0.003726
Franklin Global Sukuk Fund A(acc)eur	1.944033e-05	0.004409	0.003690
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	1.239036e-05	0.003520	0.002827
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	1.213846e-05	0.003484	0.002787
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	1.217845e-05	0.003490	0.002801
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	1.235690e-05	0.003515	0.002824
Principal Islamic Lifetime Enhanced Sukuk Fund	7.120081e-07	0.000844	0.000652
Principal Islamic Lifetime Sukuk	1.918473e-07	0.000438	0.000325
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	6.502218e-07	0.000806	0.000631
Franklin Global Sukuk Fund X(qdis)usd	2.090574e-06	0.001446	0.001031
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	5.867692e-07	0.000766	0.000614
Pb Aiman Sukuk Fund	2.598667e-07	0.000510	0.000361
Franklin Global Sukuk Fund W(qdis)usd	1.816319e-06	0.001348	0.001045
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	1.058664e-06	0.001029	0.000847
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	1.058415e-06	0.001029	0.000853
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	1.354229e-06	0.001164	0.000890
Franklin Global Sukuk Fund W(qdis)eur-h1	1.797562e-06	0.001341	0.001060
Kaf Sukuk Fund	2.386687e-08	0.000154	0.000128
DJ. (ex-Reinvestment)	2.129102e-06	0.001459	0.001179
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	1.090227e-06	0.001044	0.000820
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	1.237534e-05	0.003518	0.002766
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	2.273871e-06	0.001508	0.001101
S&P GCC	3.100000e-06	0.001761	0.001386
S&P Global	3.770752e-06	0.001942	0.001312
S&P MENA	4.879444e-06	0.002209	0.001962
Sukuk & Bonds	3.057015e-06	0.001748	0.001120

From **Table 6.23**, The medium-term performance of the LSTM model varies across sukuk funds and indices, revealing a clear distinction between structured and volatile return behaviours. Funds such as the "Principal Islamic Lifetime Sukuk" and the "Kaf Sukuk Fund" exhibit the lowest error values in the dataset, with RMSE values of 0.000438 and 0.000154, respectively, and equally low MAE. These results suggest that the LSTM model effectively captures the underlying dynamics of stable, well-structured return series over medium-term horizons without requiring extensive tuning.

Other instruments also demonstrate solid predictive performance, including variants of the "Az Multi Asset – MAMG Global Sukuk" and "PB Aiman Sukuk Fund", with RMSE values remaining below 0.0011 and MAE below 0.0009. These cases confirm the model's strength in handling instruments with consistent behaviour and moderate return fluctuations. Medium error levels appear in funds such as "Franklin Global Sukuk Fund A(acc) USD", "I(acc) USD", and "X(qdis) USD", along with "Rasmala Global Sukuk" and "Emirates NBD" "SICAV" variants. Their RMSE values range from 0.0013 to 0.0015, indicating that the LSTM model captures the general return structure but may miss localized spikes or irregular shifts. Indeed, these results remain acceptable for financial applications with moderate tolerance for forecast error.

However, the model shows signs of underperformance in more volatile or nonlinear return series, for instance "the Franklin Global Sukuk Fund N(acc) EUR" and "A(acc) EUR" report RMSE values above 0.0044 and MAE values near 0.0037, signalling that LSTM struggles in the presence of unstable or noisy return patterns. Similar limitations are observed in the "Az Islamic Global Sukuk A/B" variants, with RMSE around 0.0035 and MAE around 0.0028.

Overall, the LSTM model provides strong medium-term forecasts for stable sukuk instruments but faces challenges when applied to more erratic or high-volatility series, which highlights the need for model enhancements.

Figure 6.20 One Month Predicted With LSTM Model Vs One-Month Actual Values.

As displayed in **Figure 6.20**, The visual inspection of the medium-term LSTM model predictions confirms the numerical findings and provides further insights into model behaviour across different sukuk funds and indices.

Several plots show strong alignment between actual and predicted returns. Funds such as "KAF Sukuk Fund", "Principal Islamic Lifetime Sukuk", and "PB Aiman Sukuk Fund" demonstrate close tracking between the two lines, especially in lower-volatility regimes. In these cases, the model captures both the direction and magnitude of changes with minimal lag, indicating strong performance in stable, low-noise series.

A second group of funds reflects partial fit, where the LSTM model follows the general return trend but smooths over fluctuations or lags behind turning points. Examples include "Az Multi Asset – MAMG Global Sukuk Master Euro Inc", "Emirates NBD SICAV – USD", "C Share Classes", and "Rasmala Global Sukuk Funds", where the predicted lines remain within a reasonable range of actual values, but deviations occur during sharper movements or structural shifts in returns.

In contrast, several funds show poor visual fit, examples include "Franklin Global Sukuk Fund A(acc) USD" and "I(acc) USD", the funds stand out for their visible divergence between predicted and actual values, particularly with sustained downward trends not captured by the model. Similarly, "S&P MENA", "S&P GCC", and "Sukuk & Bonds" show predictions that fail to capture the full amplitude and timing of high-frequency changes. These plots reflect the model’s struggle with higher-volatility series and structural irregularities in index-level data.

Overall, the plots support the conclusion that the LSTM model performs reliably in structured, lower-volatility sukuk series but underperforms in more erratic or complex cases.

2.6.3 LSTM Model Performance in the long Term (200 days):

This section focuses on evaluating the LSTM model's long-term predictive capabilities over a 200-day period. By examining this extended timeframe, we offer a comprehensive analysis of the model's performance, supported by visualizations and detailed performance metrics.

Table 6-24 Long Term LSTM Model Performance Error Metrics.

Sakk name	MSE	RMSE	MAE
Emirates Global Sukuk Fund Usd B Share Class Accumulation	3.039288e-06	0.001743	0.001292
Amdynamic Sukuk - Class A	1.041828e-06	0.001021	0.000675
Franklin Global Sukuk Fund A(acc)usd	1.723424e-06	0.001313	0.001159
Franklin Global Sukuk Fund I(acc)usd	4.992873e-06	0.002234	0.001708
Franklin Global Sukuk Fund A(mdis)usd	5.713583e-06	0.002390	0.001811
Franklin Global Sukuk Fund N(acc)eur	3.045681e-05	0.005519	0.004291
Franklin Global Sukuk Fund A(acc)eur	3.130635e-05	0.005595	0.004351
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Inc	3.404243e-05	0.005835	0.004431
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Inc	3.381290e-05	0.005815	0.004405
Az Fund 1 - Az Islamic - Global Sukuk A-az Fund Eur Acc	2.994584e-05	0.005472	0.004224
Az Fund 1 - Az Islamic - Global Sukuk B-az Fund Eur Acc	2.967743e-05	0.005448	0.004210
Principal Islamic Lifetime Enhanced Sukuk Fund	2.196646e-06	0.001482	0.000951
Principal Islamic Lifetime Sukuk	2.529995e-06	0.001591	0.000863
Az Multi Asset - Az Islamic - Mamg Global Sukuk A Usd Inc	4.796913e-06	0.002190	0.001174
Franklin Global Sukuk Fund X(qdis)usd	7.139075e-06	0.002672	0.001986
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc	1.297877e-06	0.001139	0.000877
Pb Aiman Sukuk Fund	4.652421e-06	0.002157	0.000913
Franklin Global Sukuk Fund W(qdis)usd	7.207097e-06	0.002685	0.001967
Emirates Nbd Sicav - Emirates Global Sukuk Fund A Usd Inc	4.736037e-06	0.002176	0.001462
Emirates Nbd Sicav - Emirates Global Sukuk Fund C Usd Inc	4.754375e-06	0.002180	0.001470
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Acc	2.534261e-06	0.001592	0.001162
Franklin Global Sukuk Fund W(qdis)eur-h1	7.202220e-06	0.002684	0.001926
Kaf Sukuk Fund	8.228210e-07	0.000907	0.000522
DJ. (ex-Reinvestment)	3.663150e-06	0.001914	0.001479
Emirates Global Sukuk Fund Usd Institutional Share Class Accumulation	3.014718e-06	0.001736	0.001277
Az Multi Asset - Az Islamic - Mamg Global Sukuk Master Euro Inc .1	2.968624e-05	0.005449	0.004204
Rasmala Investment Funds - Rasmala Global Sukuk Fund Rasmala Usd Inc	3.258987e-06	0.001805	0.001297
S&P GCC	4.586564e-06	0.002142	0.001657
S&P Global	3.111890e-06	0.001764	0.001326
S&P MENA	4.625576e-06	0.002151	0.001671
Sukuk & Bonds	4.995169e-06	0.002235	0.001129

