

## Introduction

Several authors have studied the effect of the uncertainties of the soil parameters and the surcharge loading and or of the vertical and horizontal drains on the surface elastic settlement or on the surface consolidation settlement. Some investigators [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11] have considered the uncertain parameters as random variables. Others have modelled the uncertain parameters by random fields to take into account the soil spatial variability [12], [13], [14], [15], [16]. It should be emphasised here that most of these studies make use of Monte Carlo Simulation (MCS) methodology to compute the Probability Distribution Function (PDF) of the system response or the failure probability ( $P_f$ ). This methodology is well known to be very expensive because of the great number of calls of the deterministic model required for the probabilistic analyses especially when it comes to problems with small failure probabilities  $P_f \leq 0.001$ . Au and Beck [17] proposed an efficient approach (called Subset simulation) to calculate the small failure probabilities in the case where the uncertain parameters are modelled by random

variables. In this approach, the failure probability is expressed as a product of conditional probabilities of some chosen intermediate failure events. Thus, the problem of evaluating a small failure probability in the original probability space is replaced by a sequence of events in the conditional probability space. Except [18], [19] who applied the Subset simulation approach to one-dimensional (1D) random field problems, the Subset simulation method was mainly applied in the literature to problems where the uncertain parameters were modelled by random variables.

In this paper, the Subset simulation method was employed to perform a probabilistic study of the surface settlement in the case of coupled analysis due to a uniform surcharge loading on the ground surface with a two-dimensional (2D) spatially varying Young's modulus. Notice that, due to the layered nature of soils, their parameters generally exhibit a larger autocorrelation length in the horizontal direction compared to that in the vertical direction. Thus, the Young's modulus is considered in this article as an anisotropic random field. The Karhunen-Loeve (KL) expansion is used to discretise the random field. The deterministic model employed to compute

the system response is based on numerical simulations using the commercial software FLAC3D. One should highlight here that the soil spatial variability causes uneven loading displacement. The average value of the vertical displacement is considered herein to represent the system response. The results are equal to the average displacement under a uniform loading. After a brief description of the KL expansion and the Subset simulation method, the implementation of the latter approach in the case of a random field problem is presented. Then, the probabilistic analysis of a uniform loading resting on a spatially varying soil and the corresponding results are presented and discussed. This article ends with a conclusion.

## Section snippets

### Subset simulation

Subset simulation has been proposed by Au and Beck [17] in order to calculate the small failure probabilities  $P_f \leq 0.001$ . In such a case, the different uncertain parameters are modelled by random variables. Several authors [19], [20], [21] have used this method and have shown the tool efficiency for the computation of small failure probabilities with a reduced

simulations number compared to the crude MCS. As a result, the evaluation will require a much smaller number of iterations. More details

## Biot consolidation theory in two-dimensional (2D)

The Biot consolidation theory [25], [26] takes into account the coupling between the water and the soil skeleton (elastic porous material). The coupling in FLAC3D occurs in two directions: pore-pressure changes cause volumetric strains to occur that influence the stresses. In turn, the pore pressure is affected by the straining that takes place. The interstitial water is coupled with the solid skeleton by equilibrium conditions and also by continuity. For the equilibrium in 2D with the absence

## Expansion of Karhunen-Loeve (KL)

Several discretization methods that consist in decomposing the initial field  $\phi$  into optimal complete deterministic functions are proposed in the literature [27], [28], [29]. These methods are based on series expansions. In this paper, the expansion method of Karhunen-Loeve (KL) will be adopted.

The Karhunen-Loève expansion is an efficient approach to carry out the discretization of the random field  $\theta(X)$ , where  $X$  denotes the spatial coordinates and  $\theta$  indicates the random nature of the field.

### Deterministic analysis

The deterministic model employed in this paper is based on numerical simulations using the finite differences code FLAC3D. This code makes use of the Biot theory presented previously. Both uncoupled and coupled consolidation analyses were carried out. The computation of the deterministic surface settlement and the validation of the obtained results by comparison with those existing in literature have been validated by Houmadi et al. [8] and they are presented as follows:

Two cases of a single

### Probabilistic analysis

This section focuses on the computation of the surface settlement in the case of coupled analysis due to a uniform load  $q$  on the ground surface. A single heterogeneous soil layer

of 15 m width and 10 m depth by a unit third dimension has been considered (Fig. 3).

The target is calculating the probability to exceed an admissible vertical displacement. This probability is called the failure probability. The uncertain soil parameter in this study is Young's Modulus ( $E$ ). This parameter was considered

## Conclusions

This paper presents an alternative procedure to Monte Carlo simulation (MCS) for calculating the failure probability of a uniformly distributed load on a heterogeneous soil. This procedure is based on the Subset simulation approach that has been used mainly in the literature for calculating the failure probability for problems where uncertain parameters have been modelled as random variables. The proposed procedure was applied for a soil whose Young's modulus ( $E$ ) is a random field. This field

## CRediT authorship contribution statement

**Houmadi Youcef:** Conceptualization, Methodology, Software, Validation, Formal analysis, Resources, Data curation, Writing - original draft, Writing - review & editing,

Supervision, Project administration. **Cherif Benmoussa**

**Mohammed Yazid:** Conceptualization, Methodology,  
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**Houda:** Software, Formal analysis, Resources, Data curation,  
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Declaration of Competing Interest

None.