This paper presents a numerical investigation of a two-dimensional (2D) oscillatory flow around a cylinder of different elliptic ratios, in order to study the effect of the elliptic form of the cylinder on the vorticity field and the hydrodynamic forces that act on it. The elliptic ratio $\varepsilon\varepsilon$ was varied from 1 to 0.1, where the small axis is parallel to the flow direction, simulating cases ranging from a circular cylinder to the case of a cylinder with a profiled elliptic section. The investigations presented here are for Reynolds number $Re = 100$ and Keulegan number $KC = 5$. The numerical visualization of the flow for different elliptic ratios shows five different modes of vortex shedding (symmetric and asymmetric pairing of attached vortices, single-pair, double-pair, and chaotic), which depend on the range of the elliptic ratio. The results show that the longitudinal force increases with the reduction of the elliptic ratio. The transverse force appears from the elliptic ratio $\varepsilon\varepsilon=0.75$ and increases with the reduction of this ratio in the range of $\varepsilon 0.75 \geq \varepsilon \geq 0.4$, then decreases for $\varepsilon\varepsilon<0.4$. On the other hand, concerning the Morison coefficients the results show that the drag coefficient is sensitive to the swirling layout while the coefficient of inertia does not seem to be much affected by the geometry of the cylinder.