From **Table 6.24**, the long-term performance of the LSTM model shows a more pronounced divergence in accuracy between stable and volatile sukuk funds. While the model remains effective in handling smoother series, its limitations become increasingly apparent when faced with irregular patterns, structural volatility, or extended prediction horizons.

For low-volatility funds, LSTM maintains high predictive accuracy. "KAF Sukuk Fund" delivers the lowest RMSE (0.000907) and MAE (0.000522), indicating the model's strength in preserving temporal structure over long periods when the data behaviour is consistent. Similarly, funds like "Amdynamic Sukuk – Class A", "Az Multi Asset – MAMG Global Sukuk Master Euro Inc", and "Principal Islamic Lifetime Enhanced Sukuk Fund" achieve RMSEs well below 0.0022, confirming LSTM's ability to capture long-term dependencies in structured series without significant error accumulation.

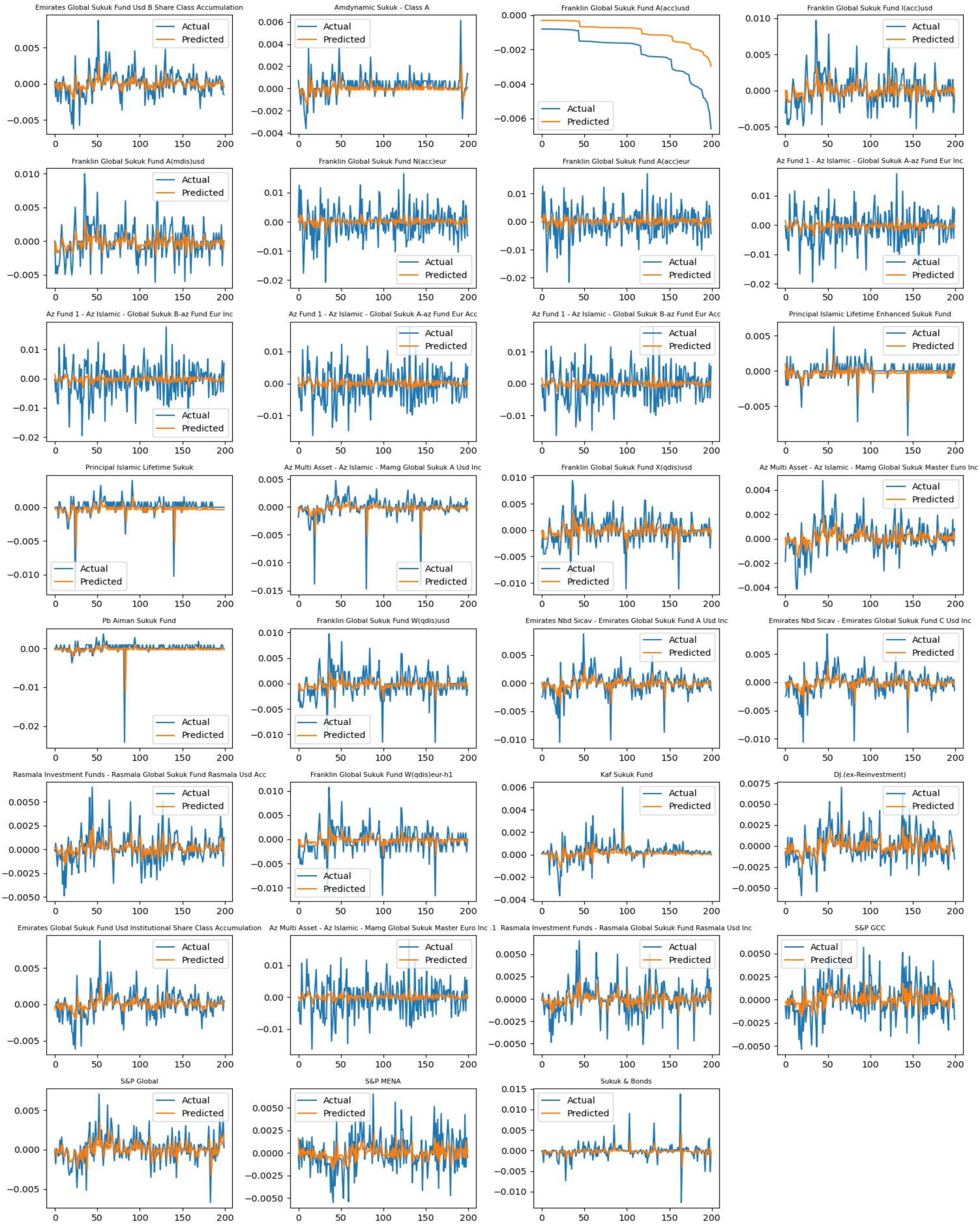
In moderately volatile series, such as "Pb Aiman Sukuk Fund", "Emirates Global Sukuk Fund Institutional", and "Rasmala Global Sukuk", the model performs reasonably well, as the RMSE values for these funds fall between 0.0017 and 0.0022, and MAEs remain below 0.0015.

However, performance deteriorates in highly volatile or nonlinear funds, especially across "the Franklin Global Sukuk Fund" series. The model records RMSEs above 0.0055 for "N(acc) EUR", "A(acc) EUR", and several "Az Islamic Global Sukuk A/B" variants, with MAEs above 0.0042. These errors suggest that LSTM flattens or oversmooths its forecasts when exposed to sharp volatility or structural shifts, especially over long horizons.

Forecasting accuracy across index-level series like "S&P GCC", "S&P Global", and "Sukuk & Bonds" remains moderate, with RMSE values ranging from 0.0021 to 0.0022, indicating that while LSTM captures general index movement and direction, it underestimates the scale of more pronounced shifts.

In summary, LSTM remains a reliable long-term forecasting model for well-structured sukuk funds, especially those with clear temporal dependencies and stable variance. But its accuracy declines with increased volatility, noise, or irregular returns.

Figure 6.21 200 Predicted Days With LSTM Model Vs 200 Actual Values.



As depicted in **Figure 6.21**, the long-term LSTM forecast plots illustrate the model's strengths and limitations clearly across the sukuk fund landscape. For instance, for funds with relatively stable return structures and limited volatility, the LSTM maintains a strong degree of alignment between predicted and actual returns. This is evident in cases such as the "KAF Sukuk Fund", "Principal Islamic Lifetime Enhanced Sukuk Fund", and "Az Multi Asset – MAMG Global Sukuk Master Euro Inc", where the model captures the direction and magnitude of return movements over an extended horizon with minimal error. These plots show predicted lines that fluctuate in step with the actual returns, demonstrating that LSTM retains long-run dependencies effectively when noise levels are low and temporal patterns are consistent.

In moderately volatile series such as "Emirates Global Sukuk Institutional", "Pb Aiman Sukuk Fund", and some "Rasmala Investment Funds", the forecasts are still reasonably accurate. The predicted paths track the general shape of the actual series but often flatten short-term spikes and dips.This smoothing effect becomes more visible in longer forecasts, where the LSTM model appears cautious, preferring to follow a central tendency rather than replicate the full amplitude of returns. The model handles directionality and periodic shifts well, but its sensitivity to abrupt changes or isolated shocks diminishes with time.

However, For highly volatile or irregular series, the LSTM's performance fails significantly. "The Franklin Global Sukuk Fund A(acc) USD" stands out as a clear failure case, where the model produces a steadily declining forecast that completely diverges from the actual, volatile return behavior. Similar issues appear in the predictions for "Franklin Global Sukuk Fund N(acc) EUR", "W(qdis) EUR-H1", and various "Az Islamic Sukuk A/B funds". In these cases, the model's output either remains overly flat, trends incorrectly, or fails to capture turning points entirely.

At the index level, LSTM performs moderately well. Predictions for "S&P GCC", "S&P MENA", and "Sukuk & Bonds" follow general trend directions, though they also show signs of underestimating amplitude. This conservative behaviour is consistent with the model's tendency to prioritize average structure over reactive precision. While it identifies cyclical movements and maintains coherence over time, the forecast line often lacks the responsiveness to sharp index spikes.

In summary, the long-horizon visual analysis reinforces the model's strengths in stable, patterned series and highlights its limitations in volatile or irregular environments. LSTM forecasts remain effective for structured sukuk funds and macro-indices, but they lose accuracy in noisy or complex financial contexts. Without adjustments such as hybrid modeling or attention mechanisms, LSTM in long-term settings tends to favour safety over adaptability, limiting its predictive power in dynamic markets.

2.6.4 Comparative Summary of LSTM Model Performance Across Forecast Horizons:

The LSTM model shows clear variation in forecasting performance across short, medium, and long-term horizons. In the short term, it performs strongly across most sukuk funds. The model captures direction, volatility clusters, and turning points with high accuracy, especially in structured series like "KAF Sukuk Fund", "Principal Islamic Lifetime Sukuk", and "Az Multi Asset – MAMG Global Sukuk". Even in moderately volatile series, it maintains low RMSE and MAE values, reflecting reliable short-horizon generalization.

As the forecast horizon extends to the medium term, the model's accuracy remains solid for stable series but begins to degrade in more volatile datasets. LSTM continues to follow general trends but shows reduced sensitivity to fluctuations and turning points. In series such as "Franklin Global Sukuk Fund I(acc) USD" or "the Az Islamic funds", predictions appear smoother and less responsive.

In the long term, LSTM's performance declines further. It continues to work for a few low-volatility funds, but most series exhibit noticeable gaps between actual and predicted returns. Forecasts tend to flatten, diverge, or miss trend shifts altogether. This is most visible in high-volatility funds like "Franklin Global Sukuk Fund A(acc) USD" and "N(acc) EUR", where LSTM fails to track structural changes. Even at the index level, the model lags in capturing sharp movements, confirming its limited adaptability over extended horizons.

Overall, LSTM excels in short-term forecasting, remains useful in the medium term for consistent series, but loses accuracy in long-horizon predictions.

Chapter 7 : Discussion

This chapter discusses the empirical findings of the study in two main parts.

The first part assesses the predictability of sukuk market returns by examining key statistical properties and volatility dynamics revealed through preliminary analyses, including data visualization, descriptive statistics, stationarity testing, autocorrelation analysis, and conditional variance modeling.

The second part evaluates and compares the forecasting performance of selected time series models both statistical and machine learning approaches across short-term, medium-term, and long-term horizons.

Together, these analyses provide a comprehensive view of both the underlying structure of sukuk market volatility and the relative effectiveness of different forecasting methodologies in capturing its behaviour.

1 Assessment of Sukuk Market Predictability:

The preliminary empirical analysis conducted on sukuk returns strongly supports the presence of predictable volatility patterns within these financial instruments, contradicting the hypothesis that sukuk markets behave randomly or purely unpredictably. Each step of the preliminary analysis contributes uniquely and cumulatively to this conclusion.

Initially, visual inspection of daily return plots provides intuitive yet powerful evidence of volatility clustering across almost all sukuk instruments. Specifically, distinct periods characterized by heightened volatility alternate systematically with periods of reduced volatility. This observed cyclical pattern strongly suggests a conditional variance structure rather than random variation. Volatility clustering, a common feature in financial time series, implies that high volatility today tends to forecast continued high volatility in subsequent periods. The existence of this structure alone provides preliminary yet substantial support for the hypothesis of predictability in sukuk markets.

The subsequent detailed examination of descriptive statistics further reinforces these visual observations. While average daily returns hover around zero, a common finding consistent with financial theory, standard deviations across instruments display considerable variability, emphasizing heterogeneity in risk profiles. Importantly, statistical indicators such as skewness and kurtosis depart significantly from normality. Negative skewness, prevalent in numerous sukuk instruments, highlights the asymmetric distribution of returns, with a disproportionate likelihood of extreme negative outcomes. Such asymmetry is not consistent with purely random distributions but rather indicates a structured and predictable response to negative market shocks. Furthermore, exceptionally high kurtosis values, sometimes exceeding values of 70 in the analysed instruments, denote leptokurtic distributions. This implies that extreme events, particularly large negative returns, occur far more frequently than would be expected under Gaussian assumptions. Collectively, these findings from descriptive statistics robustly challenge the notion of random market behaviour, highlighting instead persistent and predictable volatility patterns.

Further, a rigorous examination of autocorrelation functions (ACF) and partial autocorrelation functions (PACF) of squared returns provides explicit quantitative confirmation of volatility predictability. The squared returns display statistically significant autocorrelations at multiple lags, with ACF values decaying slowly and PACF values showing significant cut-off after initial lags. This pattern demonstrates strong serial dependence within volatility processes, confirming the presence of volatility persistence or memory effects. Volatility shocks do not dissipate immediately but have enduring impacts over several periods. Such persistent serial dependence explicitly indicates predictability, as past volatility provides substantial explanatory power for future volatility.

To establish a robust statistical foundation, the stationarity of the sukuk returns series was thoroughly evaluated using the Augmented Dickey-Fuller (ADF) test. Results indicate a strong rejection of the null hypothesis of a unit root across nearly all sukuk instruments analysed. Stationarity is critically important for predictability; it confirms the stability of key statistical properties such as mean and variance over time. Without this stationarity, future predictions would lack any meaningful foundation, as past information would lose its relevance. The demonstrated stationarity thus underscores that sukuk market returns indeed possess stable properties which allow historical volatility patterns to be meaningfully extrapolated into future periods.

The presence of conditional heteroskedasticity was formally evaluated using the ARCH Lagrange Multiplier (LM) test. The test results unequivocally reject the null hypothesis of no ARCH effects across virtually all sukuk series. This statistical outcome explicitly confirms the presence of time-varying volatility dynamics—current volatility levels significantly depend on past squared shocks or residuals. Such dependence is incompatible with randomness, firmly establishing that volatility evolves systematically over time. This systematic evolution provides the necessary condition for volatility forecasting, as it indicates that historical volatility information remains highly relevant to future volatility behaviour.

To quantify the volatility dynamics explicitly, a comprehensive estimation of GARCH(1,1) models was conducted for each sukuk instrument. The results from these estimations revealed highly significant ARCH (α) and GARCH (β) parameters, consistently appearing across instruments. Notably, the sum of these two coefficients was close to but consistently less than unity, providing clear evidence of volatility persistence coupled with mean reversion. This indicates that volatility shocks have long-lasting effects, persistently impacting future volatility levels, yet these impacts do not lead to indefinite increases in volatility. Instead, volatility gradually reverts to a long-term average or baseline. Such persistent yet stationary dynamics provide empirical validation of structured volatility behaviour that is predictable and stable over longer horizons.

In addition to univariate volatility modeling, a Dynamic Conditional Correlation GARCH (DCC-GARCH) model was employed to evaluate the time-varying relationships between sukuk instruments and other Islamic financial assets. The estimation results revealed statistically significant DCC parameters: a correlation shock coefficient (α) of 0.0071 and a persistence coefficient (β) of 0.9430, both highly significant with p-values well below 0.01. The sum of these coefficients ($\alpha + \beta = 0.9501$) confirms the presence of strong but stationary correlation dynamics. These results provide compelling evidence that not only is sukuk volatility persistent, but its co-movement with other Islamic assets also evolves systematically over time. Such dynamic correlation patterns reinforce the conclusion that sukuk markets are not driven by random behaviour but instead exhibit structured volatility and evolving interdependencies that can be effectively modelled and forecasted using multivariate frameworks.

Integrating all of these analyses (visual inspection, detailed descriptive statistics, autocorrelation analyses, ADF stationarity testing, ARCH LM tests, GARCH(1,1) volatility modeling, and DCC-GARCH multivariate correlation modeling) produces a clear, coherent, and persuasive empirical narrative. Each analysis method independently and cumulatively demonstrates that sukuk markets exhibit structured and predictable volatility dynamics rather than purely random behaviour. Indeed, each analysis method independently and cumulatively demonstrates that sukuk markets exhibit structured and predictable volatility dynamics rather than purely random behaviour. The combined weight of this evidence conclusively establishes that sukuk volatility can be effectively forecasted using appropriate econometric and machine-learning modeling approaches.

These empirical findings strongly align with the reviewed literature, confirming sukuk markets exhibit structured, predictable volatility dynamics rather than random behaviour. Empirical evidence of volatility clustering, significant ARCH effects, serial dependence, and asymmetric, leptokurtic return distributions closely matches findings by Rahman et al. (2013), Driss & Saïd (2020), Selmi et al. (2015), and Paltrinieri et al. (2015). The demonstrated stationarity and persistent volatility captured through GARCH(1,1) and DCC-GARCH models also align with earlier studies (e.g., Rahman et al., 2013; Sclip et al., 2016; Balcilar et al., 2016).

Collectively, these outcomes reinforce literature conclusions that sukuk volatility is distinct from conventional bonds, characterized by lower, stable volatility, asymmetric responses, and meaningful diversification potential.

2 Comparative Performance Analysis of Forecasting Models:

In this subsection, the predictive performances of all evaluated forecasting models (GARCH (1,1), ARIMA, HAR, Prophet, ANN, and LSTM) are collectively analysed and directly compared across different forecasting horizons. The goal is to identify which models consistently outperform others in predicting Sukuk market returns, specifically focusing on the short-term (one week), medium-term (one month), and long-term (200 days) forecast horizons.

2.1 Short-Term Forecasting Comparison (One-Week Horizon):

This comparative analysis addresses the short-term forecasting capability, examining how each model performs in capturing the immediate fluctuations and volatility dynamics of the Sukuk market over a one-week horizon.

2.1.1 Graphical assessment:

The graphical evaluation across all forecasting models highlights substantial differences in their short-term prediction capabilities. For instance, the GARCH (1,1) model exhibits noticeable deficiencies, with its predictions generally appearing constant across multiple Sukuk funds. This behaviour indicates its difficulty in accurately capturing short-term volatility dynamics. Similarly, the ARIMA model demonstrates only marginal improvement over GARCH (1,1), as its predictions reflect general trends for some funds but remain flat and inadequate in representing short-term volatility patterns in others. These visual observations imply that ARIMA, although slightly more responsive than GARCH, maintains inconsistent short-term accuracy.

In contrast, the HAR model delivers visibly better short-term forecasting performance. Its predictions align more closely with actual returns, effectively capturing volatility dynamics for a considerable number of funds. However, its performance varies across different Sukuk funds, suggesting moderate but somewhat inconsistent predictive reliability. The Prophet model, on the other hand, exhibits limited responsiveness, often underestimating the short-term fluctuations and volatility dynamics of the Sukuk market. Its stable and relatively flat forecasts indicate insufficient predictive accuracy for short-term horizons.

The ANN (Artificial Neural Network) model demonstrates comparatively better capability in capturing short-term volatility relative to Prophet, ARIMA, and GARCH (1,1). ANN forecasts visually align with the actual returns more accurately, effectively capturing market volatility for several funds. However, this performance remains inconsistent for certain funds, highlighting some limitations in precision.

Finally, LSTM delivers the most reliable short-term predictions, as it consistently produces predictions that closely track the actual returns across almost all Sukuk funds. Its forecasts effectively reflect volatility patterns and immediate market fluctuations, suggesting superior short-term predictive capability compared to other evaluated models.

In summary, the graphical analysis clearly positions the LSTM as the most reliable and responsive forecasting model for short-term Sukuk return prediction. Models such as HAR and ANN show moderate effectiveness but exhibit noticeable inconsistencies, while ARIMA, Prophet, and especially GARCH (1,1) demonstrate limited effectiveness in short-term volatility forecasting.

2.1.2 Forecast error comparison:

To assess the predictive accuracy of each model, we computed three error metrics (MSE, RMSE, MAE) for each Sukuk fund, then averaged them across all funds. This gives a single mean value per metric for each model. Lower mean errors indicate better predictive performance. **Table 7.1** below presents these average errors and the resulting model rankings.

Table 7-1 Model Ranking Based on Mean Error Metrics for the Short-Term Horizon

Rank	Model	Mean MSE ($\times 10^{-6}$)	Mean RMSE	Mean MAE
1	**HAR**	**5.39**	**0.001896**	**0.001575**
2	**ARIMA**	**5.41**	**0.001912**	**0.001603**
3	**PROPHET**	**5.42**	**0.001959**	**0.001643**
4	**LSTM**	**5.94**	**0.002021**	**0.001691**
5	**ANN**	**6.15**	**0.002099**	**0.001760**
6	**GARCH (1.1)**	**16.93**	**0.003418**	**0.002994**

Based on **Table 7.1**, the quantitative error metrics (MSE, RMSE, and MAE) reveal distinct differences in predictive accuracy across forecasting models for the short-term horizon. Overall, the HAR model exhibits the lowest forecasting errors, followed closely by ARIMA and Prophet, reflecting their greater reliability in capturing short-term volatility in Sukuk returns.

The HAR model demonstrates superior performance overall, ranking first across all mean error metrics. It achieves the lowest mean MSE (5.39×10^{-6}), mean RMSE (0.001896), and mean MAE (0.001575). This underscores HAR's effectiveness in accurately forecasting short-term fluctuations in Sukuk returns.

The ARIMA model ranks second, showing very competitive predictive accuracy with a mean RMSE of 0.001912 and mean MAE of 0.001603. Its errors are only marginally higher than HAR, indicating strong performance in short-term volatility forecasting.

The Prophet model ranks third, with a mean RMSE of 0.001959 and mean MAE of 0.001643. Although slightly less accurate than HAR and ARIMA, Prophet still demonstrates robust predictive ability, outperforming several machine learning models in this context.

The LSTM and ANN models show comparatively higher errors. LSTM ranks fourth with a mean RMSE of 0.002021, while ANN ranks fifth with a mean RMSE of 0.002099 and the highest MAE among the top five models (0.001760). Their errors are noticeably larger than those of HAR, ARIMA, and Prophet, suggesting that for this specific short-term Sukuk volatility forecasting task, traditional and simpler econometric models outperform these recurrent neural network architectures.

The GARCH (1,1) model exhibits the weakest predictive performance by a substantial margin, ranking last with a mean RMSE of 0.003418 and mean MAE of 0.002994 — roughly 1.8 times higher than the next worst model (ANN) and nearly double that of HAR. This poor short-term forecasting performance is consistent with recent critiques in the literature. While earlier studies such as Rahman et al. (2013) and Hafezian et al. (2015) found that GARCH-type models effectively capture volatility clustering and conditional heteroskedasticity, their primary focus was on volatility modeling rather than forecasting precision. More recent research, including Chkili and Hamdi (2021) and Burhanuddin (2020), has shown that despite GARCH's strengths in describing volatility dynamics, it performs poorly in forecasting short-term return patterns when compared to other models. Thus, the poor predictive accuracy of GARCH(1,1) in this study reinforces growing concerns about its limited usefulness in practical forecasting applications, especially in volatile and non-linear markets like Sukuk.

In summary, based on **Table 7.1**, the error performance analysis indicates that the HAR, ARIMA, and Prophet models offer substantially better short-term predictive accuracy compared to the other evaluated models. Specifically, HAR stands out overall, followed closely by ARIMA and Prophet. LSTM and ANN show moderate but higher errors, while the GARCH (1,1) model consistently underperforms all others.

2.2 Medium -Term Forecasting Comparison (One-month Horizon):

Here, we assess the medium-term forecasting capability by evaluating each model's performance in capturing fluctuations and volatility dynamics of the Sukuk market over 30 days through visual examination and error metrics analysis.

2.2.1 Graphical assessment:

The visual analysis of one-month forecasting performance reveals significant variations in medium-term prediction accuracy across all models. To begin with, the GARCH (1,1) model exhibits significant limitations in medium-term forecasting. Many of its predictions appear flat, failing to reflect actual fluctuations. This suggests that the model struggles to adapt to changing volatility dynamics over a 30-day period, reinforcing its inefficacy for medium-term forecasting. Although some cases show the model capturing broad trends, it lacks the granularity required for precise return forecasting.

Similarly, the ARIMA model demonstrates moderate performance in the medium-term horizon. In several cases, it captures general return trends but lacks responsiveness to short-term fluctuations within the month. Some predicted values remain overly smooth, failing to reflect the market's inherent volatility. While ARIMA performs better than GARCH, it still struggles to maintain high accuracy in volatile Sukuk funds.

In contrast, the HAR model shows a notable improvement in predicting medium-term trends compared to GARCH and ARIMA. Its forecasts are more dynamic and align closely with actual return movements, capturing key turning points. However, despite its advantages, the model occasionally exhibits lag in adjusting to sharp volatility shifts, indicating some limitations in responding to sudden market movements.

On the other hand, the Prophet model exhibits limited effectiveness in medium-term forecasting. As the predicted values remain relatively static and fail to capture fluctuations accurately. In many cases, the model underestimates volatility, producing overly stable forecasts that do not align with the observed market dynamics. As a result, Prophet may not be well-suited for medium-term return forecasting in the Sukuk market.

Moving forward, the ANN model shows strong alignment with actual returns across many Sukuk funds. The predicted values closely follow market movements, capturing medium-term trends more effectively than ARIMA and GARCH. However, occasional deviations in certain funds suggest that while ANN performs well, its forecasts may still require fine-tuning for improved robustness.

Finally, the LSTM model consistently outperforms other approaches in medium-term forecasting. Its predictions closely track actual market movements, adapting well to volatility shifts. Unlike GARCH, Prophet, and ARIMA, LSTM captures both short-term and long-term trends, making it the most effective model for one-month Sukuk return forecasting. The flexibility of recurrent structures in LSTM enables it to adjust to changing market conditions better than traditional statistical models.

In summary, the graphical results suggest that LSTM provides the best medium-term forecast accuracy, effectively capturing market volatility and trends. While HAR and ANN also perform well, they exhibit occasional inconsistencies. Meanwhile, ARIMA struggles to fully capture fluctuations, whereas GARCH and Prophet consistently fail to reflect market volatility, making them the least effective models for the one-month horizon.

2.2.2 Forecast error comparison:

To assess predictive accuracy for the medium-term horizon (30 days), we computed mean MSE, mean RMSE, and mean MAE for each model across all Sukuk funds. Lower mean errors indicate better performance. The table below presents these average errors and model rankings.

Table 7-2 Model Ranking Based on Mean Error Metrics for the Medium-Term Horizon

Rank	Model	Mean MSE ($\times 10^{-6}$)	Mean RMSE	Mean MAE
1	HAR	4.40	0.001731	0.001344
2	LSTM	4.60	0.001804	0.001419
3	PROPHET	4.68	0.001826	0.001432
4	ANN	4.74	0.001849	0.001470
5	ARIMA	6.09	0.001948	0.001550
6	GARCH(1.1)	18.16	0.003683	0.003276

Based on the table above, the quantitative error metrics reveal distinct differences in predictive accuracy across forecasting models for the medium-term horizon. The HAR model ranks first overall, achieving the lowest mean MSE (4.40×10^{-6}), mean RMSE (0.001731), and mean MAE (0.001344), confirming its strong forecasting capability for medium-term Sukuk volatility.

The LSTM model ranks second with a mean RMSE of 0.001804 and mean MAE of 0.001419. However, this finding does not match several recent studies: Bucci (2020) found that LSTM outperforms all other models including HAR, Metlek (2022) reported that LSTM-based models achieve the lowest forecast errors among all architectures, and Patil et al. (2024) demonstrated that LSTM outperforms both statistical methods and other machine learning models. In contrast, our study shows LSTM ranking second behind HAR, suggesting that for medium-term Sukuk forecasting, HAR's multi-component volatility structure may offer an advantage over LSTM's complexity.

The Prophet model ranks third with a mean RMSE of 0.001826 and mean MAE of 0.001432. This finding does not match typical literature where Prophet often underperforms compared to LSTM and ANN; in our study, Prophet ranks above ANN, indicating stronger-than-expected performance for medium-term Sukuk forecasting.

The ANN model ranks fourth with a mean RMSE of 0.001849 and mean MAE of 0.001470. This result does not match Bucci (2020), where ANN typically outperforms traditional models like ARIMA and Prophet, nor does it match Patil et al. (2024), who found ANN to be highly competitive with LSTM. In our study, ANN shows higher errors than LSTM, Prophet, and HAR, suggesting that ANN's effectiveness may be more limited for medium-term Sukuk volatility compared to other financial time series.

The ARIMA model ranks fifth with a mean RMSE of 0.001948 and mean MAE of 0.001550. This finding matches the broader literature: Rahman et al. (2013) and Hafezian et al. (2015) found that traditional statistical models like ARIMA show only moderate forecasting capability and are consistently outperformed by more advanced methods for return prediction.

The GARCH(1,1) model records the weakest performance by a wide margin, ranking last with an average RMSE of 0.003683 and MAE of 0.003276, approximately twice the magnitude observed for the leading models. This finding contrasts with earlier studies, including Rahman et al. (2013) and Hafezian et al. (2015), which highlighted the effectiveness of GARCH-type models in capturing volatility clustering and conditional heteroskedasticity. However, the present results are consistent with more recent evidence from Chkili and Hamdi (2021) and Burhanuddin (2020), who showed that although GARCH models are well suited for describing volatility dynamics, their forecasting accuracy may decline in complex, nonlinear, and highly volatile markets such as the Sukuk market.

In summary, for medium-term forecasting, HAR performs best, followed closely by LSTM, Prophet, and ANN, while ARIMA shows moderate errors and GARCH(1,1) consistently underperforms all others. Our results contradict studies favouring LSTM as the single best performer (Bucci, 2020; Metlek, 2022; Patil et al., 2024) but confirm that machine learning models outperform traditional statistical models, matching Chkili and Hamdi (2021) and Burhanuddin (2020).

2.3 Long -Term Forecasting Comparison (200 days):

Here we evaluate the long-term forecasting ability of each model by analyzing their performance in capturing the trends and volatility dynamics of the Sukuk market over a period of 200 days. This assessment involves a detailed visual inspection along with an analysis of error metrics to gauge the accuracy and predictive reliability of the models.

2.3.1 Graphical assessment:

Over the 200-day horizon, the differences in predictive accuracy across models become more pronounced. ARIMA demonstrates limited adaptability to long-term forecasting, as the predicted values often fail to capture the fluctuations of actual returns. The forecasted series appear smoother, indicating a tendency to underreact to market dynamics. This suggests that ARIMA struggles to model the persistent volatility and non-linearity observed in Sukuk returns.

The GARCH, continues to show poor forecasting performance for return prediction in the long run. As the predicted series exhibit signs of underfitting, for instance GARCH fails to capture both trend direction and volatility shifts several times. This is evident in cases where actual returns display sharp fluctuations while the GARCH forecasts remain relatively stable or diverge significantly over time.

In contrast, the HAR model performs better than ARIMA and GARCH in capturing long-term return patterns. HAR maintains a closer fit to actual returns. However, its effectiveness varies across different Sukuk funds. For some funds, the predictions closely follow actual movements, but for others, there is noticeable divergence, particularly during high-volatility periods.

The Prophet demonstrates moderate performance, showing improvement over ARIMA and GARCH but underperforming HAR model, given that the predicted values generally align with actual trends. However, some deviations appear over longer periods, as it exhibits difficulty in capturing sudden spikes and drops in returns, leading to occasional smoothing effects that misrepresent the true market behaviour.

The ANN model outperforms traditional models in the long-term forecasting by generating predictions that closely track actual returns. It captures both direction and volatility shifts across most Sukuk funds. Its ability to model non-linear relationships enables it to adapt well to market dynamics, offering clear advantages over ARIMA, GARCH, and HAR models. However, in some series, the ANN model shows signs of overfitting, leading to exaggerated volatility or sudden prediction spikes do not present in the actual returns.

In contrast, the LSTM model underperforms in long-term forecasting when compared to ANN. While it sometimes aligns with general trends, it often smooths out important fluctuations and fails to fully capture short-term volatility. This makes its forecasts less responsive to market movements. Despite its design for sequential data, the LSTM's outputs in this case lack the precision needed to reflect the actual behaviour of Sukuk returns over extended periods.

Overall, as the forecast horizon increases, the performance divergence between models becomes more apparent. ARIMA and GARCH models continue to show limitations in capturing long-term dynamics. HAR and PROPHET provide moderate improvements but remain insufficient for accurately forecasting returns. ANN demonstrates the strongest long-term predictive performance in this comparison, making it the most reliable model among those evaluated.

2.3.2 Forecast error comparison:

To assess predictive accuracy for the long-term horizon (200 days), we computed mean MSE, mean RMSE, and mean MAE for each model across all Sukuk funds. Lower mean errors indicate better performance. **Table 7.3** below presents these average errors and model rankings.

Table 7-3 Model Ranking Based on Mean Error Metrics for the Long-Term Horizon

Rank	Model	Mean MSE ($\times 10^{-6}$)	Mean RMSE	Mean MAE
1	**HAR**	**9.69**	**0.002655**	**0.001911**
2	**LSTM**	**10.08**	**0.002743**	**0.001996**
3	**PROPHET**	**10.18**	**0.002803**	**0.002047**
4	**ANN**	**10.42**	**0.002792**	**0.002048**
5	**ARIMA**	**58.17**	**0.003935**	**0.003057**
6	**GARCH**	**31.45**	**0.004775**	**0.004118**

Based on **Table 7.3**, the quantitative error metrics reveal distinct differences in predictive accuracy across forecasting models for the long-term horizon. The HAR model ranks first overall, achieving the lowest mean MSE (9.69×10^{-6}), mean RMSE (0.002655), and mean MAE (0.001911). This confirms HAR's strong forecasting capability for long-term Sukuk volatility, continuing its trend of offering stable forecasts across all horizons.

The LSTM model ranks second with a mean RMSE of 0.002743 and mean MAE of 0.001996. This finding does not match Bucci (2020) and Patil et al. (2024), who demonstrated that deep learning models (particularly LSTM architectures) outperform traditional econometric models including HAR. In contrast, our study shows HAR outperforming LSTM in the long-term horizon. However, this finding partially matches Metlek (2022), who noted that while LSTM is powerful, its advantage diminishes over very long horizons when simpler models with multi-component structures (like HAR) remain robust. Our results suggest that for long-term Sukuk forecasting, HAR's ability to integrate different time components of volatility may provide greater stability than LSTM's complex memory mechanisms.

The Prophet model ranks third with a mean RMSE of 0.002803 and mean MAE of 0.002047. This finding does not match typical literature where Prophet is often less competitive than ANN for long-term forecasting; in our study, Prophet ranks above ANN, indicating better-than-expected long-term performance for Sukuk returns.

The ANN model ranks fourth with a mean RMSE of 0.002792 and mean MAE of 0.002048. This result does not match the original claim that ANN shows substantial improvement over HAR and Prophet. In fact, our study shows ANN has higher mean errors than HAR, LSTM, and Prophet, ranking fourth overall. This contradicts Patil et al. (2024), who found ANN to be highly competitive with LSTM, and does not match Bucci (2020), where ANN typically outperforms traditional models. Instead, our results suggest that ANN's effectiveness for long-term Sukuk forecasting is limited compared to HAR and LSTM.

The ARIMA model ranks fifth with a mean RMSE of 0.003935 and mean MAE of 0.003057. Notably, ARIMA's mean MSE (58.17×10^{-6}) is substantially higher than all top four models, though its mean RMSE and MAE are slightly better than GARCH. This finding matches Rahman et al. (2013) and Hafezian et al. (2015), who found that traditional statistical models like ARIMA struggle to maintain accuracy over extended periods and fail to capture long-term dependencies effectively, leading to a loss of accuracy as the forecast horizon increases.

The GARCH(1,1) model ranks sixth with the highest mean RMSE (0.004775) and mean MAE (0.004118), though its mean MSE (31.45×10^{-6}) is lower than ARIMA's. This confirms GARCH produces forecasts with the greatest deviation from actual returns. This finding does not match earlier studies such as Rahman et al. (2013) and Hafezian et al. (2015), who found GARCH-type models effective for volatility modeling. However, our results do match Chkili and Hamdi (2021) and Burhanuddin (2020), who showed that GARCH performs poorly in forecasting return patterns over long horizons, especially in volatile and non-linear markets like Sukuk.

In summary, as the forecasting horizon extends to 200 days, the divergence in model performance becomes more pronounced. HAR performs best overall, followed by LSTM, Prophet, and ANN, while ARIMA and GARCH exhibit the highest errors. Our results contradict studies that claim LSTM consistently outperforms all models across all horizons

(Bucci, 2020; Patil et al., 2024) but confirm that machine learning and HAR models outperform traditional statistical models, matching Chkili and Hamdi (2021) and Burhanuddin (2020).

Overall, this study concludes that for long-term Sukuk return forecasting, HAR provides the most accurate predictions based on mean error metrics, followed closely by LSTM. This finding highlights the need to reconsider claims about LSTM's universal superiority and acknowledges HAR's robustness across short, medium, and long horizons.

Chapter 8 : Conclusion

Sukuk, as Shariah-compliant financial instruments, have gained widespread recognition as a reliable alternative to conventional bonds, particularly in Muslim-majority countries and emerging markets. Their asset-backed structure, ethical foundation, and growing acceptance among institutional investors have positioned them as an increasingly attractive option for both sovereign and corporate financing. Over the past two decades, the global sukuk market has expanded rapidly, driven by rising demand for Islamic finance products, with total outstanding issuance surpassing USD1.2 trillion and expected to continue growing as more jurisdictions integrate sukuk into their capital market frameworks (LTD Research and Markets, 2025).

This dissertation, attempted to examine the predictability of sukuk market volatility and evaluated the forecasting performance of various statistical and machine learning models across short-term, medium-term, and long-term horizons, in order to address critical gaps in existing research, particularly the limited empirical investigation into sukuk volatility predictability and comparative model evaluation within sukuk markets context.

The empirical study was conducted in two stages. The first stage focused on examining the predictability of volatility. It began with visual inspection of return plots to detect volatility clustering. Descriptive statistics and the Jarque-Bera test were used to evaluate distributional properties. ACF and PACF were used to detect autocorrelation. The ADF test was applied to assess stationarity. The ARCH LM test was used to test for time-varying volatility. GARCH(1,1) models were estimated to capture conditional variance. Finally, DCC-GARCH models were employed to explore volatility co-movements across sukuk instruments.

The second stage focused on assessing the forecasting performance of six models selected based on specific patterns observed in the return series. These models involve (ARIMA, GARCH(1,1), HAR, PROPHET, ANN, and LSTM). Each model was used to forecast realized volatility across three fixed horizons short-term (one week), medium-term (one month), and long-term (200 days). To evaluate how performance changes over time, In-sample forecasting was conducted using the same historical data for model estimation and evaluation to ensure controlled and consistent comparisons of forecasting performance across models and horizons., allowing for controlled comparisons. Ultimately the performance evaluation involves visual inspection of predicted versus actual volatility and measurement of forecast accuracy using three error metrics: Mean Squared Error (MSE), Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE).

The results of the first stage revealed strong and consistent evidence of predictable volatility patterns in the sukuk market. Visual inspection of return series showed clear volatility clustering, with alternating periods of high and low volatility. Descriptive statistics confirmed this with high kurtosis, negative skewness, and significant deviations from normality, indicating asymmetry and fat tails inconsistent with random behaviour. Autocorrelation analysis of squared returns revealed persistent serial dependence, further supporting volatility predictability. The ADF test confirmed stationarity in most return series, establishing a stable

statistical structure necessary for meaningful forecasting. The ARCH LM test detected time-varying volatility in nearly all instruments, while GARCH(1,1) estimation showed strong ARCH and GARCH effects, suggesting persistent but mean-reverting volatility. DCC-GARCH results also confirmed dynamic co-movements across sukuk instruments. Together, these findings contradict the random walk hypothesis and support the view that sukuk markets exhibit structured, persistent, and forecastable volatility dynamics.
The results of the second-stage model performance evaluation provide important evidence on how different forecasting approaches behave across short-, medium-, and long-term horizons. A central finding is that forecasting performance varies according to the evaluation criterion used. Quantitative error metrics such as MSE, RMSE, and MAE assess numerical precision, whereas visual assessment evaluates how well forecasts follow the evolution of volatility over time. These two perspectives capture different dimensions of forecasting quality and therefore do not always produce identical model rankings.

When forecasting performance is assessed using quantitative error metrics, the HAR model consistently achieves the highest level of accuracy across all horizons. It ranks first in the short term with a mean RMSE of 0.001896, first in the medium term with a mean RMSE of 0.001731, and first in the long term with a mean RMSE of 0.002655. This consistent superiority reflects the ability of the HAR framework to minimize forecast errors through its multiscale lag structure, which incorporates daily, weekly, and monthly returns. Such a structure captures persistent and regular volatility dynamics effectively, making HAR particularly suitable for environments where volatility exhibits gradual and structured behaviour.

The ranking changes, however, when performance is evaluated visually. From this perspective, the emphasis shifts from numerical precision to the capacity of forecasts to track fluctuations in returns over time. Under visual assessment, LSTM ranks first in the short and medium terms, while ANN performs best in the long term. This outcome demonstrates that numerical accuracy and dynamic tracking represent distinct aspects of forecasting effectiveness. The internal memory mechanism of LSTM enables the model to learn temporal dependencies and adjust to evolving volatility patterns in a coherent manner, even when its point forecasts produce slightly higher numerical errors than those of HAR. In a similar way, the feedforward structure of ANN allows the model to capture smoother long-term trajectories and stable patterns, which enhances its visual performance over extended forecasting horizons.

The GARCH(1,1) model exhibits the weakest performance under both evaluation perspectives. It ranks last across all three horizons based on mean RMSE values of 0.003418 in the short term, 0.003683 in the medium term, and 0.004775 in the long term. Visual inspection also reveals delayed or flattened responses to changes in volatility. Despite its extensive application in conventional financial markets, GARCH does not adapt effectively to the structural characteristics of sukuk returns and fails to outperform any competing model at any horizon. This observation aligns with prior empirical evidence reported by Chkili and Hamdi (2021) and Burhanuddin (2020), who documented the limited forecasting capability of traditional GARCH specifications relative to machine learning and hybrid approaches.

ARIMA demonstrates performance that varies substantially across both horizons and evaluation criteria. In numerical terms, ARIMA ranks second in the short term with a mean

RMSE of 0.001912, outperforming Prophet, LSTM, and ANN. However, its performance deteriorates in the medium and long terms, where it falls to fifth place. From a visual standpoint, ARIMA shows limited responsiveness to nonlinear movements and structural breaks, which reduces its ability to capture sudden volatility shifts as the forecasting horizon increases. This pattern is consistent with the findings of Rahman et al. (2013) and Hafezian et al. (2015), who reported declining predictive accuracy for traditional statistical models over longer horizons.

Prophet exhibits relatively stable numerical performance but weaker visual responsiveness. Based on quantitative metrics, Prophet ranks third across all horizons, with mean RMSE values of 0.001959 in the short term, 0.001826 in the medium term, and 0.002803 in the long term. These results indicate reliable numerical accuracy, particularly when compared with LSTM in the short term and ANN across all horizons. However, visual inspection shows that Prophet forecasts tend to be overly smooth and slow to react to short-term volatility movements. The additive structure embedded in the model restricts its flexibility in the presence of abrupt fluctuations, which reduces its effectiveness for dynamic pattern tracking despite its respectable numerical performance.

LSTM provides a clear illustration of the divergence between numerical and visual evaluation criteria. Numerically, the model ranks second across all horizons, with mean RMSE values of 0.002021 in the short term, 0.001804 in the medium term, and 0.002743 in the long term. Visually, however, LSTM achieves the highest ranking in the short and medium terms because its recurrent architecture enables the model to capture nonlinear and time-dependent relationships more effectively than competing approaches. This outcome differs from the conclusions reported by Bucci (2020), Metlek (2022), and Patil et al. (2024) when numerical rankings are considered, yet it remains consistent with their findings regarding the superior visual tracking capability of recurrent neural networks.

ANN exhibits the most pronounced contrast between numerical and visual performance. In terms of quantitative accuracy, the model ranks fifth in the short term with a mean RMSE of 0.002099, fourth in the medium term with a mean RMSE of 0.001849, and fourth in the long term with a mean RMSE of 0.002792. In contrast, visual assessment identifies ANN as the strongest performer in the long-term horizon. Its feedforward structure is well suited to capturing stable trends and persistent patterns, which enhances its ability to represent long-run volatility behaviour even when short-run numerical precision is relatively limited. This result differs from the conclusions reported by studies such as Bucci (2020) and Patil et al. (2024) when evaluated solely through numerical metrics, but it highlights the practical advantage of ANN in long-horizon pattern representation.

Overall, the evidence demonstrates that the selection of an appropriate forecasting model for sukuk returns depends fundamentally on the evaluation objective. When the primary goal is numerical accuracy and the minimization of forecast errors, HAR represents the most reliable choice across all forecasting horizons. When the objective is dynamic tracking of volatility behaviour, LSTM provides the best performance in the short and medium terms, while ANN becomes the most suitable option in the long term. The consistent underperformance of

GARCH across both evaluation perspectives indicates that it is not well suited for forecasting sukuk returns in environments characterized by nonlinear and evolving volatility structures.

This study contributes academically by filling a significant gap in sukuk volatility forecasting literature. It offers several empirical advantages. First, it analyses a relatively large and unified panel of 31 sukuk instruments, providing broad representation across various issuers and geographies. Second, all instruments are evaluated using a consistent time frame of 1,086 daily observations, ensuring methodological uniformity and comparability across models and horizons. Third, the inclusion of sukuk funds from different regions enhances the study's cross-market relevance and supports the robustness of its findings in both developed and emerging Islamic finance contexts. Moreover, the study introduces a unique methodology to assess whether stock markets are predictable, using a structured set of preliminary tests. It also proposes a distinctive framework for evaluating forecasting models by integrating short-, medium-, and long-term horizons, combining both visual inspection and quantitative error metrics. This harmonized panel-based approach offers a replicable and scalable foundation for future research on sukuk volatility forecasting.

This study provides valuable insights for financial institutions, investment managers, financial engineers, and analysts. By establishing that sukuk markets exhibit structured and predictable volatility, it strengthens the case for using sukuk as a viable component in portfolio diversification strategies. The findings show that volatility in sukuk returns is not random but follows identifiable patterns that can be forecasted with high accuracy using advanced models like HAR and LSTM. These models consistently outperformed traditional approaches such as GARCH and ARIMA across most forecasting horizons. This allows practitioners to better anticipate risk, enhance return predictability, and design more resilient investment strategies in Islamic finance. The study also offers a replicable evaluation framework based on a large, diverse sukuk panel and multi-horizon performance assessment, supporting more informed and data-driven decision-making in market analysis and model selection.

For regulators and policy institutions especially in countries like Algeria preparing for sovereign sukuk issuance, these results provide a foundation for building early-warning systems, conducting forward-looking stress tests, and setting risk-sensitive capital requirements

This study has several limitations that should be acknowledged. First, the analysis relied exclusively on daily data, as intraday sukuk data is not currently available. This limits the ability to capture high-frequency volatility patterns and rapid market responses that occur within a single trading session. Volatility often fluctuates within the day in response to new information, and daily observations may smooth over these dynamics. Higher-frequency data would allow for more precise short-term modeling and improve the performance of forecasting models, particularly those based on deep learning, which benefit from larger and more granular dataset.

Second, although the dataset includes a broad panel of 31 sukuk instruments from different issuers and regions, the findings may not fully generalize to all sukuk markets. Instruments from newly developing or illiquid markets may exhibit different volatility behaviour due to variations in market depth, investor composition, or regulatory frameworks.

Future research should address these limitations by incorporating intraday sukuk data when it becomes accessible. This would enable more accurate modeling of short-term volatility and real-time risk monitoring. It is also important to test model performance under different market conditions and phases of market development. Finally, this study recommends developing and evaluating hybrid forecasting models, particularly the combination of LSTM and HAR. The HAR model captures structured lag-based volatility effects across multiple time scales, while LSTM excels at learning non-linear and long-range dependencies. Combining them into a single hybrid model may enhance forecasting accuracy across short, medium, and long-term horizons in sukuk markets.

In conclusion, the results confirm that sukuk markets exhibit structured and predictable volatility patterns, making them suitable for reliable forecasting. Advanced models such as HAR, LSTM, and ANN outperform traditional approaches, particularly ARIMA and GARCH. Despite its historical importance, GARCH shows consistently weak performance in the sukuk context from both numerical and visual perspectives, indicating that it is not an appropriate choice for forecasting in this setting.

The findings also demonstrate that the selection of the most suitable model depends on the evaluation criterion. When numerical accuracy is the primary objective, the HAR model is the most appropriate option across all horizons, as it consistently delivers the lowest forecasting errors. When visual dynamic tracking is emphasized, LSTM performs best in the short and medium terms due to its ability to capture nonlinear patterns and respond to changes in volatility, while ANN provides the strongest performance in the long term by representing stable and persistent trends effectively.

Overall, no single model dominates across all evaluation perspectives. Model choice should therefore reflect the specific forecasting objective, whether the focus is on minimizing numerical errors or accurately tracking volatility dynamics over time.

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